Geometry is all about measuring lines, angles, surfaces, solids, velocities and their interrelationships. In this study, you will act as a consultant, designer-planner, and builder. The projects will range from designing a tree fort in your back yard to planning the construction of a sidewalk and home on the hilly streets of San Francisco to charting the path of the earth around the sun. In the process you will learn the principles as well as the vast usage of geometry in everyday life. Geometry is used by graphic animators, artists, photographers, interior designers, engineers, architects, builders, construction teams, surveyors and doctors just to name a few.

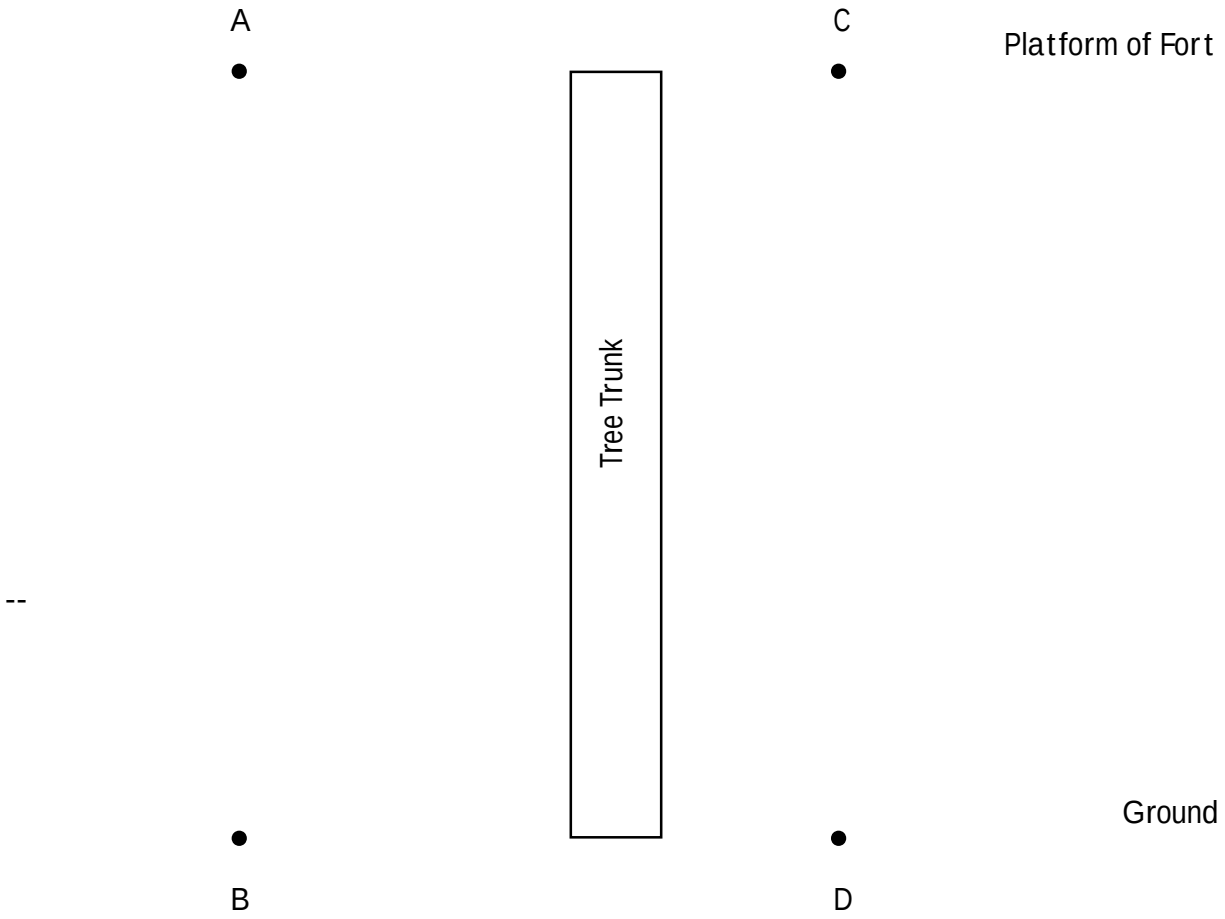
Chapter 1: A Portion of a Line

EXPLORING THE IDEA

Your little brother asks you to help him make a tree fort. He wants the platform of the fort to be 8 feet above the ground. You decide to make a rough sketch of what it would look like.

Draw a straight line going from Point A to Point C. This will represent the platform for the fort.

Next, draw a straight line going through Points B and D. Extend the line beyond these two points. This will represent the ground.



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Darken the section of the line between B and D.

The draw

EXPLAINING THE IDEA

The darkened portion of the line connecting Point B to Point D is called a

LINE SEGMENT

and is represented by drawing a line over the letters representing the endpoints.

A line segment is simply a piece of the line.
It has two ENDPOINTS. In this case Point B and Point D.
The LINE SEGMENT is named by its ENDPOINTS

$\overline{BD}$ is read line segment BD.

How would you symbolize the line segment connecting Point A to Point C? _____ .

EXPANDING THE IDEA Part 1

Next, connect Point A to Point B with a straight line.

Measure the distance between A and B: _____ inches

Now, connect Point C to Point D with a straight line.

Measure the distance between C and D: _____ inches

Line Segment	Measurement
$\overline{AB}$	
$\overline{CD}$	

How do these two distances compare? EQUAL NOT EQUAL (Circle your answer)

Write an equation representing the relationship between $\overline{AB}$ and $\overline{CD}$.

_____

How would you describe the relationship between AC and BD? _____

The distance between these two line segments is equal. No matter how far you extend these two lines they would always be an equal distance apart. When two line segments are equally spaced they are said to be

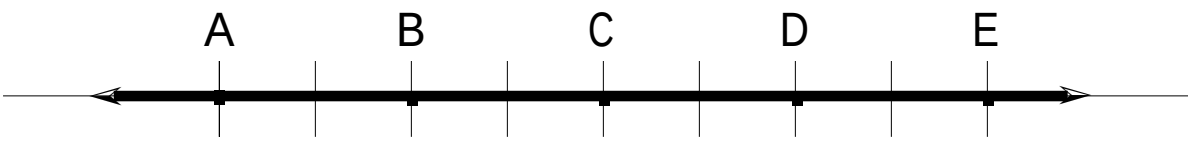
PARALLEL

and is represented by this symbol: ||

In the above problem it would be written AC || BD and is read line segment AC is parallel to line segment BD.

EXPANDING THE IDEA Part 2

Finding the distance of each of the following line segments …

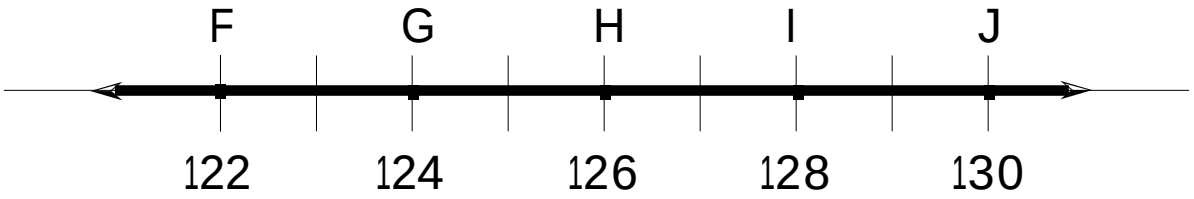


The length of $\overline{AB}$ is 2 units .

The length of $\overline{AC}$ is _____ .

The length of $\overline{BE}$ is _____ .

The length of $\overline{CE}$ is _____ .



The length of $\overline{FH}$ is found by 126 - 122 = ☐

The length of $\overline{GJ}$ is found by _____ .

The length of $\overline{FJ}$ is found by _____ .

What mathematical operation is used to determine the distance of a line segment?

Addition? Subtraction? Multiplication? Division?

The diagram above shows a line segment $\overline{AB}$. The numerical value of endpoint B is represented by the lower case letter b. The numerical value of the endpoint A is represented by the lower case letter a. The equation used to determine the length of line segment $\overline{AB}$ is …

$\overline{AB}$ = _____

Each point on a line has exactly one real number associated with it.
This number is called the

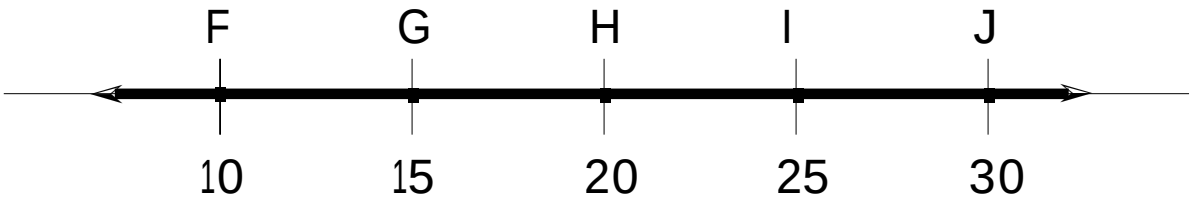
COORDINATE.

The distance between two points is determined by subtracting the smaller coordinate from the larger coordinate. In the above example, the distance between A and B is

$\overline{AB} = b - a$

EXPANDING THE IDEA Part 3

Finding the Distance between two or more Line Segments …



15. The length of FH is equal to FG + GH which is equal to (15 - 10) + (20 - 15) = ☐

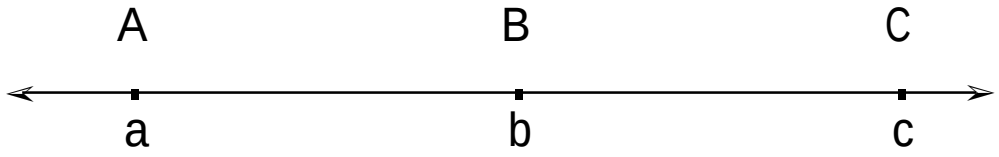
16. The length of GI is equal to _____ + _____ which is equal to _____ .

17. The length of HJ is equal to _____ + _____ which is equal to _____ .

18. The length of FI is equal to _____ + _____ + _____ which is equal to _____ .

19. What mathematical operation is used to determine the distance between two or more line segment?

Addition? Subtraction? Multiplication? Division?

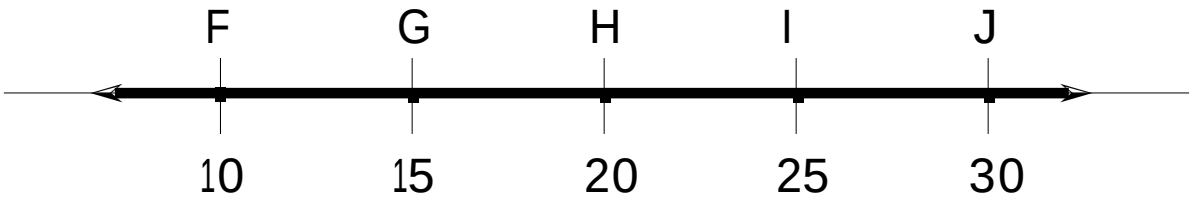


20. The diagram above shows three line segments: AB, BC, and AC. Write the general equation expressing the length of line segment AC:

AC = _____ + _____

EXPANDING THE IDEA Part 4

Finding the Distance Half Way Between Endpoints…

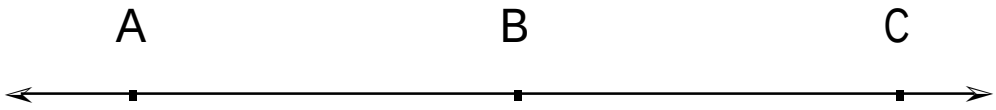


21. What is the point halfway between the line segment $\overline{FJ}$? _____ .

The distance from F to H is equal to the distance from H to J.

22. What is the point halfway between the line segment $\overline{GI}$ _____ .

23. Explain how the point halfway between the endpoints can be found.



24. B is halfway between $\overline{AC}$. That means that AB = BC.
Write a math equation showing the relationship between AB and AC :

AB = _____

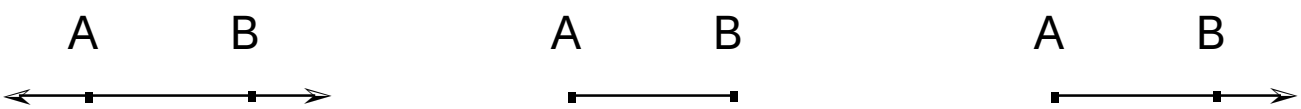
The point halfway between the two endpoints of a line segment is called the

MIDPOINT.

The midpoint is equal to one-half the distance of the line segment.
The midpoint can be determined by using either one of two methods:
METHOD 1:
Find one - half of the line segment and move that many spaces from either endpoint.
METHOD 2:
Find the average of the values for the two endpoints and divide by 2.

Expanding the Idea Part 5

Look closely at the three diagrams below …



25. Describe how the three diagrams are different …

26. Give the definition of a LINE:

27. Give the definition of a LINE SEGMENT:

28. How is the diagram on the far right different from a LINE and from a LINE SEGMENT?

A portion of a line that has only one endpoint and continues forever in one direction is called a

RAY.

A ray is named by the letter of its endpoint and any other point along the ray.

The symbol $\rightarrow$ written above the two letters is used to represent a ray.

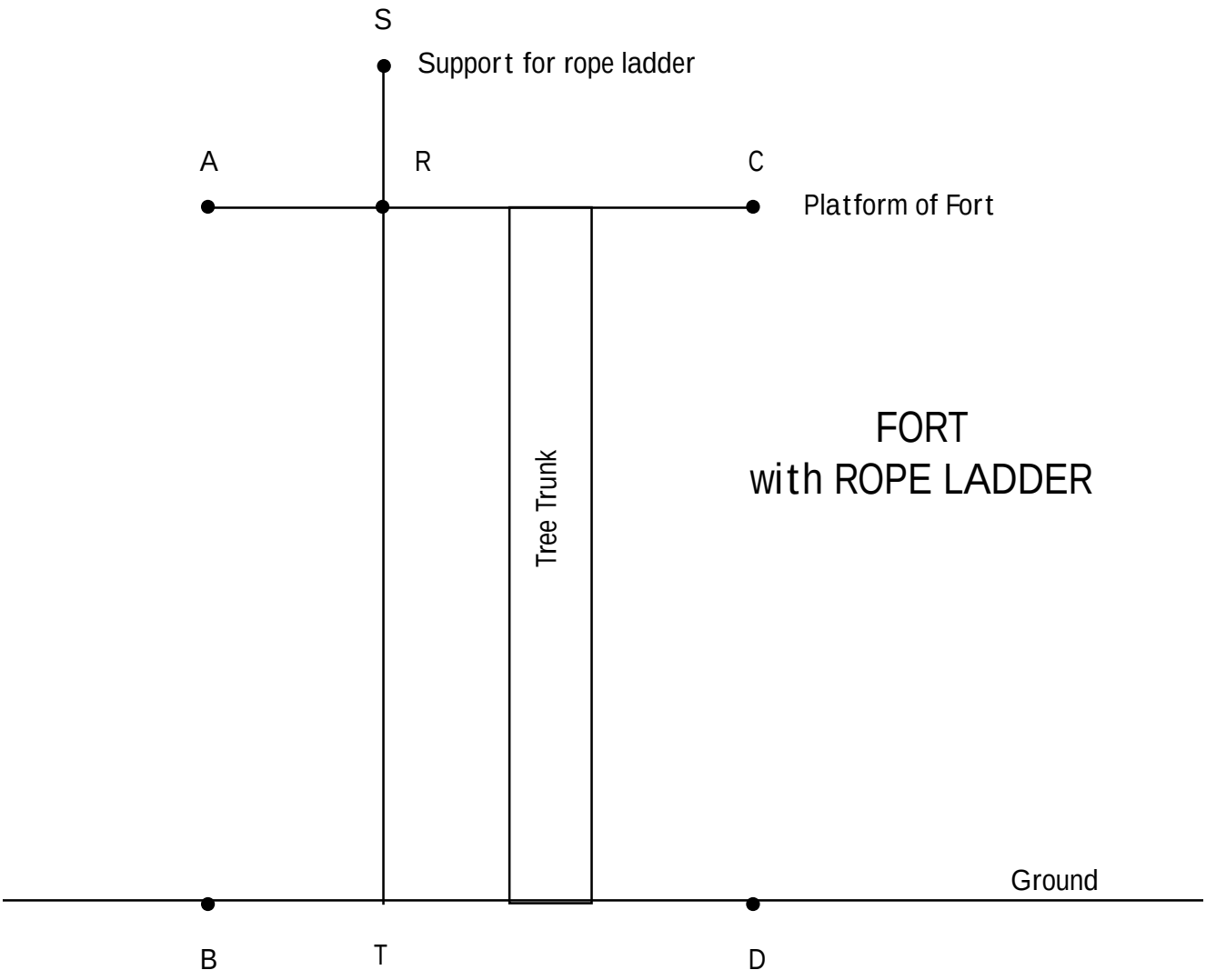
29. Using the letters A and B, represent the LINE SEGMENT: _____

30. Using the letters A' and B', represent the RAY: _____

Chapter 2: Angle

EXPLORING THE IDEA

The platform of the fort will be 8 feet above the ground. This sketch shows the fort using the rope ladder. The line segment ST represents the rope.



The rope ($\overline{ST}$) drops straight down through an opening in the fort platform to the ground
Using your protractor measure the angles the rope makes with the ground and the platform. You will make six measurements. If you need help in using a protractor, turn to Appendix A. The angle is measured in degrees and is symbolized with a small superscript circle to the right of the numerical value. For example, a measurement of 15 degrees would be written 15 º.

The angle the rope ($\overline{ST}$) makes with the ground ($\overline{BD}$):

On the left side of the rope: _____ ^{o}

On the right side of the rope: _____ ^{o}

The angle the rope ($\overline{ST}$) makes with the fort platform ($\overline{AC}$):

Below the Platform:

On the left side of the rope: _____

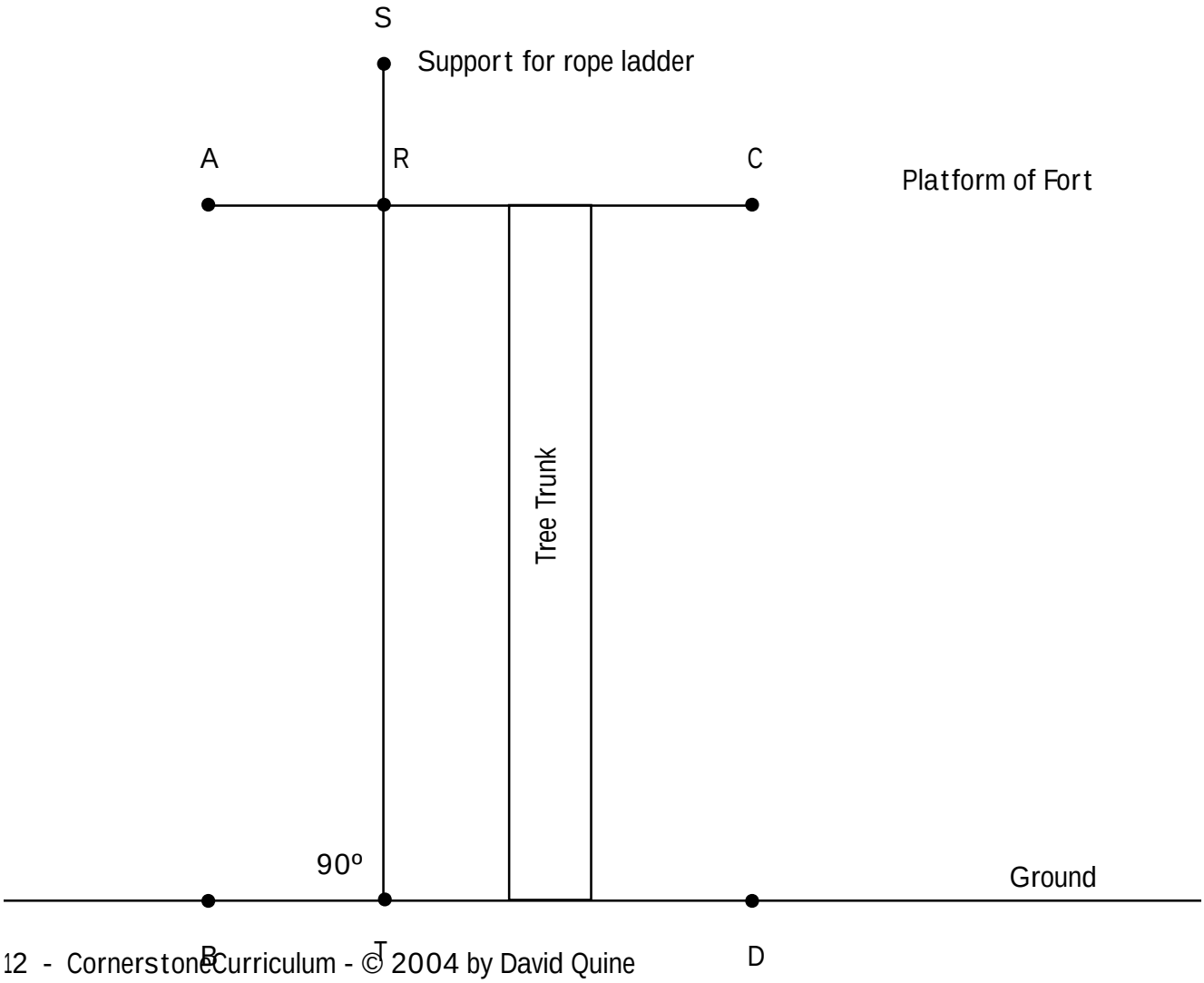
On the right side of the rope: _____

Above the Platform:

On the left side of the rope: _____

On the right side of the rope: _____

Record your measurements on the drawing below by writing the value in each of the six corners. The first one is done for you.



EXPLAINING THE IDEA

When two lines meet or intersect, they form an Angle. The symbol for angle is $\angle$.
The notation m $\angle$ means the measure of the angle.

Several angles are special angles in geometry. 90º is one of those special angles. When two lines or line segments meet and when the angle they form is a 90º angle, this angle is called a

RIGHT ANGLE.

When the angle of 90º is formed , the lines or line segments are said to be

PERPENDICULAR.
It is represented by this symbol: $\perp$

In the problem on the previous page, AC $\perp$ BD and is read line segment AC is perpendicular to line segment BD because the angle they form is 90º.

Find several other line segments on the previous page that are perpendicular and list them below. For example,

AR $\perp$ SR	_____	_____
SR $\perp$ CR	_____	_____
_____	_____	_____

EXPANDING THE IDEA Part 1

The angle that is made by the intersection of BT and RT is referred to as $\angle$BTR.
Line segment BT and line segment RT have a common endpoint.
The common endpoint of an angle is called the

VERTEX

and is the middle letter in the angle name.
This designation allows everyone to know which point is the common endpoint.

The angle that is made by the intersection of DT and RT is referred to as $\angle$DTR.

What is the vertex of this angle? _____

Circle the letter that represents the vertex in he following angles?

$\angle$TRC $\angle$ARS. $\angle$DTR. $\angle$SRC $\angle$ART

EXPANDING THE IDEA Part 2

What is the value of ∠ CRS ? _____

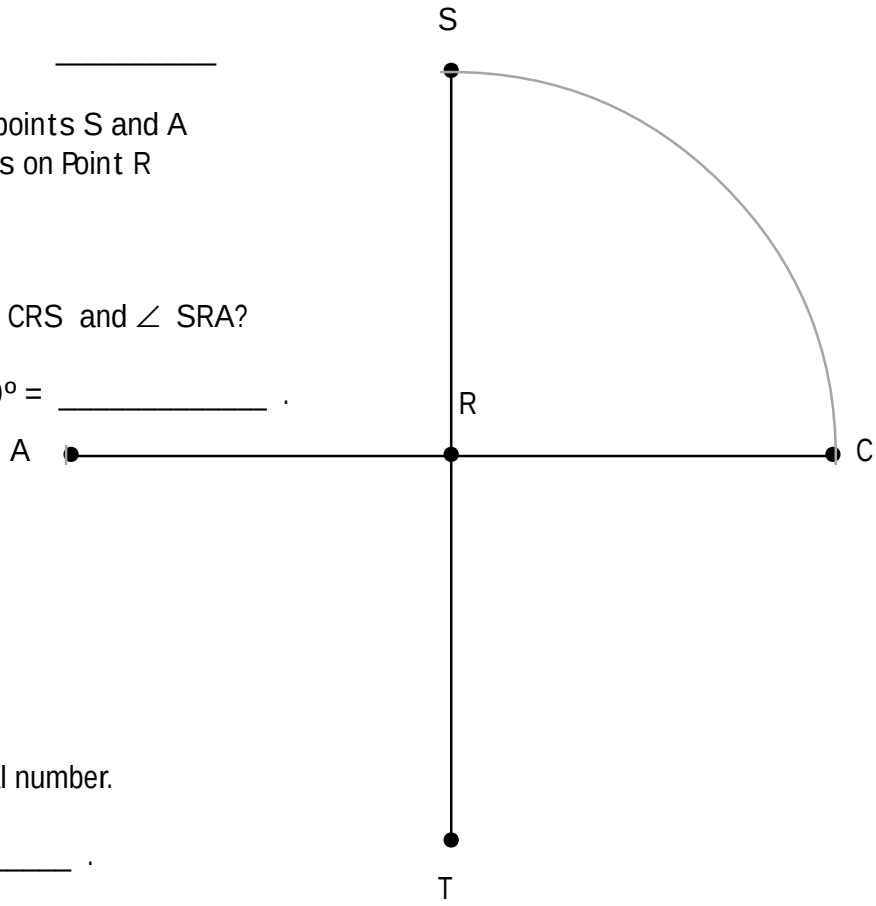
Using a compass connect points C and S by placing the tip of the compass on Point R and the pencil point on Point C. Rotate the compass to point S following the shaded curve.

What is the value ∠ SRA ? _____

Using a compass again connect points S and A
by placing the tip of the compass on Point R
and the pencil point on Point S.
Rotate the compass to point A.

What is the combined value of ∠ CRS and ∠ SRA?

∠ CRS + ∠ SRA = 90º + 90º = _____ .



Using your protractor
measure ∠ CRA.

What is that value? _____

∠ CRA is a LINE SEGMENT.
A straight line is 180º.

Angles may be added like any real number.

Predict the value of ∠ CRT. _____ .

What is the value ∠ ART ? _____

Using a compass again connect points A and T.

If you traveled along the curve from Point C through Points S and A to Point T,
how many degrees would your move? _____ .

What is the combined value of ∠ CRS and ∠ SRA and ∠ ART?

∠ CRS + ∠ SRA = 90º + 90º + 90º = _____ .

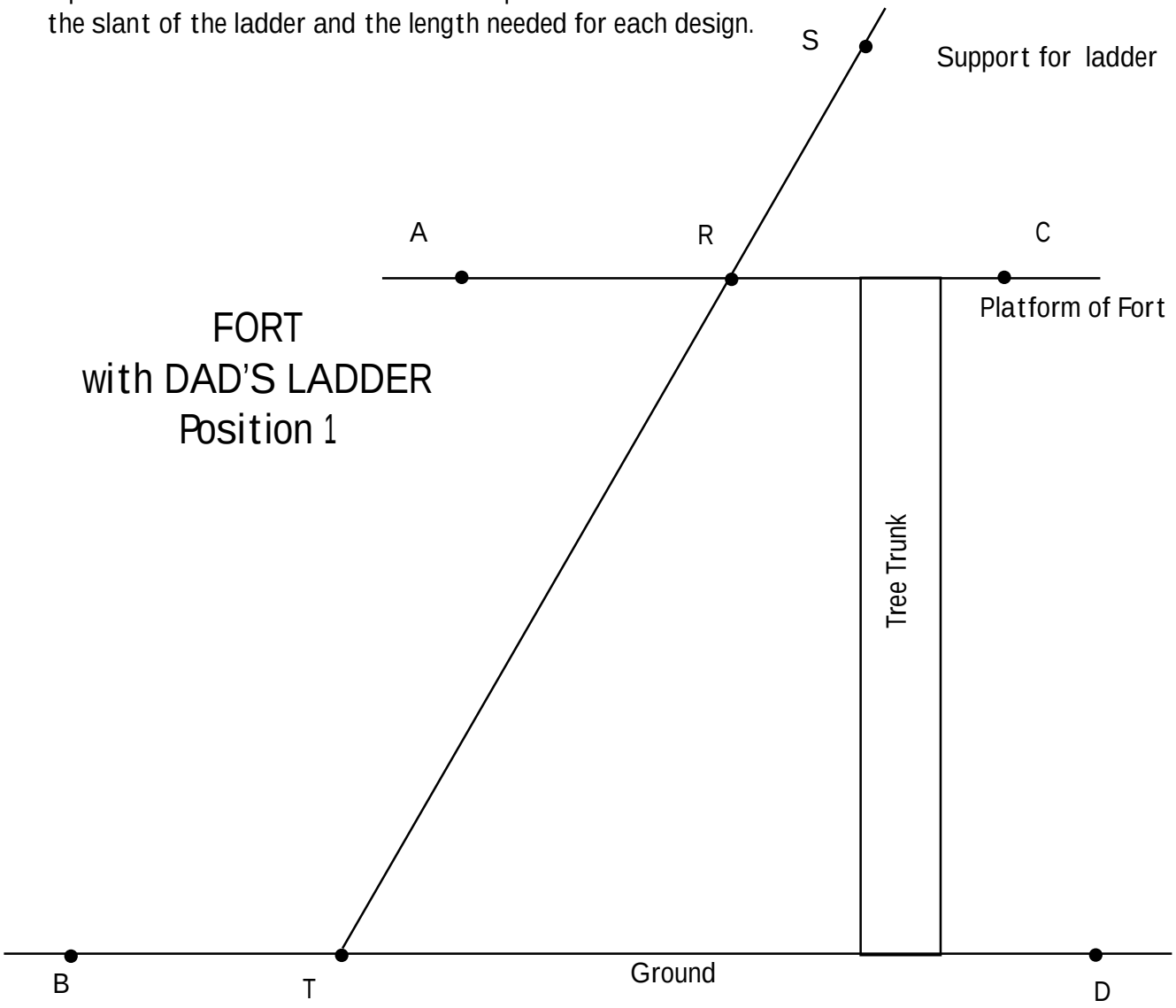
If you traveled along the curve from Point C through Points S, A, T and on to Point C, how far would your travel? _____ . Using a compass again connect points T and C.

What shape did you just construct? _____ .

How many degrees in a circle ? _____ .

EXPLORING THE IDEA

The next series of sketches show the fort design using your dad's ladder. The line segment $\overline{ST}$ represents the ladder. The ladder will be placed in several different locations. You need to find out the slant of the ladder and the length needed for each design.



The ladder, represented by $\overline{ST}$, is placed at a slant to both the ground and fort platform. Using your protractor measure the angles the ladder makes with the ground.

$\angle BTR =$ _____ ^{o}

PREDICTION

Before you measure the other angle ($\angle DTR$) what do you think this angle would be? _____

Now, using your protractor, measure angle DTR.

$\angle DTR =$ _____ ^{o}

Was your prediction correct? YES NO

What is the angle of a straight line? _____

Measure ∠ BTD.

∠ B T D = _____

Is this angle what your expected? YES NO

What is the sum of these two angles?

∠ B T R + ∠ D T R = _____ + _____ = _____

Because you know that the angle of a straight line is 180º,
if you know one of the angles along this line ,which arithmetic operation do you need to apply to find the other angle? ADDITION SUBTRACTION

180º - the known angle = the unknown angle

Now look at the angle the ladder makes with the fort platform..

How many angles does it make? _____

Measure ∠ ARS.

∠ A R S = _____

Does the line segment AC contain Point R? YES NO

Is line segment AC a straight line? YES NO

Knowing angle ARS, predict angle SRC. _____

If you need help in making this prediction, use the relationship

180º - the known angle = the unknown angle

Now, using your protractor, measure ∠ SRC.

∠ SRC = _____

Is this angle what you expected? YES NO

∠ A R S + ∠ S R C = _____ + _____ = _____

Now, turn your attention to the other two angles (∠ ART + ∠ TRC) formed by the intersection of the ladder with the platform.

Predict the sum of the following two angles:

∠ ART + ∠ TRC = _____ .

Is it necessary to measure both angles to know both angles? YES NO

If you measure ∠ ART how could you determine ∠ TRC ? _____
_____

Now, measure ∠ ART.

∠ A R T = _____

Determine ∠ TRC without measuring the angle.

∠ T R C = _____

Now, using your protractor, measure ∠ TRC .

Does your measurement match your prediction? YES NO

What is the sum of these two angles?

∠ A R T + ∠ T R C = _____ + _____ = _____

You have now measured the four angles around the intersection of the ladder and the fort platform.

PREDICTION
What measurement would you predict if you added these four angles? _____

Place the values for each measurement in the space provided below and find the sum.

∠ A R S + ∠ S R C + ∠ A R T + ∠ T R C
_____ + _____ + _____ + _____ = _____

Did your prediction match the measurements? YES NO

Below is a simple diagram of the Fort Problem with the corresponding degrees.

Look closely at the four angles in the top portion of the diagram.

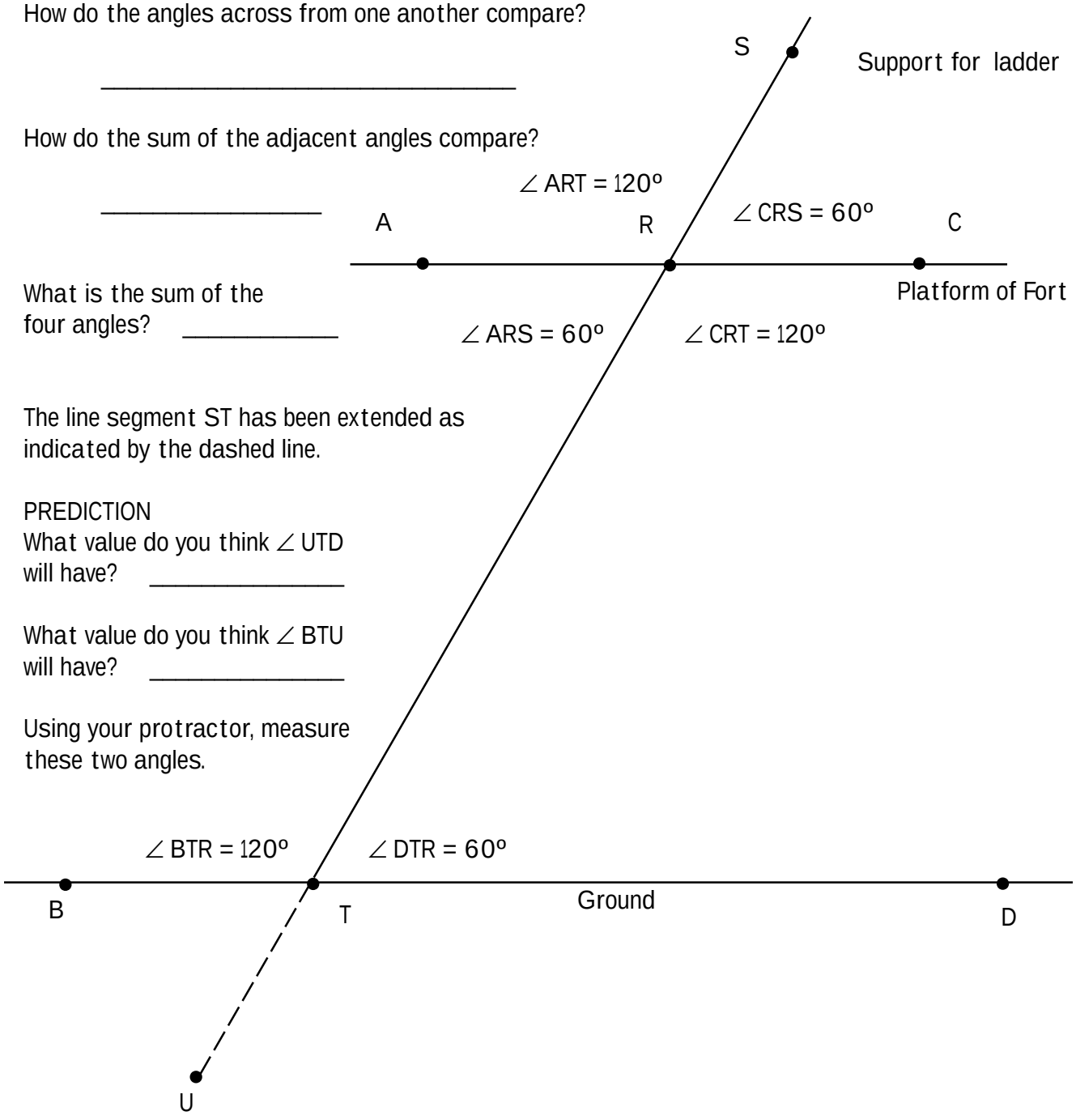
How do the angles across from one another compare?

_____

How do the sum of the adjacent angles compare?

_____

What is the sum of the four angles? _____



The line segment ST has been extended as indicated by the dashed line.

PREDICTION

What value do you think ∠ UTD will have? _____

What value do you think ∠ BTU will have? _____

Using your protractor, measure these two angles.

Were your predictions correct? YES NO

In the diagram above, highlight all the measurements equal to 120º in blue.
In the diagram above, highlight all the measurements equal to 60º in red.

Do you see a pattern? If so, explain the pattern. _____
_____ .

EXPLORING THE IDEA

In this diagram the position of the ladder has changed.

Using a protractor, carefully and accurately measure ∠ ARS. _____

PREDICTION
From your knowledge of angles, predict all the remaining angles.
Refer back to the highlighted markings on the previous diagram.
After making all your predictions,
measure each angle.

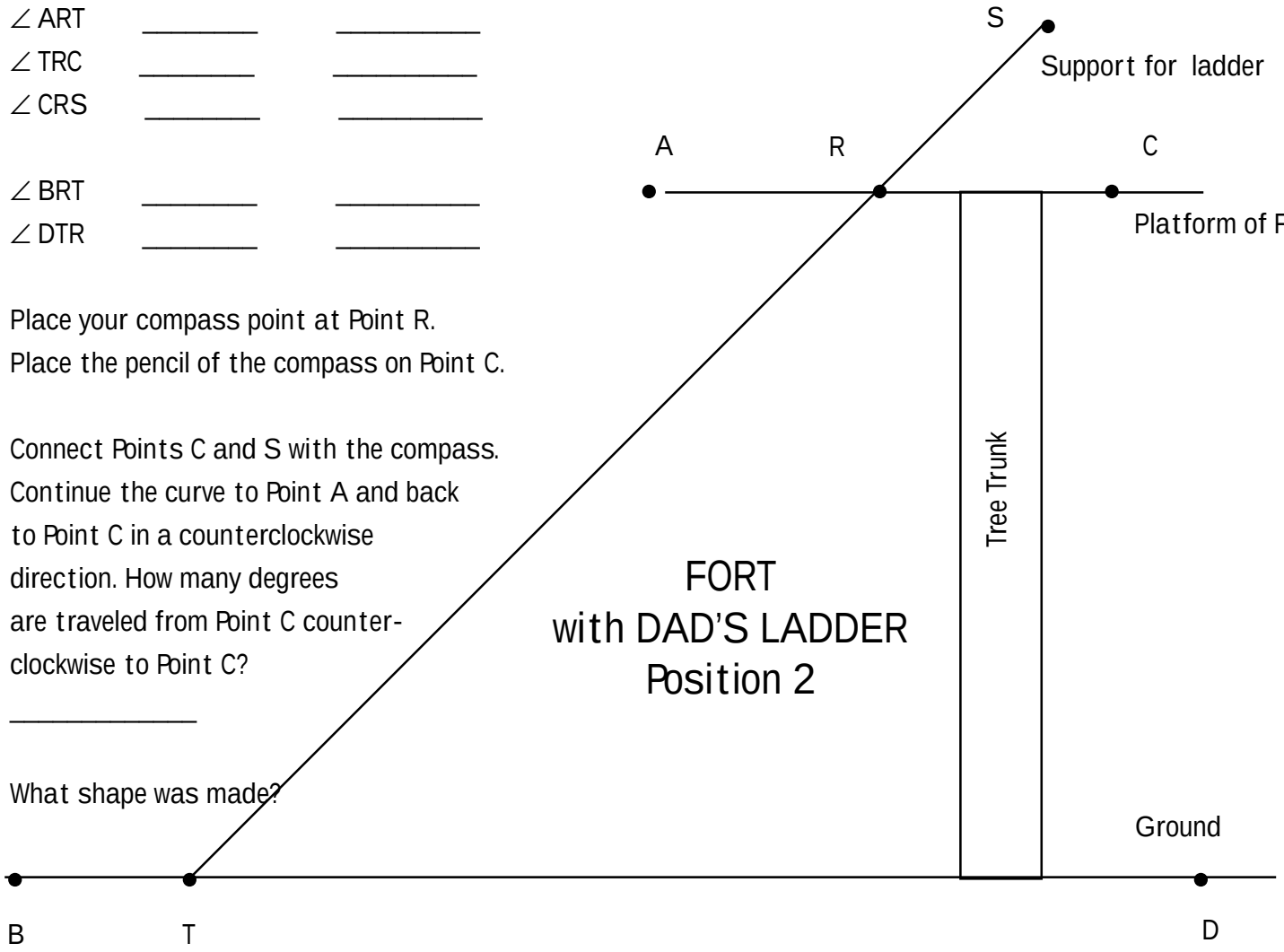
ANGLE:	PREDICTION	MEASUREMENT
∠ SRA		135º
∠ ART	_____	_____
∠ TRC	_____	_____
∠ CRS	_____	_____
∠ BRT	_____	_____
∠ DTR	_____	_____

Place your compass point at Point R.
Place the pencil of the compass on Point C.

Connect Points C and S with the compass.
Continue the curve to Point A and back
to Point C in a counterclockwise
direction. How many degrees
are traveled from Point C counter-
clockwise to Point C?

_____

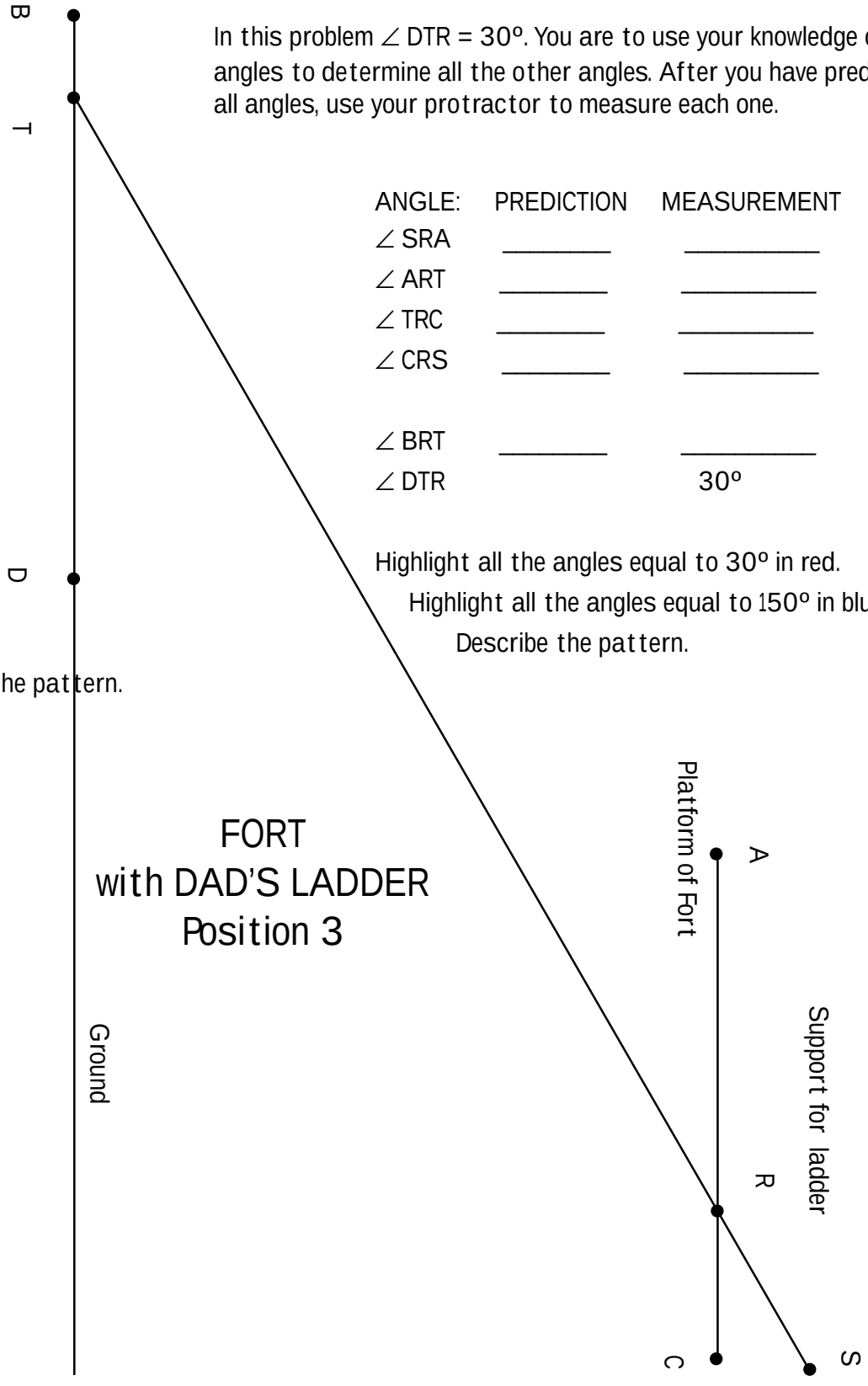
What shape was made?



Highlight all the angles equal to 45º in the above diagram red.
Highlight all the angles equal to 135º in the above diagram in blue.
Describe the pattern.

EXPLORING THE IDEA

In this diagram the position of the ladder has changed again. You will need to turn the page sideways to properly view the tree fort.



FORT
with DAD'S LADDER
Position 3

In this problem ∠ DTR = 30°. You are to use your knowledge of angles to determine all the other angles. After you have predicted all angles, use your protractor to measure each one.

ANGLE:	PREDICTION	MEASUREMENT
∠ SRA	_____	_____
∠ ART	_____	_____
∠ TRC	_____	_____
∠ CRS	_____	_____
∠ BRT	_____	_____
∠ DTR		30°

Highlight all the angles equal to 30° in red.

Highlight all the angles equal to 150° in blue.

Describe the pattern.

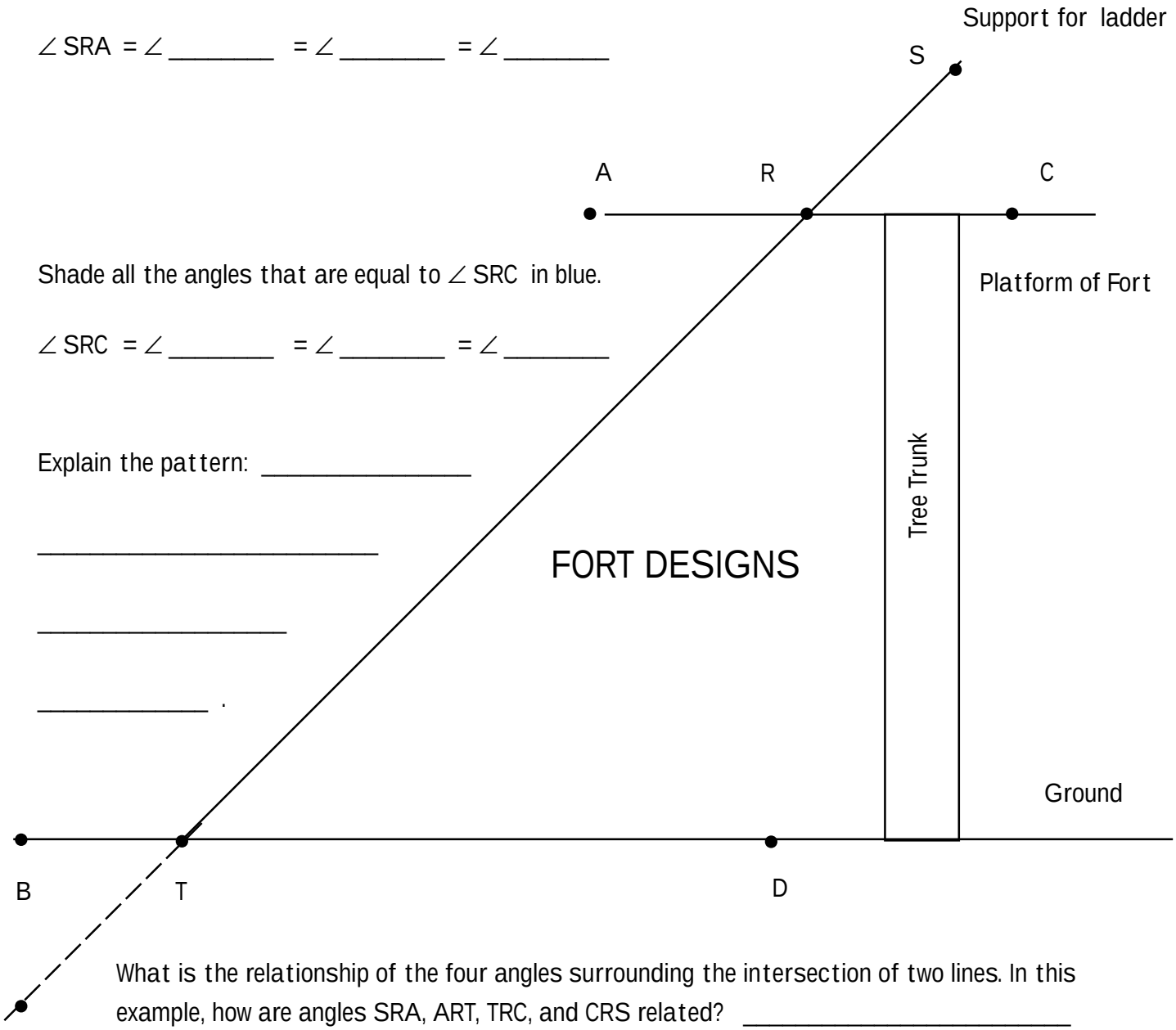
Describe the pattern.

EXPLORING THE IDEA

When you start with two parallel lines and a third line that cuts across them, what are some of the relationship that exist among the various angles that are formed? Patterns sometimes emerge as you look for similarities and then look for the important contrasts. Look at the diagram below.

Shade all the angles that are equal to ∠ SRA in red.

∠ SRA = ∠ _____ = ∠ _____ = ∠ _____



Shade all the angles that are equal to ∠ SRC in blue.

∠ SRC = ∠ _____ = ∠ _____ = ∠ _____

Explain the pattern: _____

_____

_____

_____ .

What is the relationship of the four angles surrounding the intersection of two lines. In this example, how are angles SRA, ART, TRC, and CRS related? _____

_____

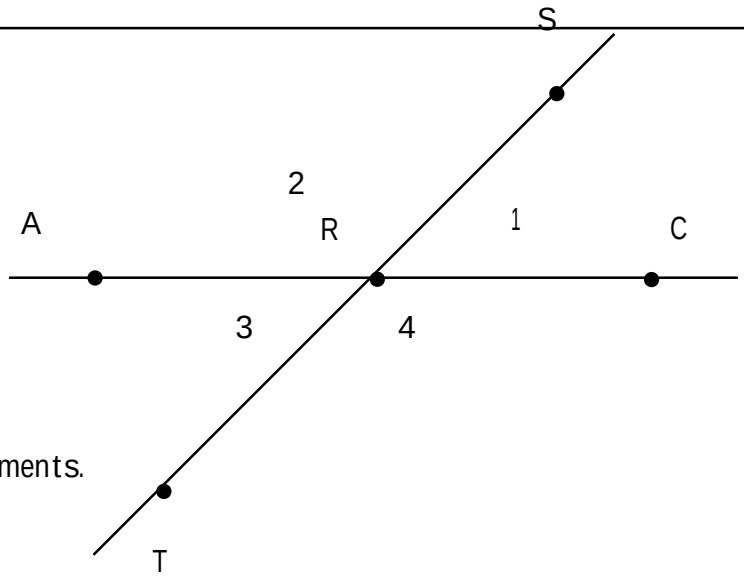
_____ .

If you were given the measurement of ∠ UTD, could you determine the value of ∠ SRC? YES NO

EXPLAINING THE IDEA

When two lines intersect, four angles are formed.
State the relationship of angles that are across from each other....

Rename ∠ CRS as ∠ 1.
Rename ∠ SRA as ∠ 2.
Rename ∠ ART as ∠ 3.
Rename ∠ TRC as ∠ 4.



Measure each of the angles.
Be very precise in your measurements.

∠ 1 = _____

∠ 2 = _____

∠ 3 = _____

∠ 4 = _____

Look at the relationships between the four angles.

What do you observe? _____

Shade the equal angles with the same color.

When two lines intersect, four angles are formed.
State the relationship of angles that are across from each other....

When two lines intersect, four angles are formed.
The angles that are across from each other are called

VERTICAL ANGLES
and all vertical angles are EQUAL.

Since ∠ SRA is across from ∠ TRC,

Then ∠ SRA and ∠ TRC are vertical angles;

Therefore, ∠ SRA = ∠ TRC,

Name the vertical angle to ∠ ART.… _____

Complete the following statements:

Since ∠ _____ is across from ∠ ART,

Then ∠ _____ and ∠ ART are _____ angles;

Therefore, ∠ _____ = ∠ ART.

Complete the following statements:

Since ∠ BTU is across from ∠ _____ .,

Then ∠ BTU and ∠ _____ are _____ angles;

Therefore, ∠ BTU = ∠ _____

Another way to think of this is to work backwards. Sometimes in geometry it is important to prove something true For example,

To prove that ∠ UTD = ∠ BTR, the thinking goes something like this.…

Since ∠ UTD is across from ∠ BTR;

Then ∠ UTD and ∠ BTR are Vertical Angles; and

Since Vertical Angles are equal

Therefore, ∠ UTD = ∠ BTR

Place the data from Naming The Idea activity on the chart below.

	ANGLE	TOTAL	GRAND TOTAL
∠ 1			
∠ 2			
∠ 3			
∠ 4			

Place the point of your compass at Point R and the compass pencil at Point C.
Rotate the pencil to Point S and then on to Point A.

How many degrees has your pencil traveled? _____ .

Add ∠ 1 to ∠ 2 and place the total in the top box labeled TOTAL.

Next, place the point of your compass at Point R and the compass pencil at Point A.
Rotate the pencil to Point T and then on to Point C.

How many degrees has your pencil traveled? _____ .

Add ∠3 to ∠ 4 and place that total in the middle box labeled TOTAL.

What was the TOTAL of these two angles? _____

A STRAIGHT LINE has an angle of 180º.

Add all four angles and place that value in the space on the right labeled GRAND TOTAL.
How many degrees was it? _____ .

What shape was created? _____ .

A number of degrees in a CIRCLE is equal to 360º.

EXPANDING THE IDEA Part 1

Materials:
- Pencil
- Protractor

Using your pencil make line segment AB … and line segment CD.
How many angles were formed? _____ .

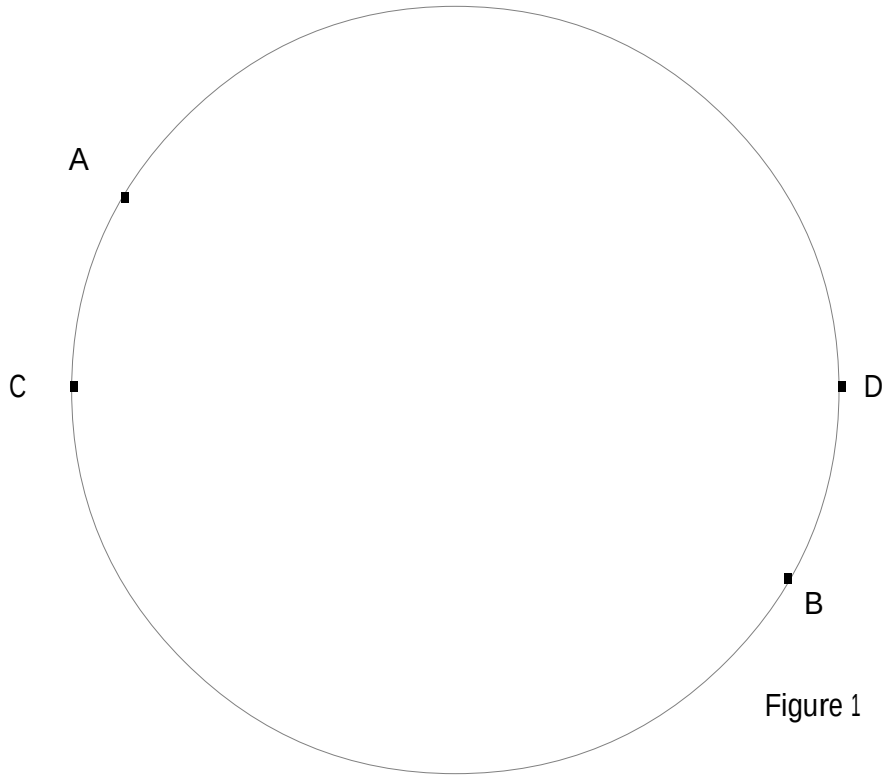


Figure 1

Starting with the top angle and moving clockwise, label the angles 1, 2, 3, and 4.

Measure each of the angles. Be very precise in your measurements.

∠ 1 = _____

∠ 2 = _____

∠ 3 = _____

∠ 4 = _____

Look at the relationships between the four angles.
What do you observe? _____

Shade the equal angles with the same color.

The angles which are across from one another are called _____ angles.

Vertical angles are _____ .

Using your pencil make line segment AB….and line segment CD.
How many angles were formed?

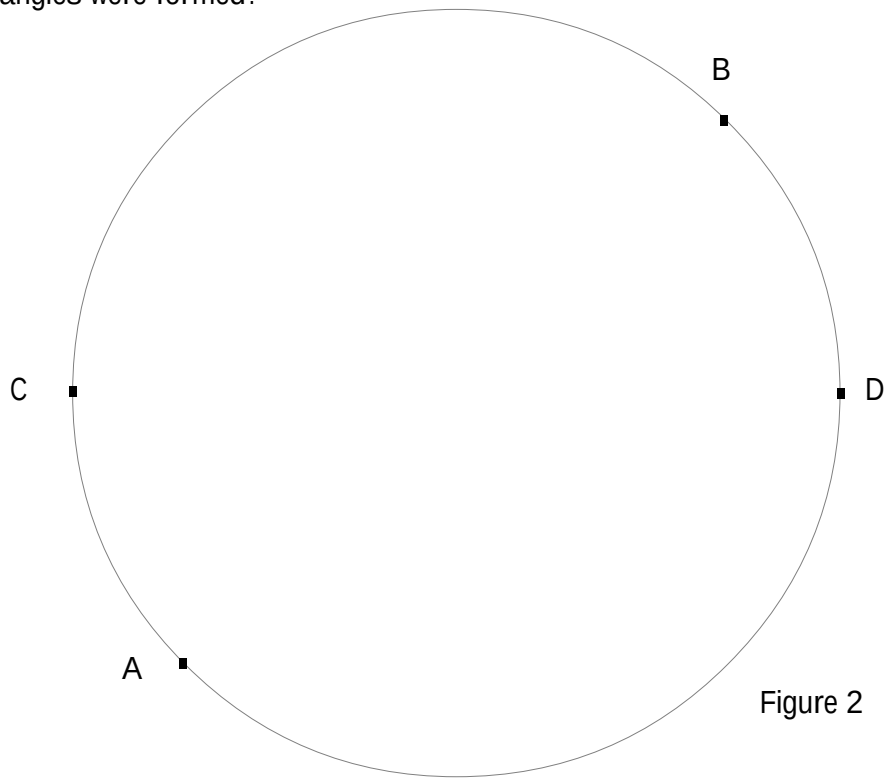


Figure 2

Starting with the top angle and moving clockwise, label the angles 1, 2, 3, and 4.

Measure each of the angles. Be very precise in your measurements.

∠ 1 = _____

∠ 2 = _____

∠ 3 = _____

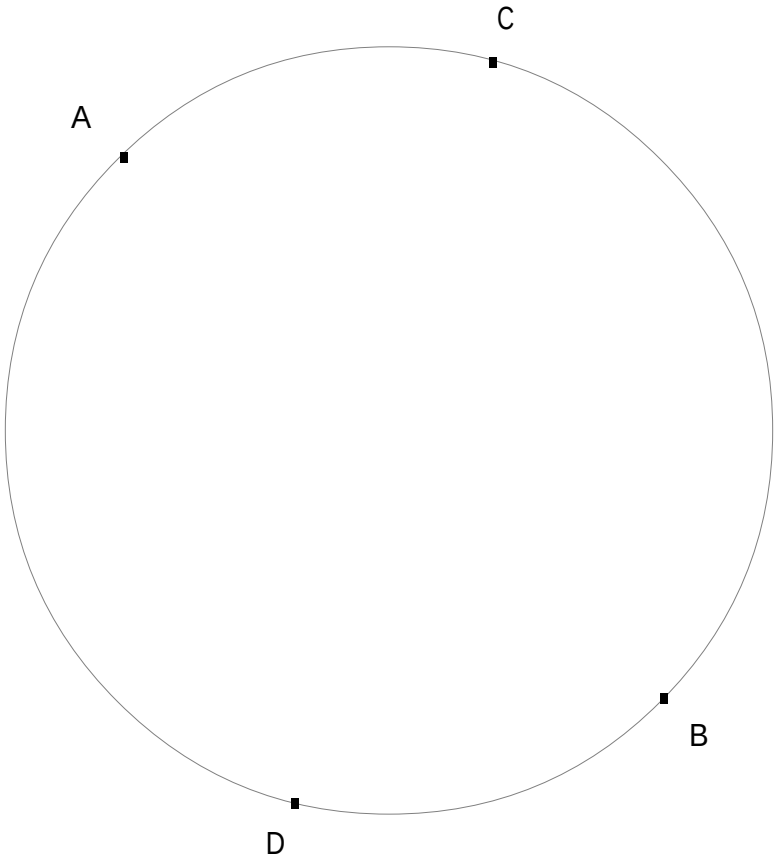
∠ 4 = _____

Look at the relationships between the four angles.

What do you observe? _____

Shade the equal angles with the same color.

Using your pencil make line segment AB….and line segment CD.
How many angles were formed?



Starting with the top angle and moving clockwise, label the angles 1, 2, 3, and 4.

Measure each of the angles. Be very precise in your measurements.

∠ 1 = _____

∠ 2 = _____

∠ 3 = _____

∠ 4 = _____

Look at the relationships between the four angles.

What do you observe? _____

Shade the equal angles with the same color.

When two lines intersect, four angles are formed.
State the relationship of angles that are across from each other….

_____

_____

_____

_____

If you know one of the angles, is it possible to determine the other three angles? Yes No

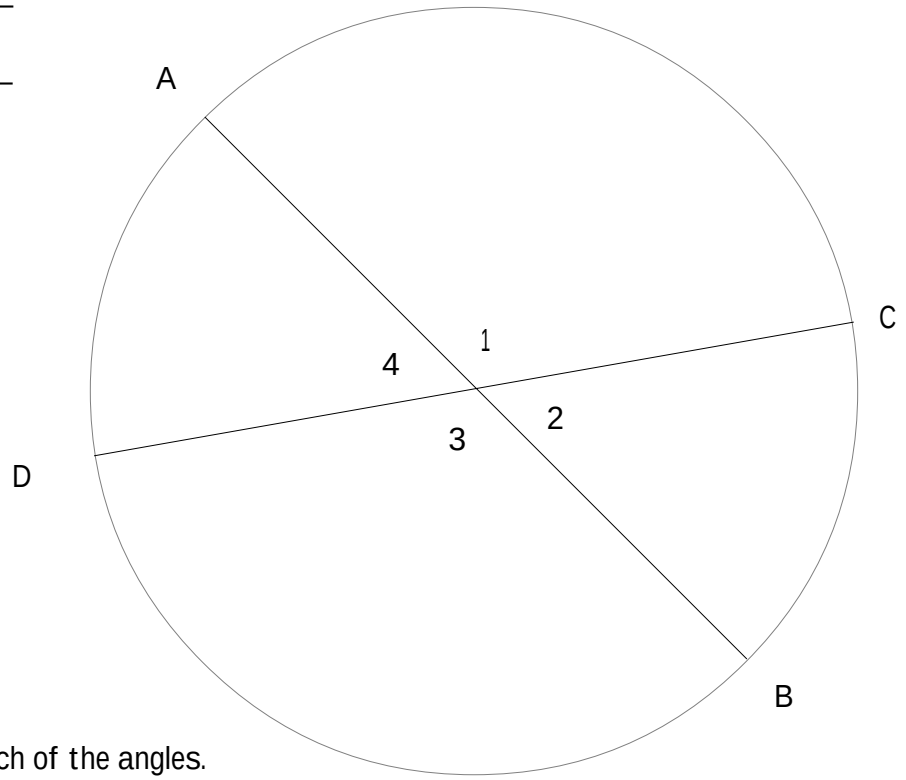
IF Angle 1 is equal to 125º, predict each of the other angles.

∠ 1 = 125º

∠ 2 = _____

∠ 3 = _____

∠ 4 = _____



Carefully measure each of the angles.

∠ 1 = 125º

∠ 2 = _____

∠ 3 = _____

∠ 4 = _____

Was your prediction correct? Yes No

If not, why not?

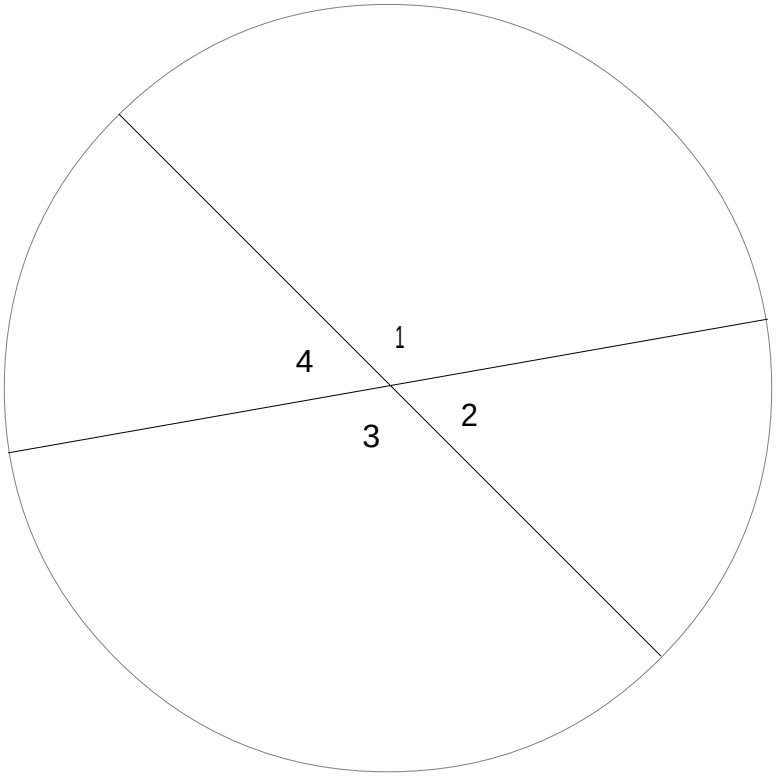
_____

_____

_____

Conclusions …

When two lines intersect …



- Four angles are formed.

- Vertical Angles are equal.

$\angle 1 = \angle 3$

$\angle 2 = \angle 4$

- The sum of the angles that are adjacent is equal to 180º.

$\angle 1 + \angle 2 = 180º$

$\angle 2 + \angle 3 = 180º$

$\angle 3 + \angle 4 = 180º$

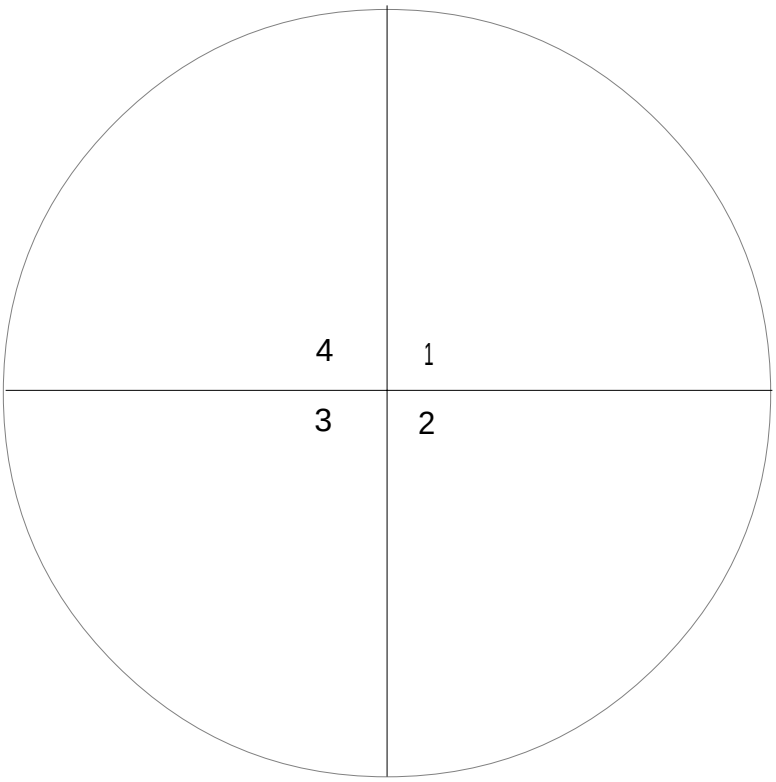
$\angle 4 + \angle 1 = 180º$

- The sum of all four angles is equal to 360º

$\angle 1 + \angle 2 + \angle 3 + \angle 4 = 360º$

EXPANDING THE IDEA - Part 2

Measure ∠ 1 _____ .



Predict

∠ 2 = _____

∠ 3 = _____

∠ 4 = _____

Measure angles 2, 3, and 4.

∠ 2 = _____

∠ 3 = _____

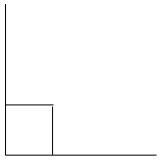
∠ 4 = _____

Did your predictions match your measurements?
If not, why not?

A 90º angle is called a

RIGHT ANGLE.

A RIGHT ANGLE is designated as follows:



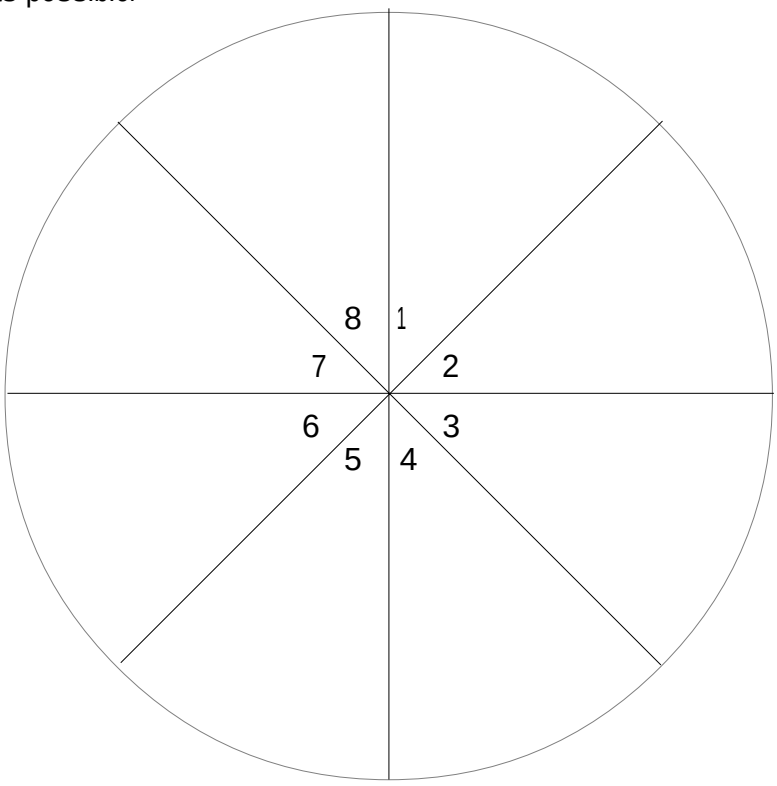
The two lines are said to be

PERPENDICULAR. and is represented by ⊥

Complete the chart below:

ANGLE	PREDICTION	ACTUAL MEASUREMENT
∠ 1 + ∠ 2 =		
∠ 2 + ∠ 3 =		
∠ 3 + ∠ 4 =		
∠ 4 + ∠ 1 =		
∠ 1 + ∠ 2 + ∠ 3 + ∠ 4 =		

Measure each angle as precisely as possible.



∠ 1 = _____

∠ 2 = _____

∠ 3 = _____

∠ 4 = _____

∠ 5 = _____

∠ 6 = _____

∠ 7 = _____

∠ 8 = _____

PREDICT the sum of …

∠ 1 + ∠ 2 + ∠ 3 + ∠ 4 = _____

ADD the sum of …

∠ 1 + ∠ 2 + ∠ 3 + ∠ 4 = _____

Did your predictions match your measurements? If not, why not?

PREDICT the sum of …

∠ 1 + ∠ 2 + ∠ 3 + ∠ 4 ∠ 5 + ∠ 6 + ∠ 7 + ∠ 8 = _____

ADD the sum of …

∠ 1 + ∠ 2 + ∠ 3 + ∠ 4 ∠ 5 + ∠ 6 + ∠ 7 + ∠ 8 = _____

Did your predictions match your measurements? If not, why not?

How many degrees are there in a complete circle? _____

Using your compass:

Place the compass point at Point C.

Place the compass pencil at Point R.

Rotate the pencil around Point C one complete turn.

What shape did you make? _____

How many degrees did your pencil travel? _____

Draw several line segments that pass through Point C and have endpoints on the circle.

Measure the angles that are formed.

Add all the angles together.

The sum of the angles is equal to how many degrees? _____

The sum should have been 360º.

If your measurements are not precisely 360º, at least they should have been very, very close.

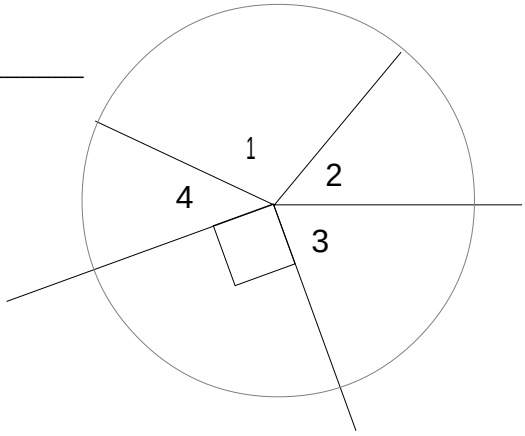
The total number of degrees in a circle is

360º

Using your knowledge of the number of degrees in a circle, and using your knowledge of the symbol for a right angle,

PREDICT the sum of …

$\angle 1 + \angle 2 + \angle 3 + \angle 4 =$ _____



Explain your reasoning:

_____
_____
_____
_____
_____
_____

Measure each angle:

$\angle 1 =$ _____

$\angle 2 =$ _____

$\angle 3 =$ _____

$\angle 4 =$ _____

CALCULATE the sum of …

$\angle 1 + \angle 2 + \angle 3 + \angle 4 =$ _____

Did your predictions match your measurements? If not, why not?

_____
_____
_____
_____

EXPANDING THE IDEA - Part 3

Measure

$\angle$ 1 = _____

$\angle$ 2 = _____

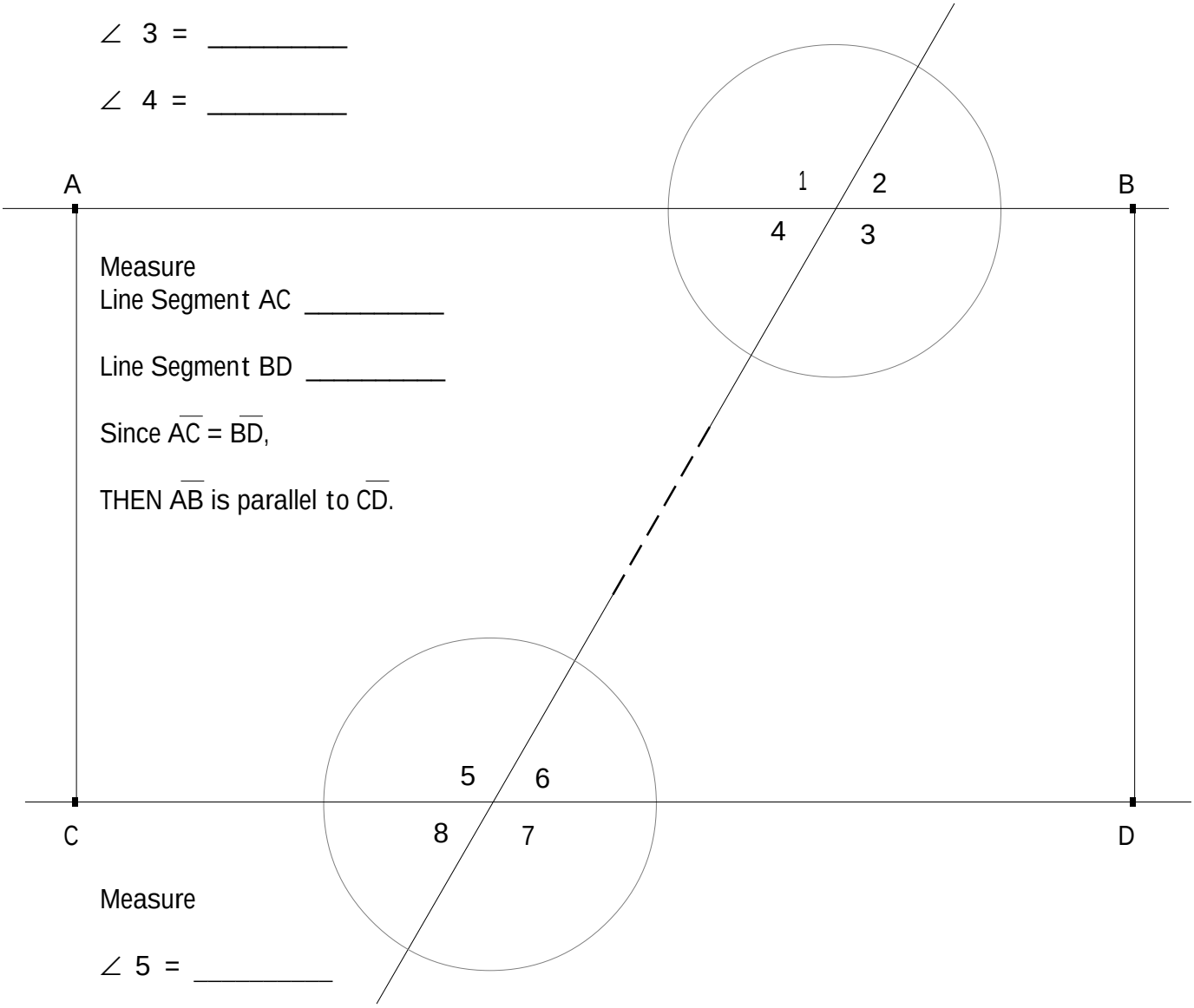
$\angle$ 3 = _____

$\angle$ 4 = _____

Which angles (1, 2, 3, 4) are VERTICAL ANGLES"

$\angle$ 1 and $\angle$ _____ $\angle$ 2 and $\angle$ _____

What do you know about VERTICAL ANGLES?



Measure
Line Segment AC _____

Line Segment BD _____

Since $\overline{AC} = \overline{BD}$,

THEN $\overline{AB}$ is parallel to $\overline{CD}$.

Measure

$\angle$ 5 = _____

$\angle$ 6 = _____

$\angle$ 7 = _____

$\angle$ 8 = _____

Which angles (5, 6, 7, 8) are VERTICAL ANGLES?

$\angle$ 5 and $\angle$ _____ $\angle$ 6 and $\angle$ _____

Connect the two diagonal lines in the above problem.

In the figure on the previous page, line segments AB and CD are parallel, and there is a diagonal line that cuts across the parallel lines.

Shade all the angles that are equal to the 'BIG ANGLE', $\angle$ 1 , with one color.

Shade all the angles that are equal to the 'small angle', $\angle$ 2 , with a different color.

Summarize the relationships among the 'BIG ANGLES':

$\angle$ 1 = $\angle$ _____ = $\angle$ _____ = $\angle$ _____

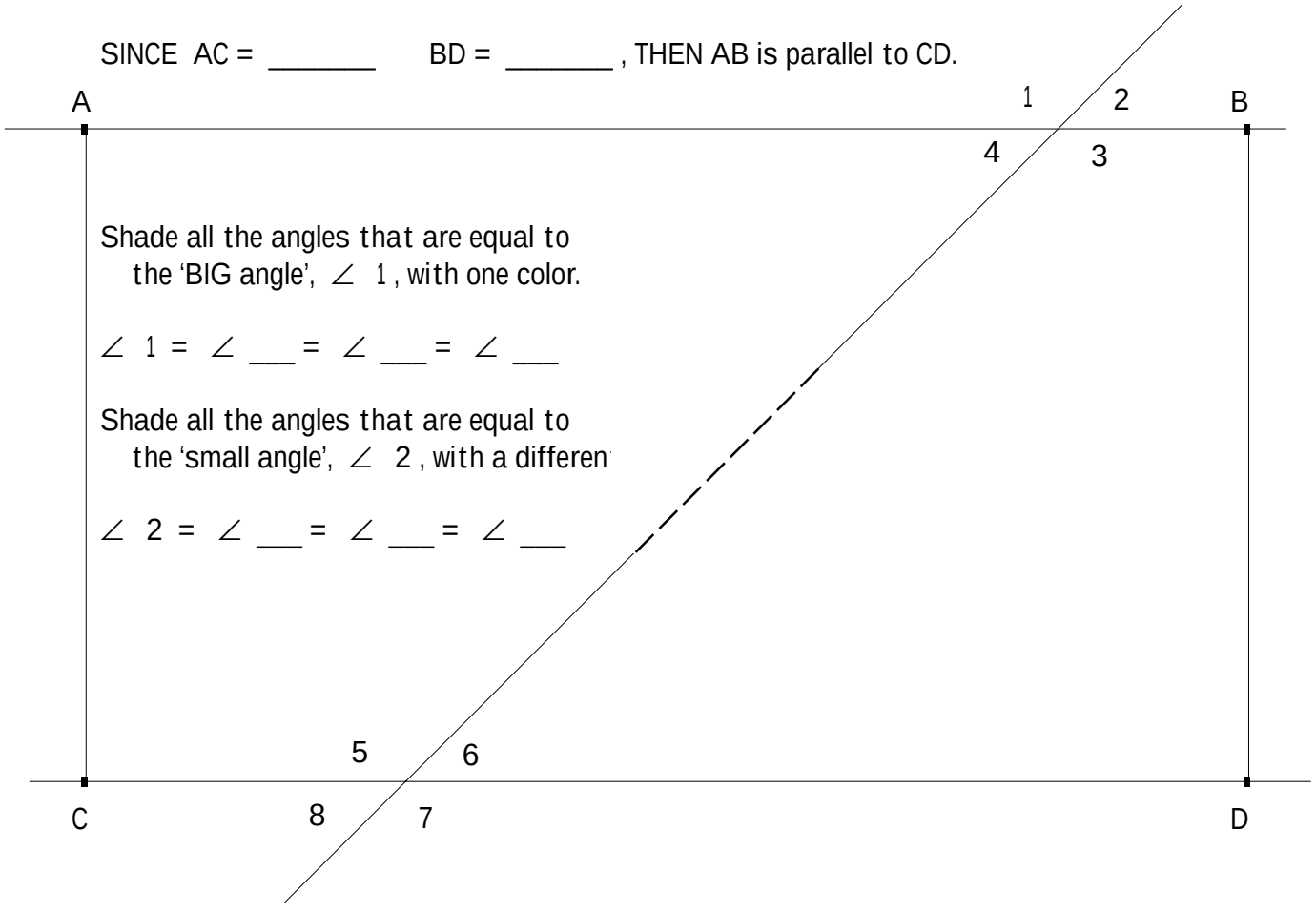
Summarize the relationships among the 'small angles':

$\angle$ 2 = $\angle$ _____ = $\angle$ _____ = $\angle$ _____

Does this relationship always exist?

Make the following measurements

SINCE AC = _____ BD = _____ , THEN AB is parallel to CD.



Shade all the angles that are equal to the 'BIG angle', $\angle$ 1 , with one color.

$\angle$ 1 = $\angle$ ___ = $\angle$ ___ = $\angle$ ___

Shade all the angles that are equal to the 'small angle', $\angle$ 2 , with a differen

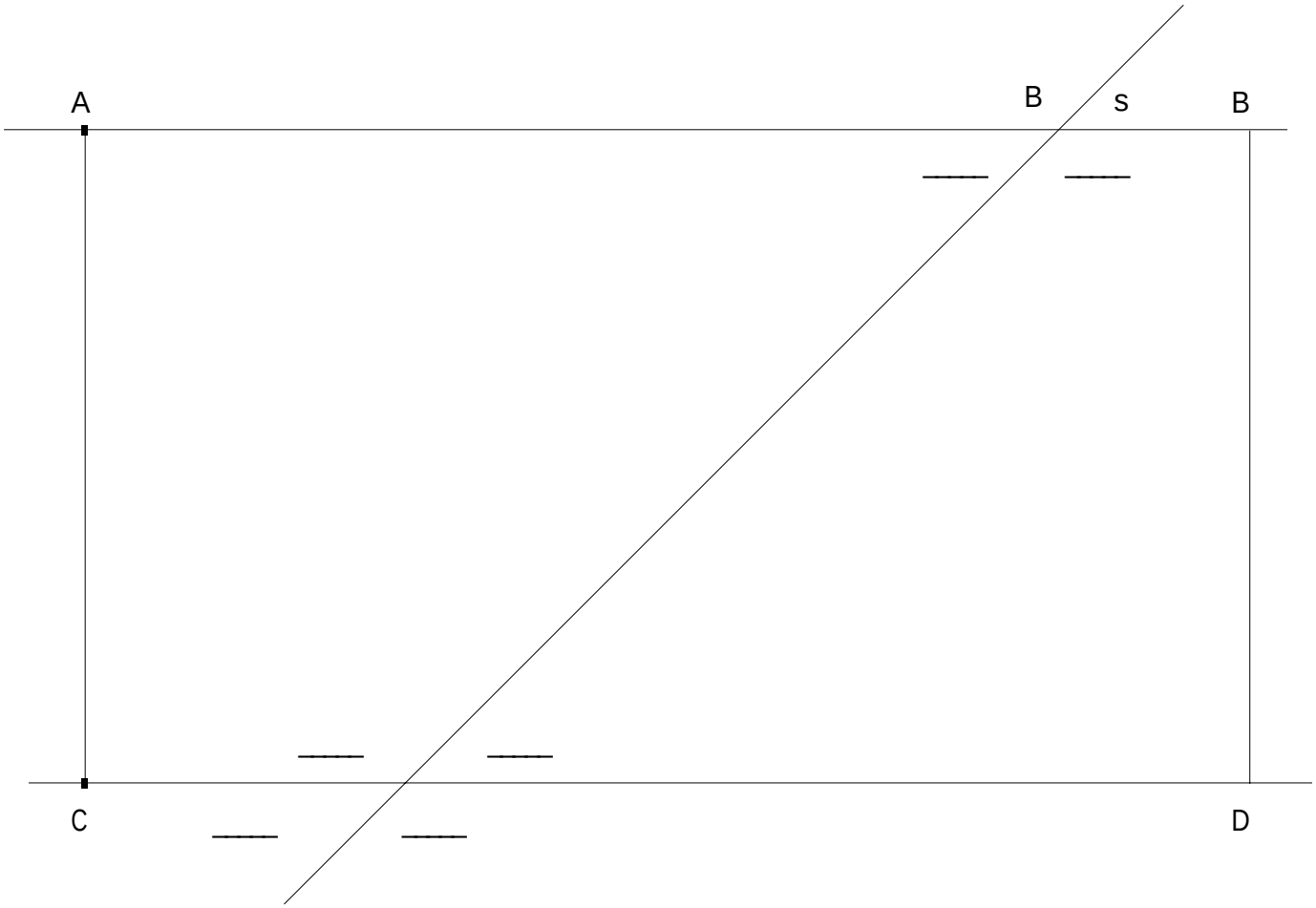
$\angle$ 2 = $\angle$ ___ = $\angle$ ___ = $\angle$ ___

When a diagonal line cuts across two parallel lines,
what is the relationship between the BIG ANGLES?

When a diagonal line cuts across two parallel lines,
what is the relationship between the SMALL ANGLES?

In the figure below one angle is marked 'B' (for BIG angle)
and the other angle 's' (for the small angle.)

Label the other angles 'B' or 's' based upon the pattern.



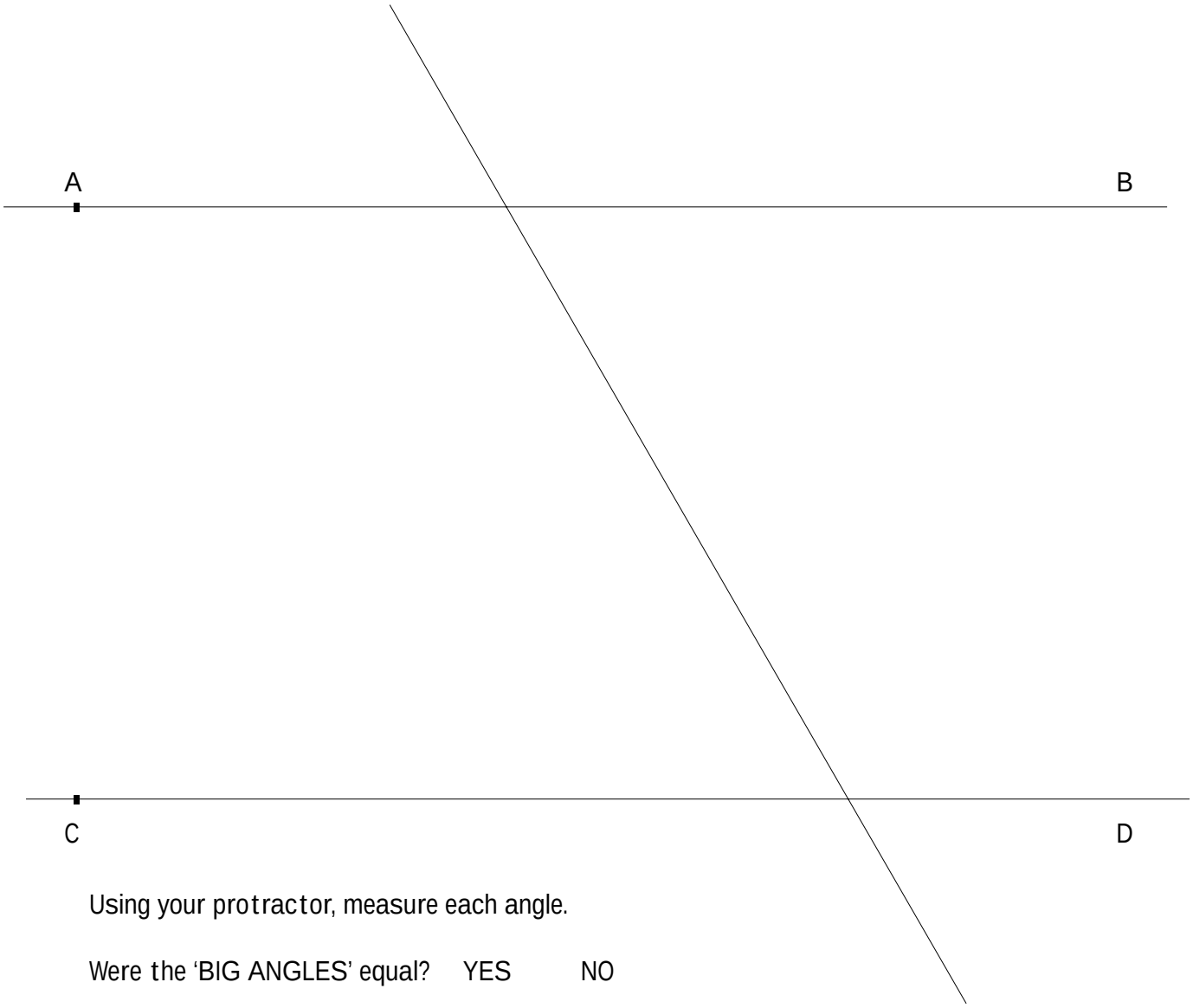
Line segments AB and CD are parallel and are cut by a diagonal line.

Shade the 'BIG ANGLES' in one color.

Shade the 'small angles' in another color.

Without measuring …
How do you think the 'BIG ANGLES' are related? Equal or Not Equal

How do you think the 'small angles' are related? Equal or Not Equal



Using your protractor, measure each angle.

Were the 'BIG ANGLES' equal? YES NO

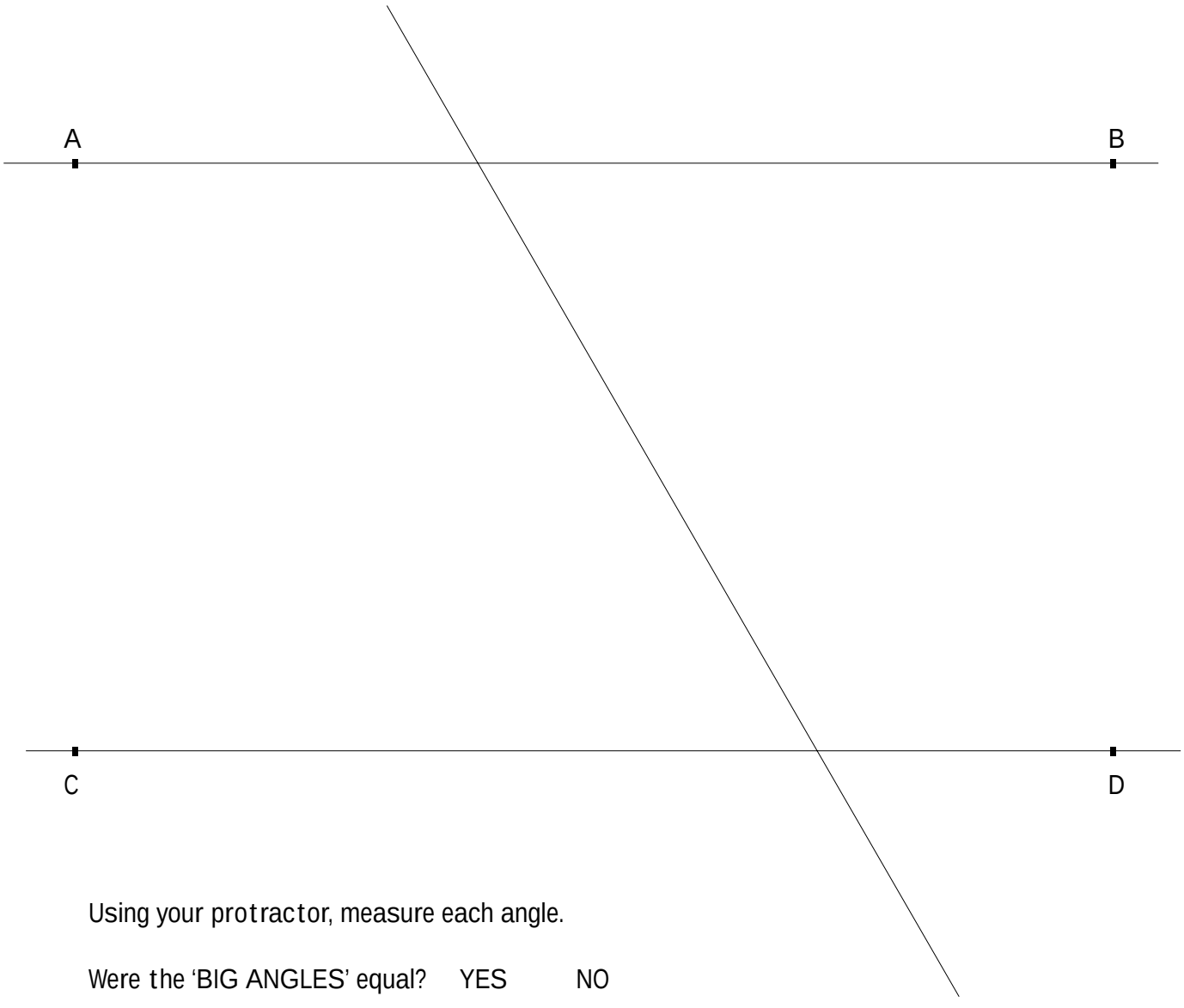
Were the 'small angles' equal? YES NO

Shade the ‘BIG ANGLES’ in one color.

Shade the ‘small angles’ in another color.

Without measuring …
How do you think the ‘BIG ANGLES’ are related? Equal or Not Equal

How do you think the ‘small angles’ are related? Equal or Not Equal



Using your protractor, measure each angle.

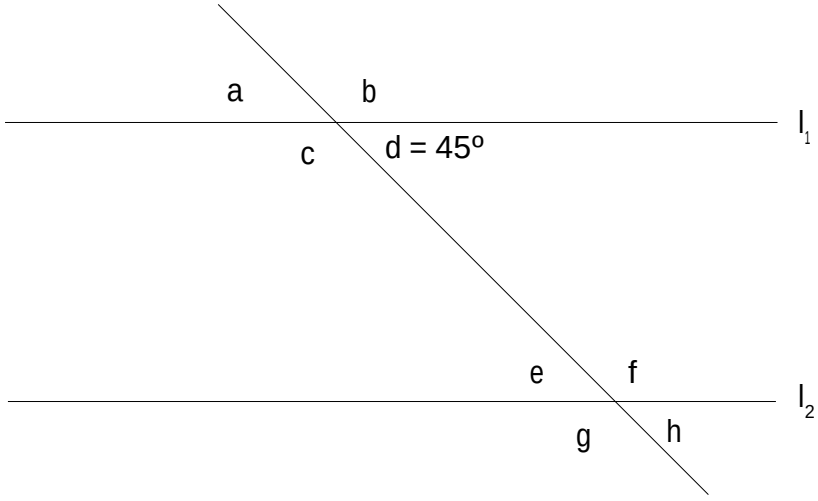
Were the ‘BIG ANGLES’ equal? YES NO

Were the ‘small angles’ equal? YES NO

In the diagram below lines l_1 and l_2 are parallel and are cut by a diagonal line.
Of the eight angles formed only one angle is known.
Is it possible to determine all the angles knowing the one angle?

Use what you know about 'BIG ANGLES' and 'small angles' , about the degrees of a straight line, and the degrees in a circle to predict all the unknown angles.

PREDICT each angle:



∠ a = _____

∠ b = _____

∠ c = _____

∠ d = 45º

∠ e = _____

∠ f = _____

∠ g = _____

∠ h = _____

MEASURE each angle:

∠ a = _____

∠ b = _____

∠ c = _____

∠ d = 45º

∠ e = _____

∠ f = _____

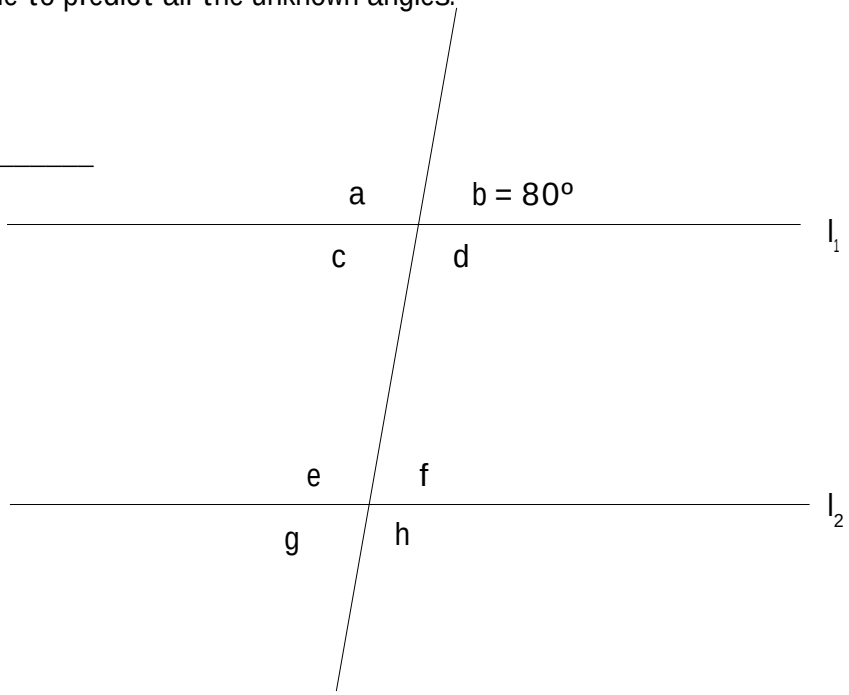
∠ g = _____

∠ h = _____

Were your predictions correct or incorrect?

In the diagram below lines l_1 and l_2 are parallel and are cut by a diagonal line.
Of the eight angles formed only one angle is known.
Is it possible to determine all the angles knowing the one angle?

Use what you know about 'BIG ANGLES' and 'small angles' , about the degrees of a straight line, and the degrees in a circle to predict all the unknown angles.



PREDICT each angle:

SINCE ∠ a + ∠ b = _____

and since ∠ b = 80º

THEN ∠ a = _____

MEASURE

∠ a = _____

Was your prediction correct?

YES NO

How are ∠ b and ∠ c related? _____

SINCE ∠ b = 80º,

THEN ∠ c = _____ .

PREDICT
SINCE ∠ b + ∠ d = _____

and since ∠ b = 80º

THEN ∠ d = _____

MEASURE

∠ d = _____

Was your prediction correct?

YES NO

PREDICT
SINCE ∠ b and ∠ c are 'small angles' (or VERTICAL ANGLES),
and since ∠ b = 80º

THEN ∠ c = _____

MEASURE

∠ c = _____

Was your prediction correct?

YES NO

PREDICT
SINCE ∠ c and ∠ f are 'small angles' (or VERTICAL ANGLES),
and since ∠ c = 80º

THEN ∠ f = _____

MEASURE

∠ f = _____

Was your prediction correct?

YES NO

PREDICT
SINCE ∠ f and ∠ g are 'small angles' (or VERTICAL ANGLES),
and since ∠ f = 80º

THEN ∠ g = _____

MEASURE

∠ g = _____

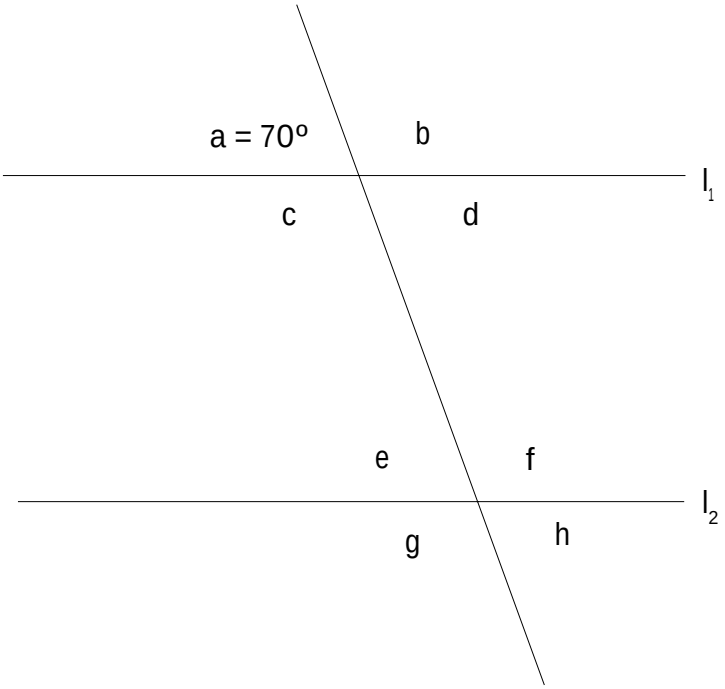
Was your prediction correct?

YES NO

The diagram below is of two parallel lines cut by a diagonal line.
Of the eight angles formed only one angle is known.
Is it possible to determine all the angles knowing the one angle?

PREDICT

$\angle$ h = _____



Explain how you made this prediction.
On what basis do you think so?

_____
_____
_____
_____

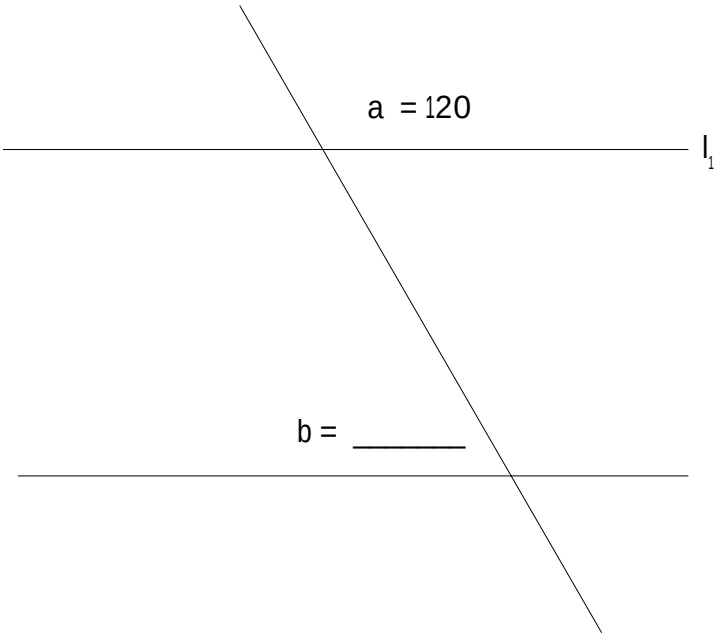
MEASURE

$\angle$ h = _____

The diagram below is of two parallel lines cut by a diagonal line.
Of the eight angles formed only one angle is known.
Is it possible to determine all the angles knowing the one angle?

PREDICT

$\angle$ b = _____



Explain how you made this prediction.
On what basis do you think so?

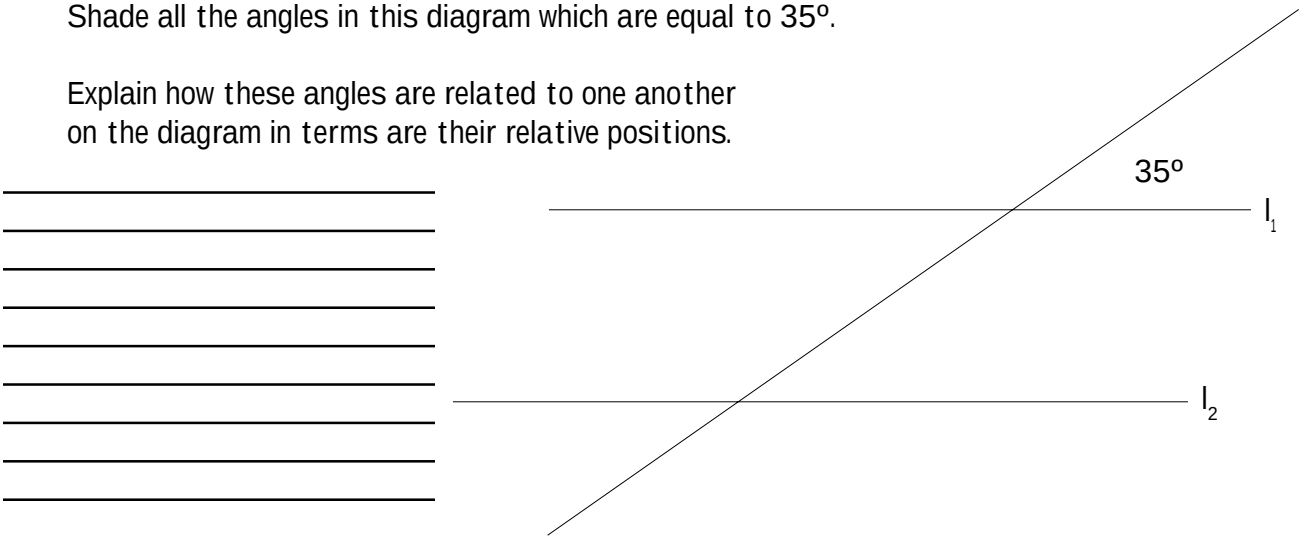
_____
_____
_____
_____

MEASURE

$\angle$ b = _____

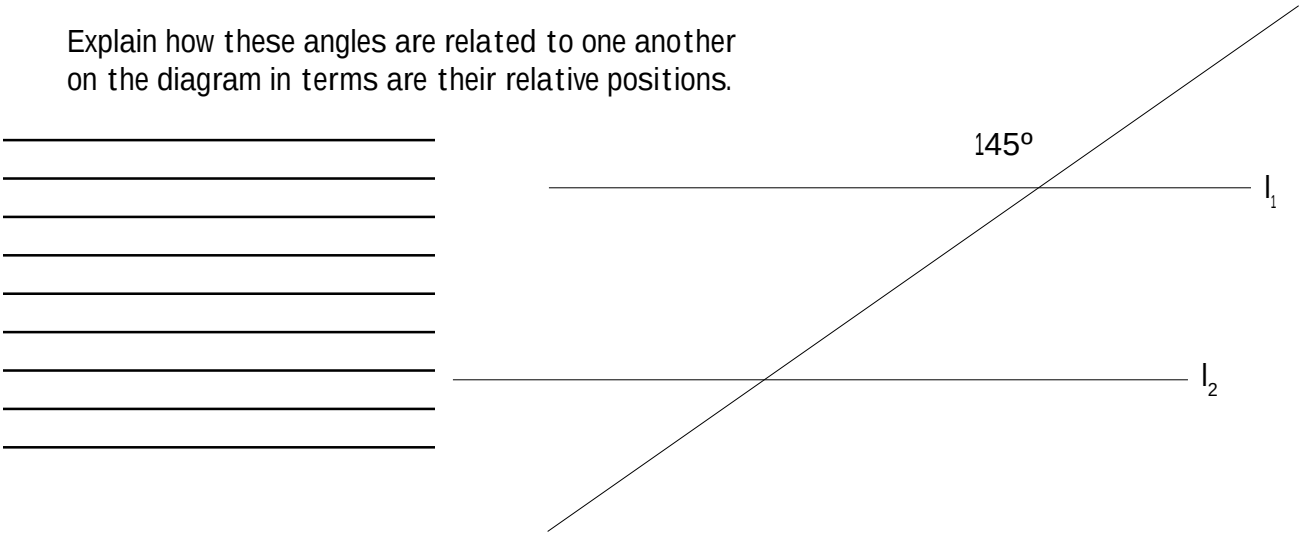
Shade all the angles in this diagram which are equal to 35º.

Explain how these angles are related to one another on the diagram in terms are their relative positions.



Shade all the angles in this diagram which are equal to 145º.

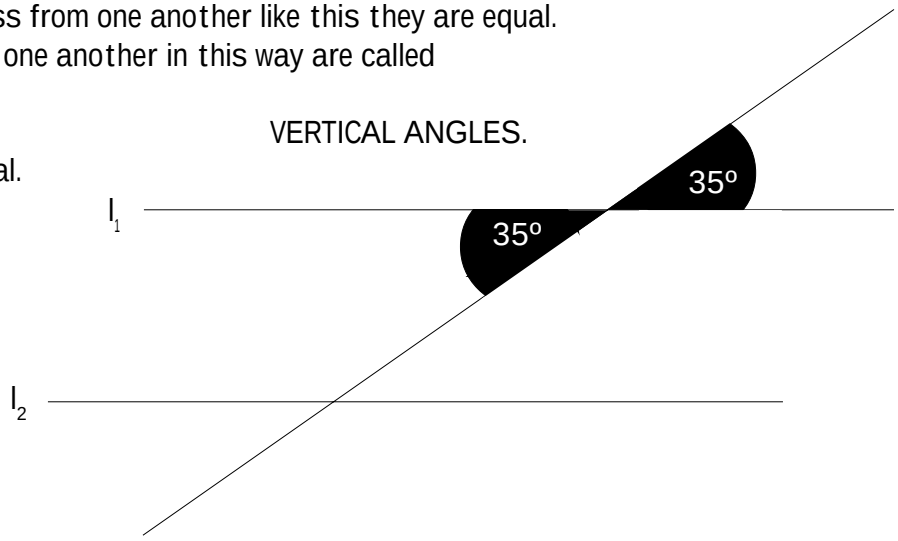
Explain how these angles are related to one another on the diagram in terms are their relative positions.



When two angles are across from one another like this they are equal.
Angles which are opposite one another in this way are called

VERTICAL ANGLES.

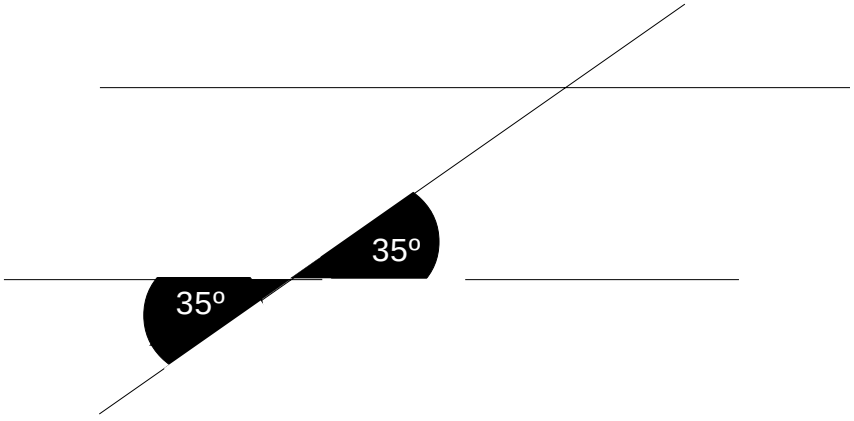
All vertical angles are equal.



Since these two angles are opposite on another also …

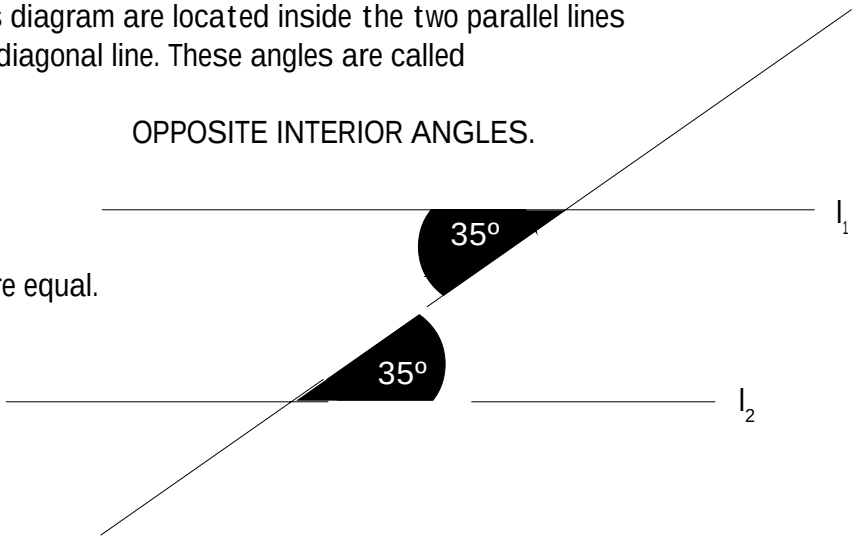
Then they are also VERTICAL ANGLES. …

Therefore they too must be _____ .



The two angles shaded in this diagram are located inside the two parallel lines and on opposite sides of the diagonal line. These angles are called

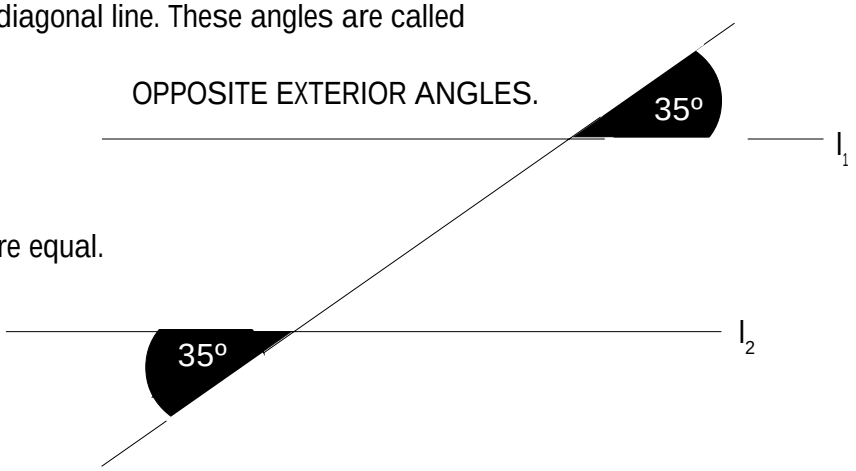
OPPOSITE INTERIOR ANGLES.



All opposite interior angles are equal.

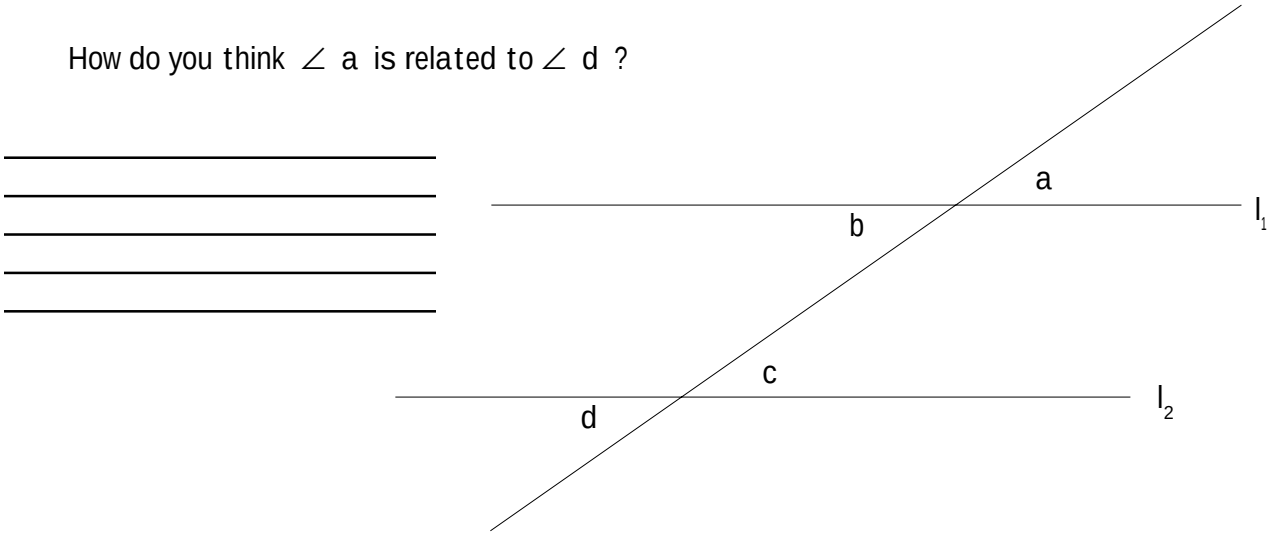
The two angles shaded in this diagram are located outside the two parallel lines and on opposite sides of the diagonal line. These angles are called

OPPOSITE EXTERIOR ANGLES.



All opposite exterior angles are equal.

How do you think ∠ a is related to ∠ d ?



THE RELATIONSHIP _____	THE REASON _____
SINCE ∠ a = ∠ b	because VERTICAL ANGLES ARE EQUAL ….
SINCE ∠ b = ∠ c	because OPPOSITE INTERIOR ANGLES ARE EQUAL ….
THEN ∠ a = ∠ c	because both of these angles are equal to ∠ b ….
SINCE ∠ c = ∠ d	because VERTICAL ANGLES ARE EQUAL ….
THEREFORE ∠ a = ∠ d	because both angles are equal to ∠ c .

MEASURE
Use your protractor to verify these relationships.

∠ a = _____

∠ b = _____

∠ c = _____

∠ d = _____

Do these measurements confirm these relationships? YES NO

Chapter 3: The Triangle

EXPLORING THE IDEA

Back to the Fort....
After much discussion and consideration, your little brother decides that he wants to use both the rope ladder and dad's ladder in his fort. The diagram below represents this relationship.

FORT DESIGNS
with rope and ladder combination

Darken the line segments $\overline{DT}$, $\overline{DR}$, and $\overline{RT}$.

What are three characteristics of this shape?

1 - This shape is completely closed.

2 - The number of sides is _____ .

3 - The number of interior angles is _____ .

4 - Measure each of the interior angles:

m∠ 1 = _____

m∠ 2 = _____

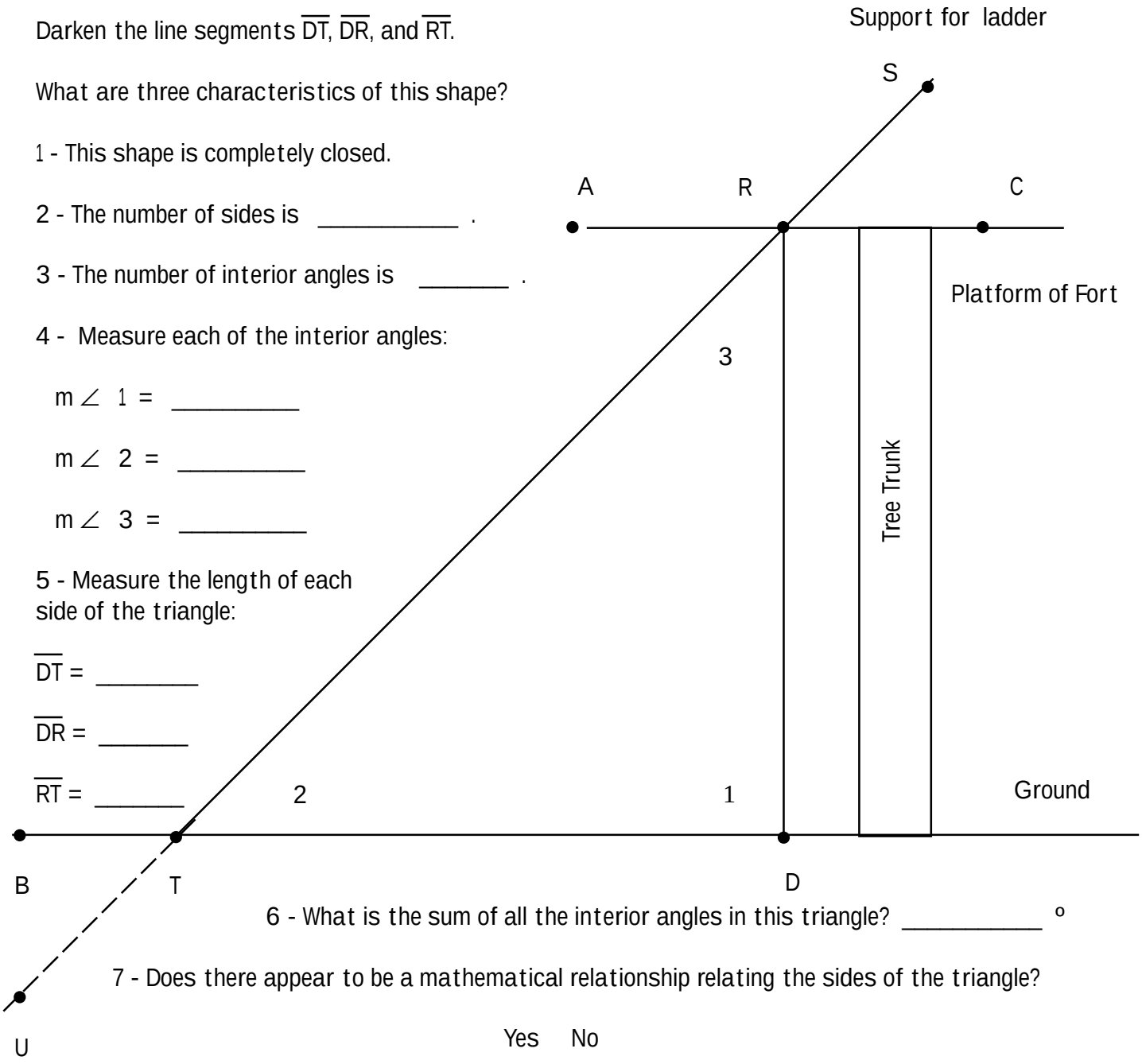
m∠ 3 = _____

5 - Measure the length of each side of the triangle:

$\overline{DT}$ = _____

$\overline{DR}$ = _____

$\overline{RT}$ = _____



6 - What is the sum of all the interior angles in this triangle? _____ °

7 - Does there appear to be a mathematical relationship relating the sides of the triangle?

Yes No

EXPLAINING THE IDEA

The word TRIANGLE means 'THREE ANGLES.

Every triangle has three interior angles.

Consequently, every triangle has three sides.

The symbol used to represent a triangle is $\triangle$.

There are several other characteristics of triangles that are of interest:

1 - the measurement of each angle;

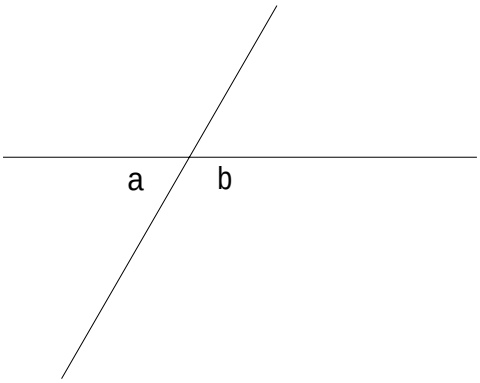
2 - the shape of the triangles;

3 - the length of the sides.

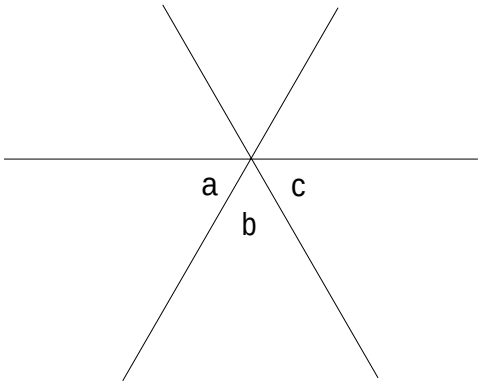
These characteristics are closely interrelated

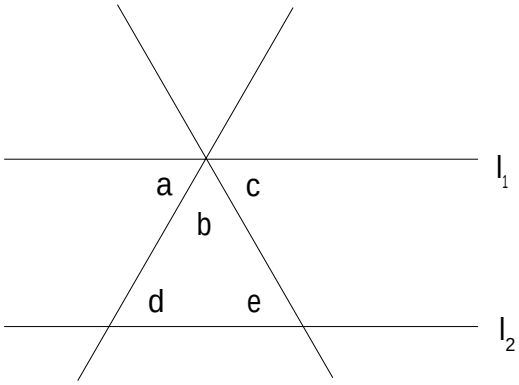
Is it possible to determine the measurement of the three angles without using a protractor?

Is it possible to determine the lengths of the three sides of a triangle without a ruler?



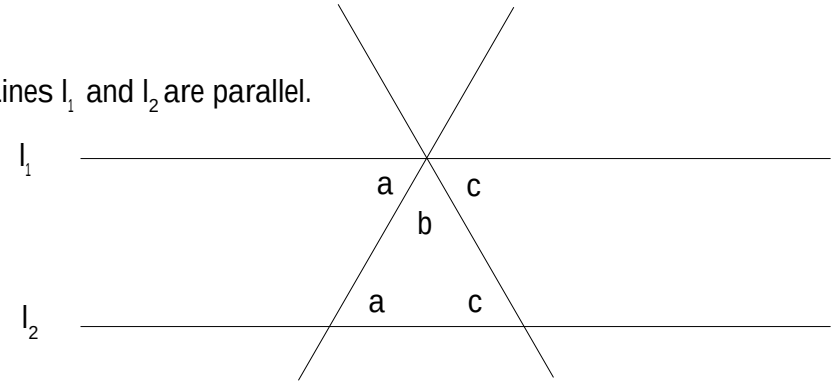
The angle of a straight line is 180°





Lines l_1 and l_2 are parallel.

Lines l_1 and l_2 are parallel.



Triangles come in a variety of shapes and sizes. Will the interior angles always add to 180º?

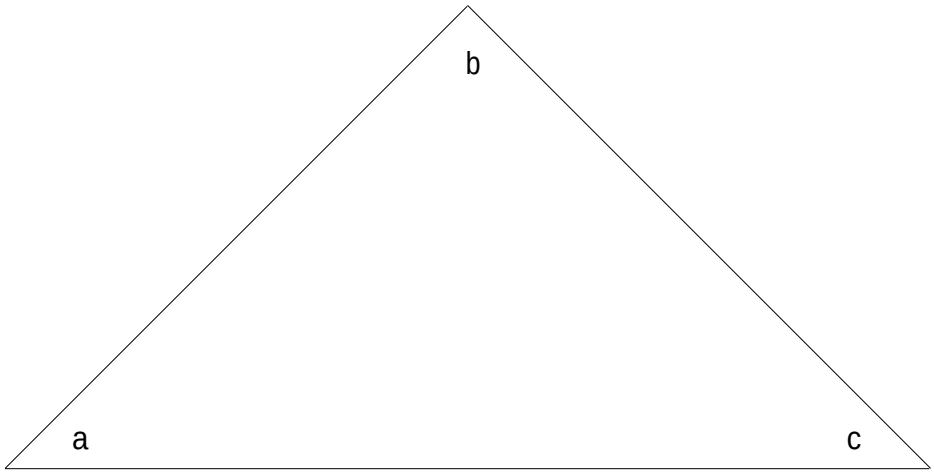
Using your protractor precisely measure the angles of the following five triangles.

Triangle 1

$\angle a =$ _____

$\angle b =$ _____

$\angle c =$ _____



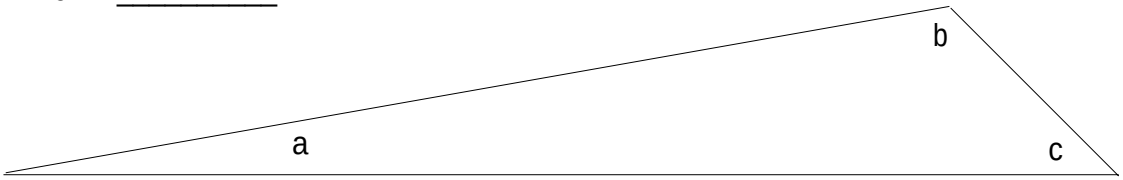
$\angle a + \angle b + \angle c =$ _____ + _____ + _____ = _____ .

Triangle 2

∠ a = _____

∠ b = _____

∠ c = _____

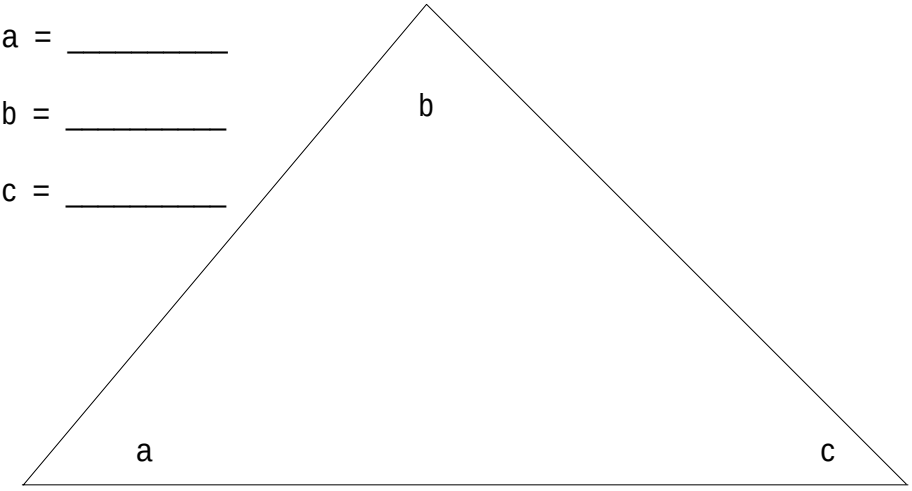


Triangle 3

∠ a = _____

∠ b = _____

∠ c = _____

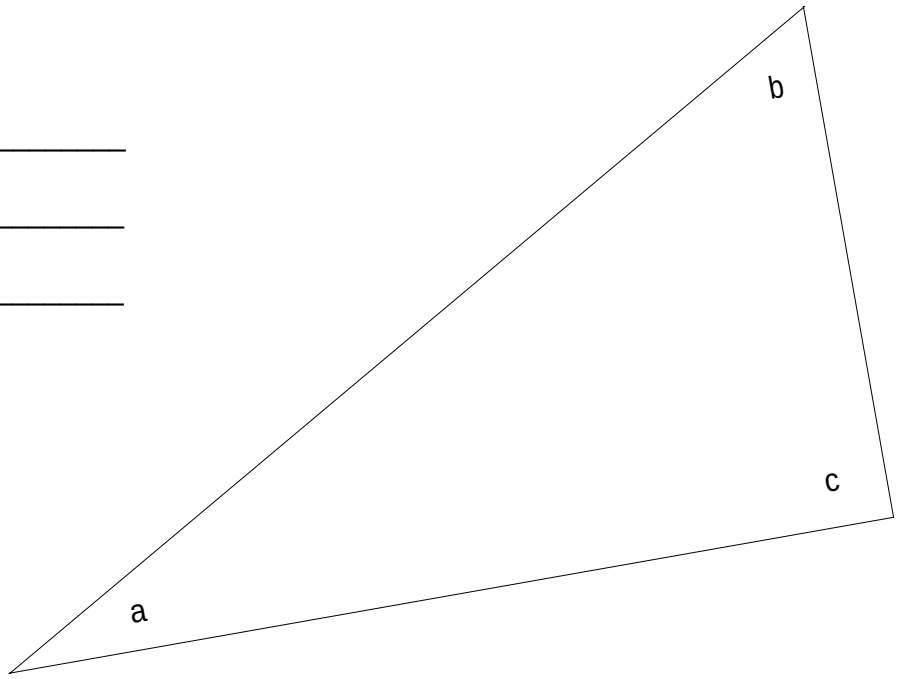


Triangle 4

$\angle$ a = _____

$\angle$ b = _____

$\angle$ c = _____

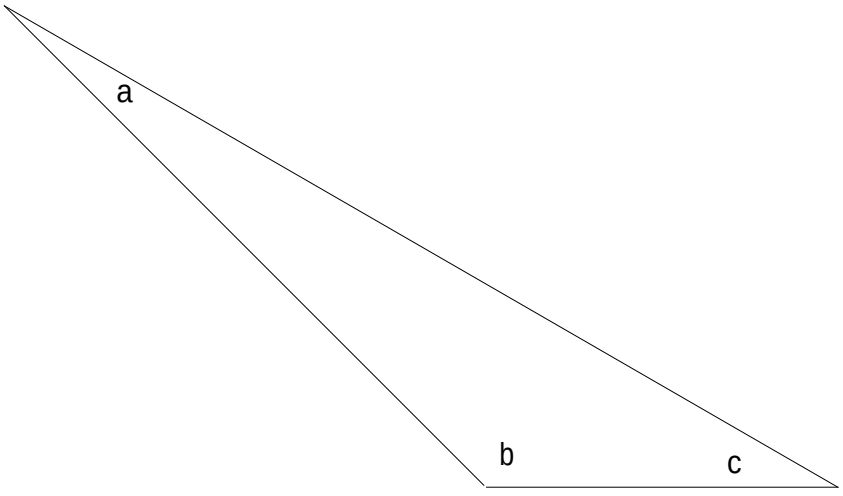


Triangle 5

$\angle$ a = _____

$\angle$ b = _____

$\angle$ c = _____



Transfer your measurements to the chart below:

Triangles

	1	2	3	4	5
∠ a					
∠ b					
∠ c					
∠ a + ∠ b + ∠ c					

Characteristics of Triangles

A triangle has how many sides? _____

A triangle has how many interior angles? _____

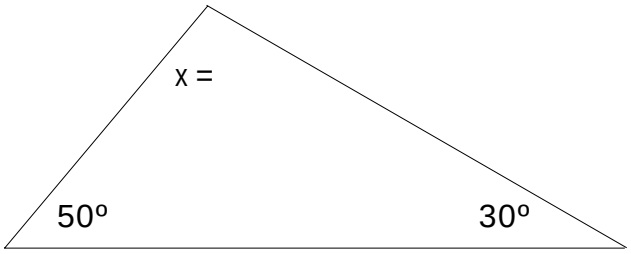
The some of the interior angles of a triangle is _____ ?

EXPANDING THE IDEA - Part 2

Using what you know about angles of triangles and straight lines, predict the missing angles.

ANGLE
Prediction

1. ∠ x = _____



State the principle that you used to make your prediction:

Now actually measure the angle using your protractor.

ANGLE
Measurement

∠ x = _____

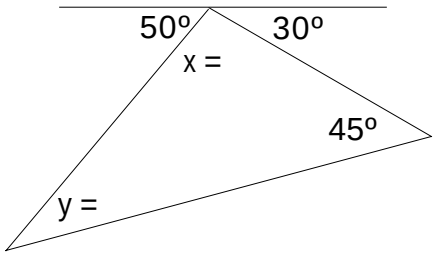
Was your prediction correct? Yes No

If your prediction was not correct, explain why?

ANGLE
Prediction

2. ∠ x = _____

∠ y = _____



What do you know about the angle of a straight line that will help you determine the value of angle x?

Now actually measure the angles using your protractor.

ANGLE
Measurements

∠ x = _____ ∠ y = _____

Was your prediction correct? Yes No

If your prediction was not correct, explain why?

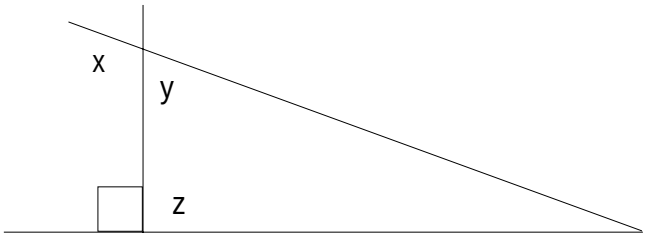
_____

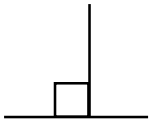
_____

ANGLE
Predictions

3. Predict the sum of

∠x + ∠y + ∠z = _____



This symbol,  , indicates an angle of 90º.

Use your knowledge of the angle of a straight line to help.

Now, actually measure the angles using your protractor.

Was your prediction correct? Yes No

ANGLE
Measurement

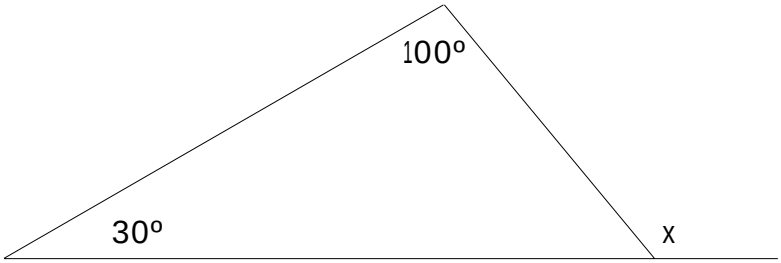
If your prediction was not correct, explain why?

∠x + ∠y + ∠z = _____ _____

_____

ANGLE
PREDICTION

4. $\angle$ x = _____



First, find the value of the unknown interior angle of the $\triangle$.

Second, use your knowledge of the straight line to calculate m $\angle$ x .

Now actually measure the angles using your protractor.

ANGLE
Measurement

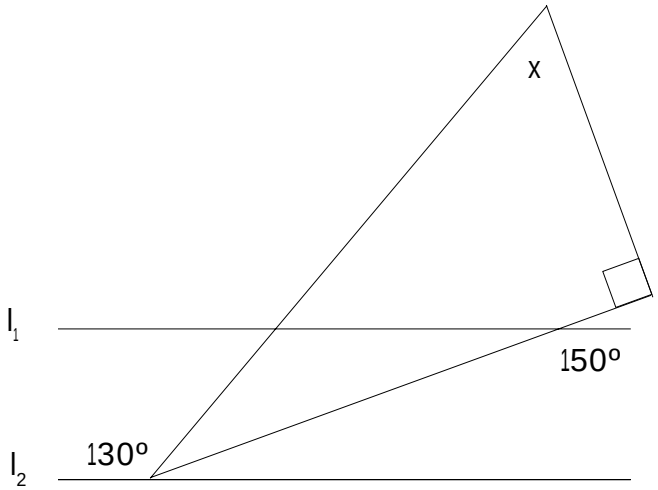
$\angle$ x = _____

Was your prediction correct? Yes No

If your prediction was not correct, explain why?

Lines l_1 and l_2 are parallel.
ANGLE PREDICTIONS

5. $\angle$ x = _____



Hint 1:
think about opposite interior angles of a line intersecting two parallel lines.

Hint 2:
think about the sum of all interior angles of a triangle.

Now, actually measure the angles using your protractor.

ANGLE
Measurement

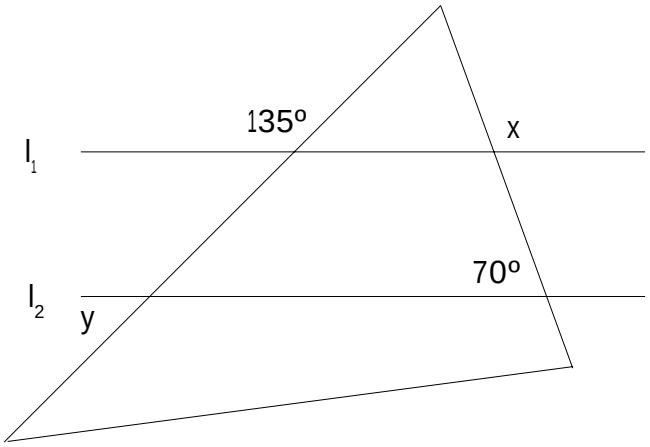
$\angle$ x = _____

Lines l_1 and l_2 are parallel.
ANGLE PREDICTION

6. Find $\angle x + \angle y =$ _____

Shade the other angles equal to 135º in one color and the angles equal to 70º in a different color.

How will that help you?



Now actually measure the angles using your protractor.

ANGLE
MEASUREMENTS

$\angle x =$ _____ $\angle y =$ _____ $\angle x + \angle y =$ _____

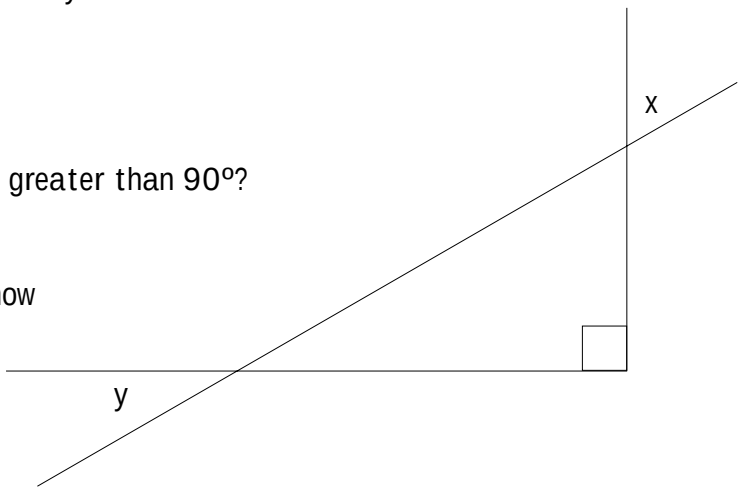
Was your prediction correct? Yes No

If your prediction was not correct, explain why?

ANGLE
Prediction

7. Is $\angle x + \angle y$ less than, equal to, or greater than 90º?

To solve this problem, recall what you know regarding Vertical Angles. Right Angles, and the Interior Angles of a triangle.



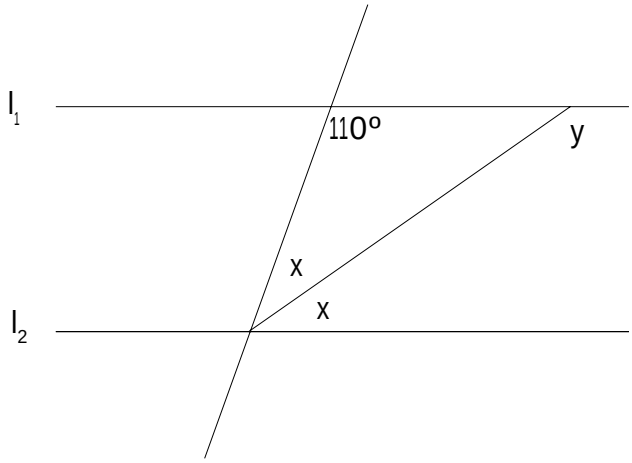
Now actually measure the angles using your protractor.

ANGLE
Measurements

$\angle x =$ _____ $\angle y =$ _____ $\angle x + \angle y =$ _____

Was your prediction correct? Yes No

Lines l_1 and l_2 are parallel.
ANGLE PREDICTION



8. ∠ x = _____

∠ y = _____

Shade other angles that are equal to 110°.

What is the angle of a straight line? _____ ° .

How will this help you determine angle x?

What is the sum of the interior angles of a triangle? _____ ° .

How will that help you determine angle y?

Now actually measure the angles using your protractor.

ANGLE
MEASUREMENT

∠ x = _____ ∠ y = _____

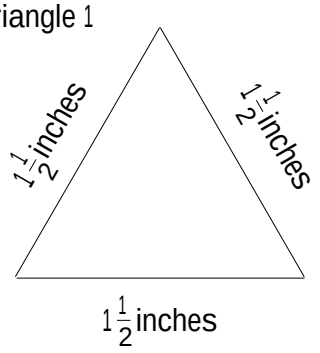
Was your prediction correct? Yes No

If your prediction was not correct, explain why?

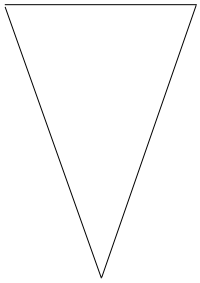
Expanding the Idea - Part 3

Observe each triangle below
With a ruler measure the sides of each triangle.
The first triangle is done for you.

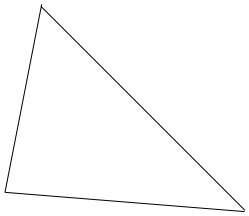
Triangle 1



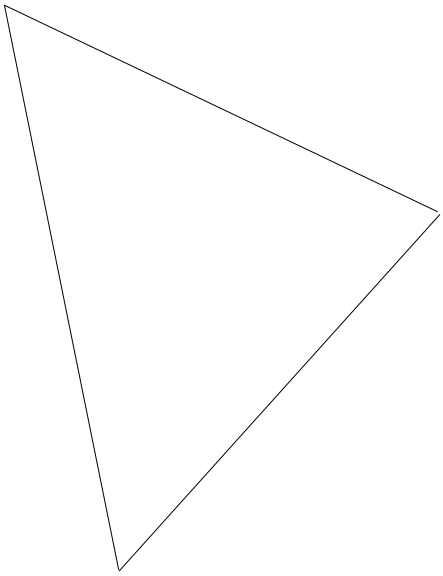
Triangle 2



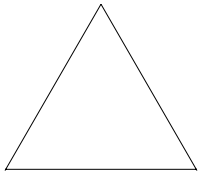
Triangle 3



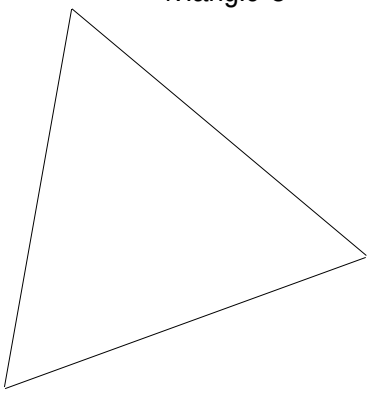
Triangle 4



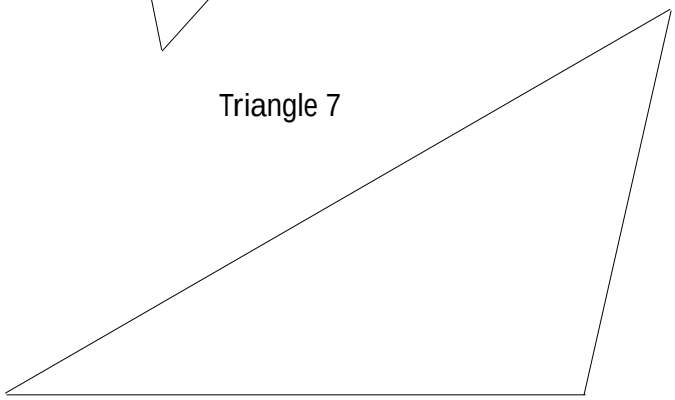
Triangle 5



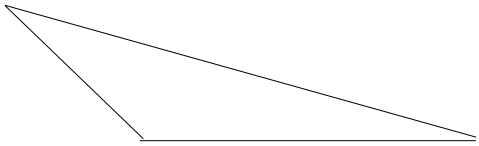
Triangle 6



Triangle 7



Triangle 8



Which triangles were alike?

Triangle 1 was like … _____

How were these triangles alike?

_____
_____
_____
_____

Triangle 2 was like … _____

How were these triangles alike?

_____
_____
_____
_____

Triangle 3 was like … _____

How were these triangles alike?

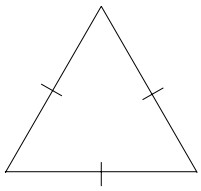
_____
_____
_____
_____

Triangles can be grouped according to their sides.

A triangle with all three sides equal in measurement is called an

EQUILATERAL TRIANGLE.

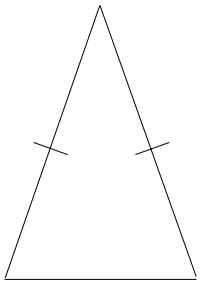
The triangle is marked to show that all three sides are equal.



A triangle with two sides equal in measurement is called a

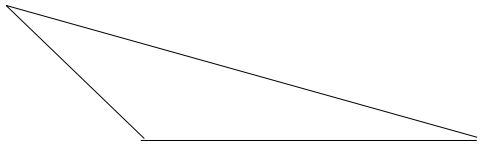
ISOSCELES TRIANGLE.

The triangle is marked to show that all three sides are equal.



A triangle with no sides equal in measurement is called a

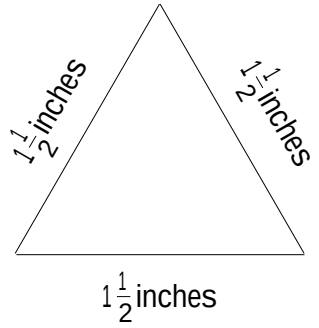
SCALENE TRIANGLE.



Mark the sides which are equal in length with the slash marking.
Identify each triangle as Equilateral, Isosceles, or Scalene.

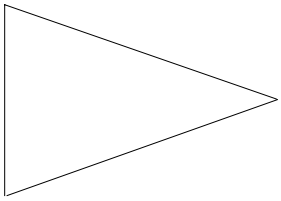
Triangle 1 is an

_____ triangle.



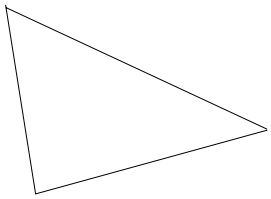
Triangle 2 is an

_____ triangle.



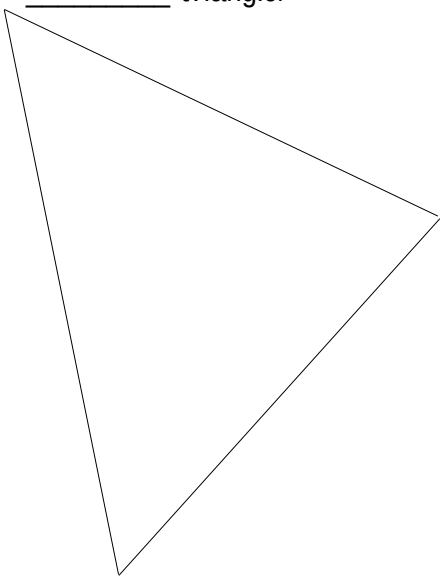
Triangle 3 is an

_____ triangle.



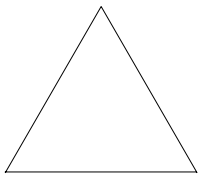
Triangle 4 is an

_____ triangle.



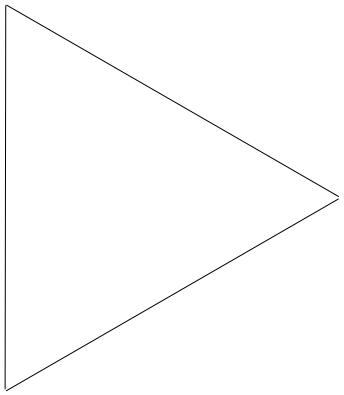
Triangle 5 is an

_____ triangle.



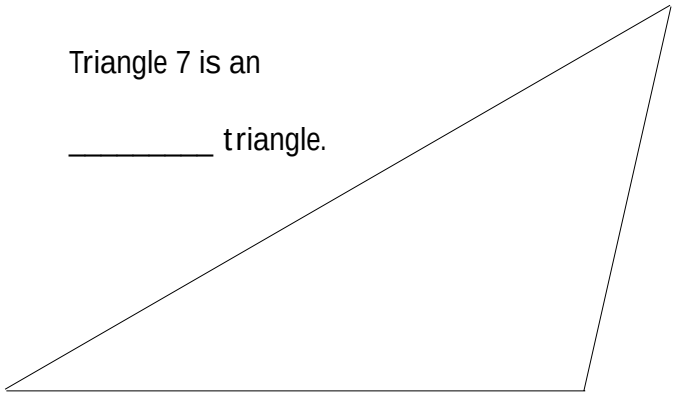
Triangle 6 is an

_____ triangle.



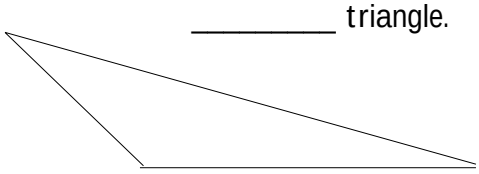
Triangle 7 is an

_____ triangle.



Triangle 8 is an

_____ triangle.



Draw three examples of each of the three kinds of triangles.

EQUILATERAL TRIANGLES

ISOSCELES TRIANGLES

SCALENE TRIANGLES

Expanding the Idea - Part 4

Did you find it difficult to construct any of the triangles on the previous page?

Constructing a specific triangle requires that you use your knowledge of the characteristics of each triangle.

Rest ate the major characteristic of an equilateral triangle:

The problem is to construct an equilateral triangle using only a straight edge (if you use a ruler turn it over so that you are unable to read the markings) and a compass.

Connect Point A to Point B using your straight edge.

Is it possible to make line segment XY exactly the same length as line segment AB without using a ruler to measure the distance? You may use your compass and straight edge.

Do you need some help?
Using your compass measure the distance between Point A and Point B by placing the endpoint of the compass on Point A and the pencil point on POINT B.

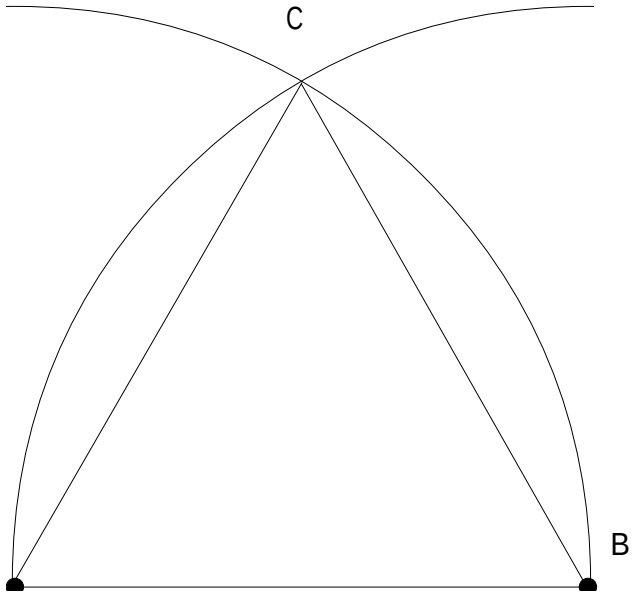
Next, without expanding the distance of the compass place the end point of the compass on Point X. Now draw an arc (a part of a circle). Every point along the arc is equal distance from Point X which is the same distance from Point X as Point B is away from Point A. You can now use your straight edge to connect Point X any where along the arc. Mark this as Point Y.

Next, using your ruler measure the line segment AB and line segment XY.
Are they equal or not equal?

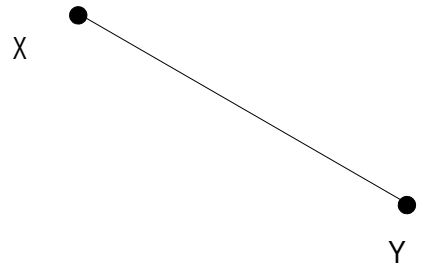
Now you know how to make a line of equal distance to another line.
Because an equilateral triangle is a triangle with all three sides equal, it is now possible to draw an equilateral triangle with only using a straight edge and a compass. Complete the diagram below by making an equilateral triangle ABC.

Do you need some help?
See the directions on the following page.

Place the end point of the compass at Point A and the pencil point at Point B.
Moving counterclockwise draw an arc (less than a half circle).
Next, place the end point of the compass at Point B and the pencil point at Point A.
Moving clockwise draw an arc (again less than a half a circle).
Where the two arc meet is Point C.
Connect Point A to Point C and connect Point C to Point B.
With a ruler measure each side of the triangle.
Are the three sides equal in length? Yes No
If not, explain why not?




X
Y


X
Y

Restate the major characteristic of an isosceles triangle:

The problem is to construct an isosceles triangle using only a straight edge (if you use a ruler turn it over so that you are unable to read the markings) and a compass.

Connect Point A to Point B using your straight edge.
Let this be the side of the triangle that is not equal to the other two sides
and let this side of the triangle be shorter than the other two sides.
Using your compass find Point C.
Then connect line segments AC and CB.

Do you need some help?
Line segment AB is to be shorter than line segments AC and BC
Line segments AC and BC must be equal.
Using your compass measure a distance greater than line segment AB by placing the endpoint of the compass on Point A and the pencil point some distance beyond POINT B.

Moving counterclockwise draw an arc (less than a half circle).
Next, without expanding or shortening the distance of the compass place the end point of the compass at Point B and the pencil point beyond Point A.
Moving clockwise draw an arc (again less than a half a circle).

Where the two arc meet is Point C.
Connect Point A to Point C and connect Point C to Point B.
With a ruler measure each side of the triangle.

Is line segment AC and line segment CB equal or not equal?

Is line segment AB less than, greater than, or equal to line segments AC and CB?

Given the Point X and the Point Y construct another isosceles triangle XYZ.
Let this be the side of the triangle that is not equal to the other two sides and let this side of the triangle be longer than the other two sides.

With a ruler measure each side of the triangle.

Is line segment XZ and line segment YZ equal or not equal?

Is line segment XY less than, greater than, or equal to line segments XZ and YZ?

Rest ate the major characteristic of a scalene triangle:

The problem is to construct a scalene triangle using only a straight edge (if you use a ruler turn it over so that you are unable to read the markings) and a compass.

Connect Point A to Point B using your straight edge.
Let line segment AB be the longest side of the triangle.
Using your compass find a Point C such that AC is not equal to AB, AC is not equal to CB, and CB is not equal to AB.
Then connect line segments AC and CB.

With a ruler measure each side of the triangle.

Is line segment AB and line segment AC equal or not equal?

Is line segment AB and line segment CB equal or not equal?

Is line segment AC and line segment CB equal or not equal?

Restate the major characteristic of each of the three triangles:

EQUILATERAL TRIANGLES

ISOSCELES TRIANGLES

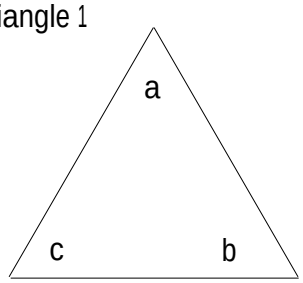
SCALENE TRIANGLES

Expanding the Idea - Part 5

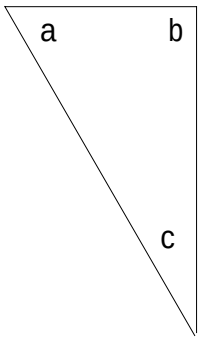
In addition to grouping triangles according to the measurement of the length of the sides, triangles are also grouped according to the measure of their angles.

Observe each triangle below.
With a protractor measure the angles of each triangle.
Record your data in the chart on the next page.

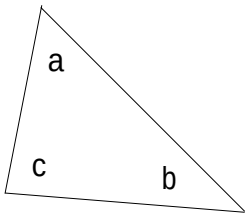
Triangle 1



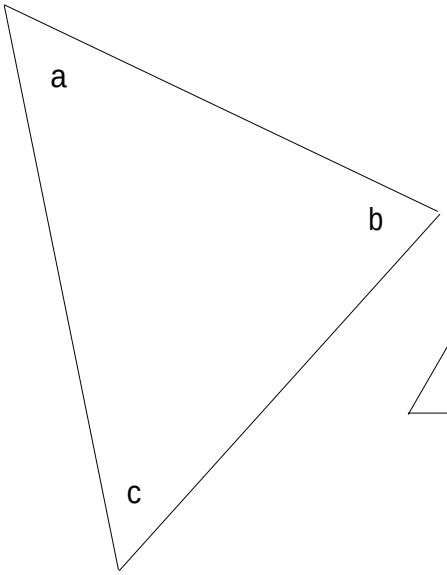
Triangle 2



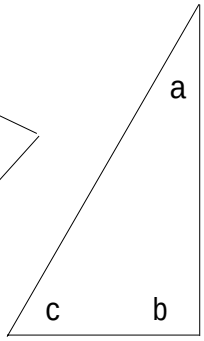
Triangle 3



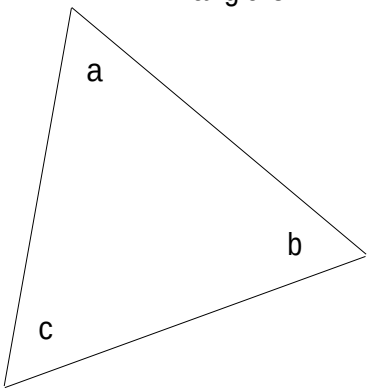
Triangle 4



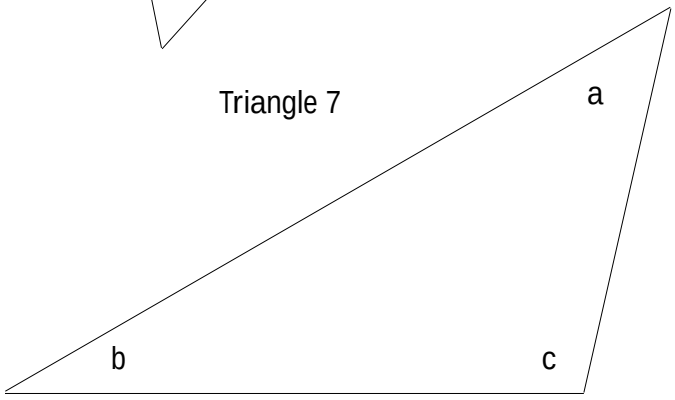
Triangle 5



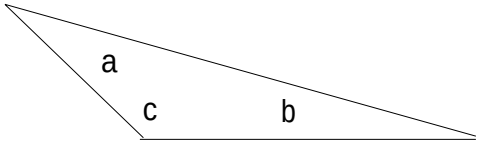
Triangle 6



Triangle 7



Triangle 8



	Angle a	Angle b	Angle c
Triangle 1			
Triangle 2			
Triangle 3			
Triangle 4			
Triangle 5			
Triangle 6			
Triangle 7			
Triangle 8			

Place the triangles into four groups based upon similar angles.

List Triangles that go together:

Describe the characteristic you used to group the triangles. (HINT: Watch for angles that are equal to 90º, less than 90º, or greater than 90º.)

GROUP 1:

GROUP 2:

GROUP 3:

GROUP 4:

Just as triangles can be grouped based upon the length of the sides, they can also be classified based upon the interior angles of the triangle.

A triangle that has a right angle (90º) in its interior is called a

RIGHT TRIANGLE.

A triangle having an interior angle greater than 90º but less than 180º is called an

OBTUSE TRIANGLE.

A triangle having all interior angles less than 90º is called an

ACUTE TRIANGLE.

A triangle having all interior angles of equal measurement is called an

EQUIANGULAR TRIANGLE.

In the chart on the previous page, name each of the 8 triangles based upon the interior angles of each triangle.

Triangles may be grouped or classified in one of two ways —
They may be grouped according to …

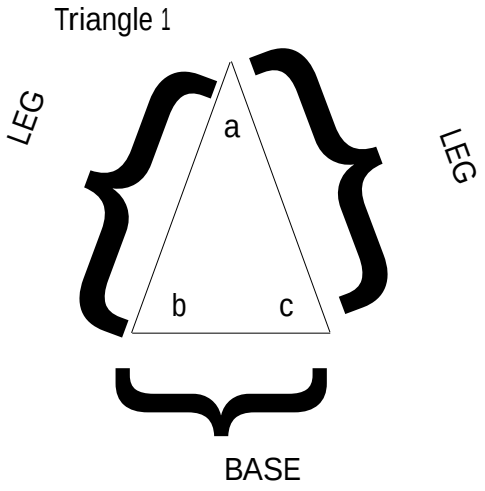
1 - _____ , or

2 - _____ .

Expanding the Idea - Part 6

Special Names for Sides of Triangles

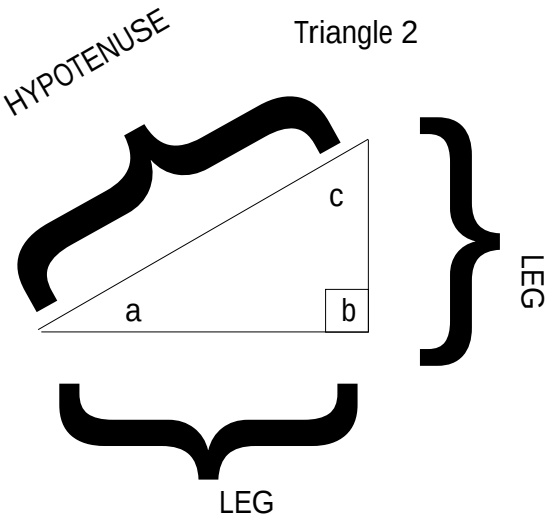
Triangle 1 is an ISOSCELES TRIANGLE.



The short side is called the BASE.

The two equal sides are called the LEGS.

Triangle 2 is a RIGHT TRIANGLE.



In a right triangle, the side opposite the right angle is called the HYPOTENUSE.

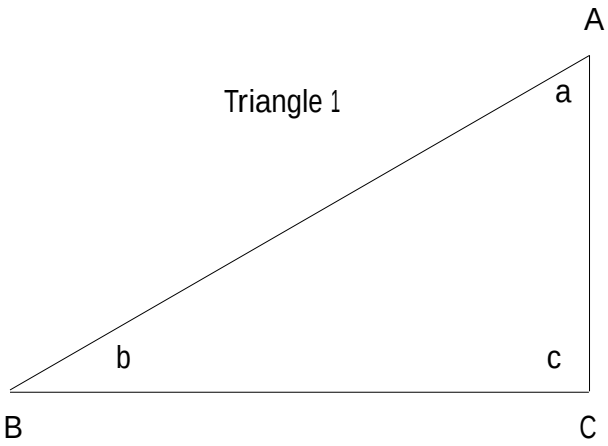
The other two sides are called LEGS

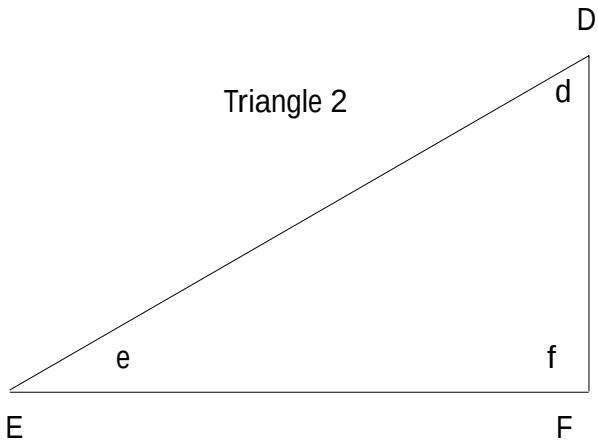
Chapter 4: The Same

Observing the Idea

Materials
protractor
ruler

Using your ruler and protractor precisely measure the sides and angles of the two triangles that are drawn below.





Triangle 1

AB = _____

BC = _____

AC = _____

∠ a = _____

∠ b = _____

∠ c = _____

Triangle 2

DE = _____

EF = _____

EF = _____

∠ d = _____

∠ e = _____

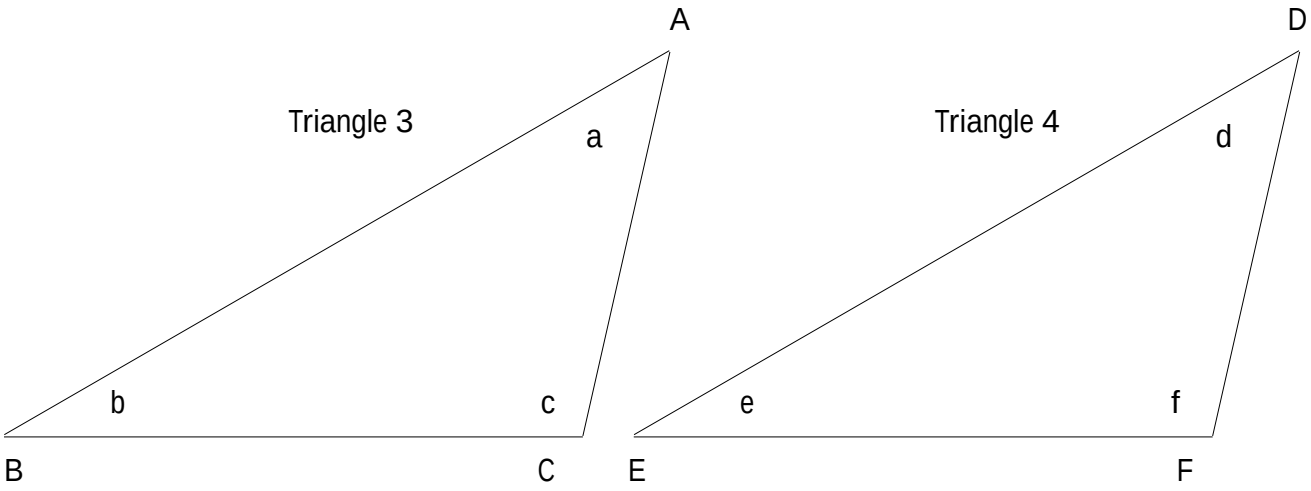
∠ f = _____

Using tracing paper, trace △ABC . Then lay it over △ DEF.
How are Triangles 1 and 2 alike?

_____

_____

Using your ruler and protractor precisely measure the sides and angles of these two triangles that are drawn below.



Triangle 3

AB = _____

BC = _____

AC = _____

∠ a = _____

∠ b = _____

∠ c = _____

Triangle 4

DE = _____

EF = _____

EF = _____

∠ d = _____

∠ e = _____

∠ f = _____

Make the following comparisons: Write = or ≠ in each box.

AB ☐ DE

BC ☐ EF

AC ☐ EF

∠ a ☐ ∠ d

∠ b ☐ ∠

∠ c ☐ ∠ f

Using tracing paper, trace △ABC . Then lay it over △ DEF.
How are Triangles 3 and 4 alike?

Naming the Idea

Some triangles are exactly the same size and shape. When the three sides and the three angles of one triangle have the same measurements as the three sides and three angles of a second triangle, the two triangles are called

CONGRUENT TRIANGLES.

The symbol to show that two triangles are congruent is $\cong$.

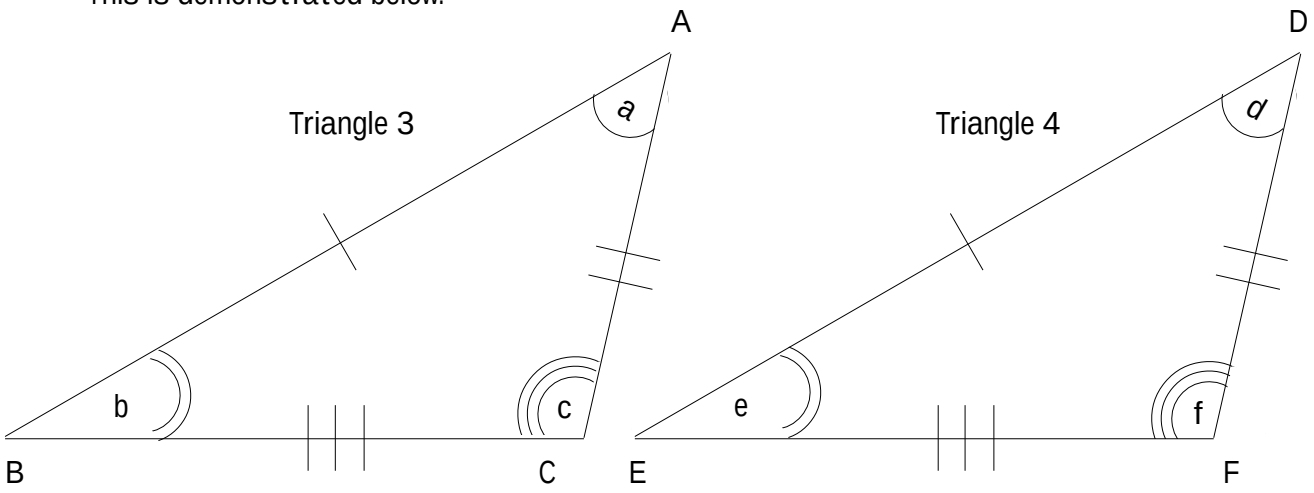
In the previous problem $\triangle ABC \cong \triangle DEF$.

The parts of the two triangles that have the same measurements are called

CORRESPONDING PARTS.

It naturally follows that Corresponding Parts of Congruent Triangles are equal.

To indicate which sides are the same , small markings are placed upon the corresponding sides.
Straight lines are used to show congruent sides.
Curved lines indicate congruent angles.
This is demonstrated below.



The single line crossing AB indicates that it is congruent to DE.
The double line crossing AC indicates _____ .
The triple line crossing BC indicates _____ .

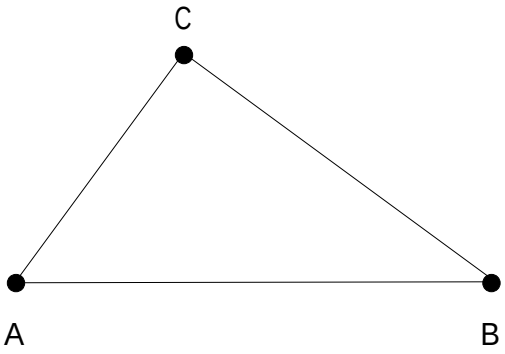
The single curved line indicates that $\angle$ a is congruent to $\angle$ d
The double curved line indicates _____ .
The triple curved line indicates _____

Expanding the Idea — Part 1

Is it possible to construct a triangle congruent to a given triangle within tracing it?

Using only a straight edge and a compass construct $\triangle XYZ$ in such a way that it is congruent to $\triangle ABC$.

Begin by constructing line segment XY that is congruent to line segment AB.
Using your knowledge of constructing lines, continue constructing $\triangle XYZ$.

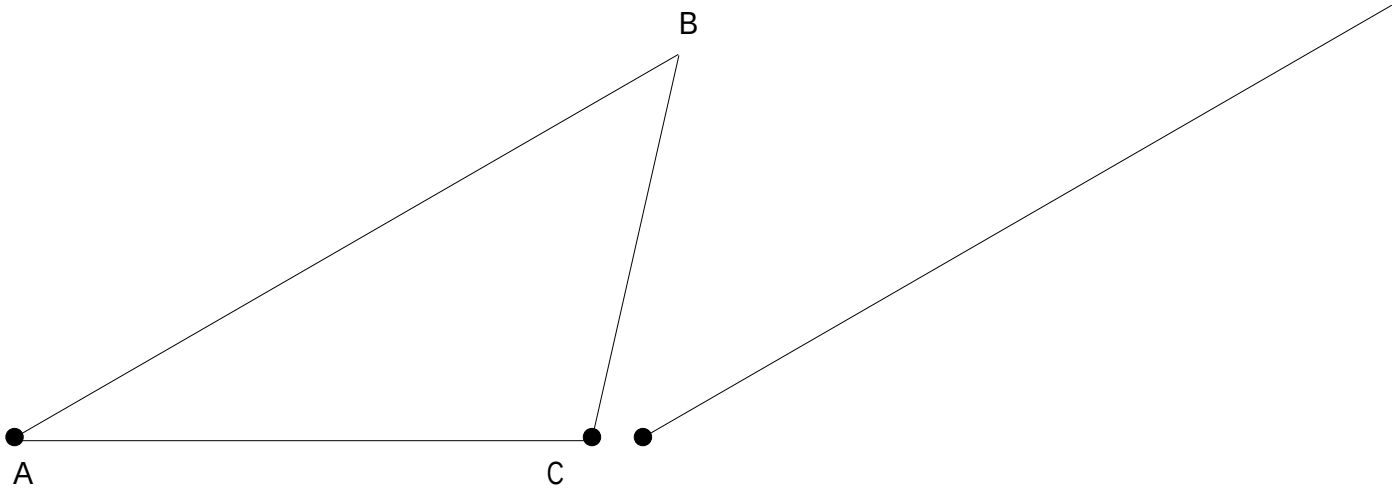


Measure the corresponding sides and angles of $\triangle ABC$ and $\triangle XYZ$.
Are the two triangles congruent? Yes No
How do you know?
Using tracing paper, trace $\triangle ABC$. Set that image on top of $\triangle XYZ$. Are they identical?

Do you need help making your construction?
Place the endpoint of the compass at Point A and the pencil point at Point B.
Move the compass point to Point X. Strike an arc across the line segment.
Where the arc crosses the line segment becomes Point Y.
Move the compass point back to Point A and the pencil point to Point C.
Move the compass point back to Point X. Strike an arc above the line segment XY.
Move the compass point to Point B and the pencil point to Point C.
Move the compass point to Point Y. Strike an arc so that this arc crosses the other arc.
The place where the two arcs intersect becomes Point Z.
Connect Points X and Z. Connect Points Z and Y.
Measure the three corresponding sides and angles of $\triangle ABC$ and $\triangle XYZ$.
Are the two triangles congruent? Yes No

Using only a straight edge and a compass construct $\triangle DEF$ in such a way that it is congruent to $\triangle ABC$.

Begin by constructing line segment XY that is congruent to line segment AB.
Using your knowledge of constructing lines, continue constructing $\triangle DEF$.



Measure the corresponding sides of $\triangle ABC$ and $\triangle DEF$.
Measure the corresponding angles of $\triangle ABC$ and $\triangle DEF$.
Are the two triangles congruent? Yes No
How do you know?
Using tracing paper, trace $\triangle ABC$. Set that image on top of $\triangle DEF$. Are they identical?

Do you need help making your construction?
On the line with endpoint D construct a segment congruent to line segment AB.
Label the other endpoint E.
Draw an arc with center D and radius congruent to line segment AC.
Draw an arc with center E and radius congruent to line segment BC.
Make the two arcs intersect. Label the point where they intersect Point F.
Connect DF and EF.

You now have constructed $\triangle DEF$ that is congruent with $\triangle ABC$.

Expanding the Idea — Part 2

Is it always necessary to measure all six pairs of corresponding parts of a triangle to determine if the triangles are congruent?

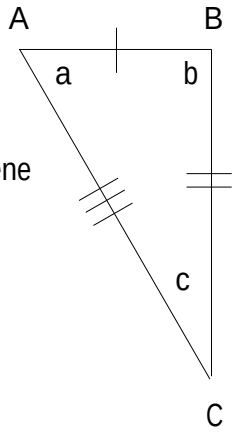
PREDICTION
IF each side of one triangle is congruent to the corresponding side of another triangle,
THEN what would you predict regarding the corresponding angles?

Equal Not Equal I Do Not Know

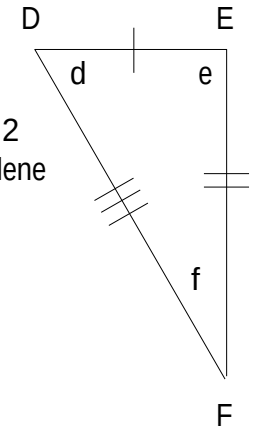
MEASURE
Using a ruler measure the sides of each triangle to determine if they are equal.
Put = or ≠ in each box below.

AB ☐ DE BC ☐ EF AC ☐ DF

Triangle 1 is a scalene triangle.



Triangle 2 is a scalene triangle.



The single line perpendicular to AB and DE indicates these corresponding segments are equal.
The double line perpendicular to BC and EF indicates these corresponding segments are equal.
The triple line perpendicular to AC and DF indicates these corresponding segments are equal.

For the triangles to be congruent the three corresponding angles must also be equal.

MEASURE
Measure the corresponding angles of each triangle to determine if they are equal.
Put = or ≠ in each box below.

∠ a ☐ ∠ d ∠ b ☐ ∠ e ∠ c ☐ ∠ f

THEREFORE it can be concluded that △ABC is congruent to △DEF.

This may be written as △ABC ≅ △DEF.

PREDICTION
Is this relationship true for the other types of triangles?
IF each side of one triangle is congruent to the corresponding side of another triangle,
THEN what would you predict regarding the corresponding angles?

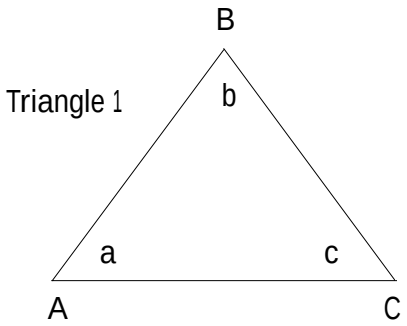
Equal Not Equal I Do Not Know

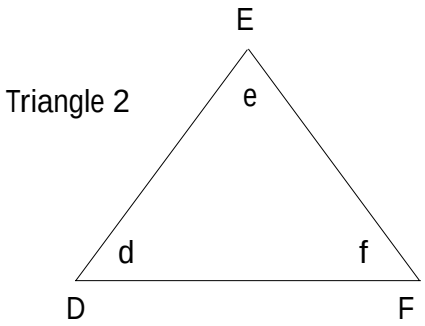
MEASURE
Measure the sides of each triangle to determine if the corresponding sides are equal.
Put = or ≠ in each box below.

AB ☐ DE BC ☐ EF AC ☐ DF

Mark the corresponding sides that are equal as on the previous page.

Triangle 1 is an _____ triangle. Triangle 2 is an _____ triangle.





MEASURE
Measure the corresponding angles of each triangle to determine if they are equal.

∠ a ☐ ∠ d ∠ b ☐ ∠ e ∠ c ☐ ∠ f

SINCE the corresponding sides are equal,
THEN the corresponding angles are also equal,

THEREFORE it can be concluded that △ABC ☐ △ DEF.

From these two examples, what pattern do you see?

IF each side of one triangle is _____

THEN the corresponding angles are _____

THEREFORE, the triangles are _____ .

TESTING YOUR IDEA

IF each side of one equilateral triangle is congruent to the corresponding side of another equilateral triangle, THEN what would you predict regarding the corresponding angles?

Equal Not Equal I Do Not Know

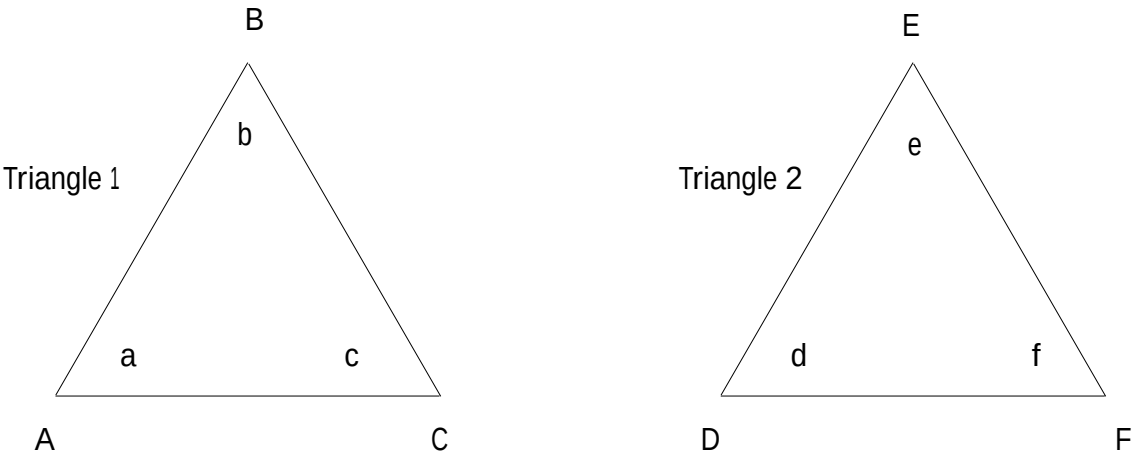
MEASURE

Measure the sides of each triangle to determine if the corresponding sides are equal.
Put = or ≠ in each box below.

AB ☐ DE BC ☐ EF AC ☐ DF

Mark the corresponding sides that are equal as on the previous page.

Triangle 1 is an _____ triangle. Triangle 2 is an _____ triangle.



MEASURE

Measure the corresponding angles of each triangle to determine if they are equal.

∠ a ☐ ∠ d ∠ b ☐ ∠ e ∠ c ☐ ∠ f

THEREFORE it can be concluded that △ABC ☐ △ DEF.

THE IDEA

IF each side of one triangle is congruent to the corresponding side of another triangle,
THEN the corresponding angles are also equal,
THEREFORE, the triangles are CONGRUENT.

This is known as the

SIDE - SIDE - SIDE RULE.

APPLYING the IDEA

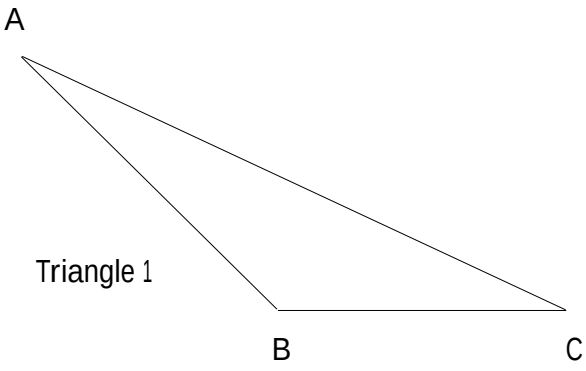
To determine if two triangles are congruent is it always necessary to measure all the sides and all the angles to see if there are equal? Yes No I Do Not Know

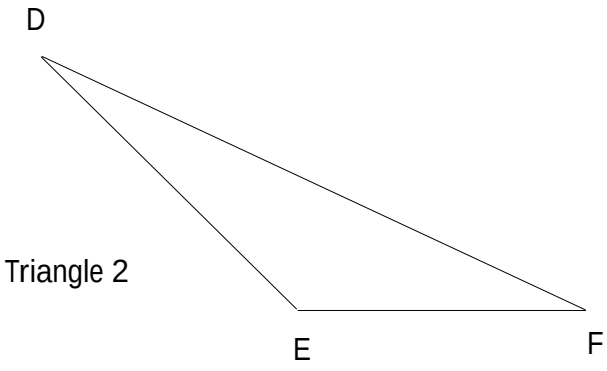
IF each side of one triangle is congruent to the corresponding side of another triangle,
THEN corresponding angles are

Equal Not Equal I Do Not Know

THEREFOR the two triangles are _____ .

Determine if the two triangles below are congruent by measuring the sides.





MEASURE

Measure the sides of each triangle to determine if the corresponding sides are equal.
Put = or ≠ in each box below.

AB ☐ DE BC ☐ EF AC ☐ DF

Mark the corresponding sides that are equal as on the previous page.

Triangle ABC is _____ to Triangle DEF

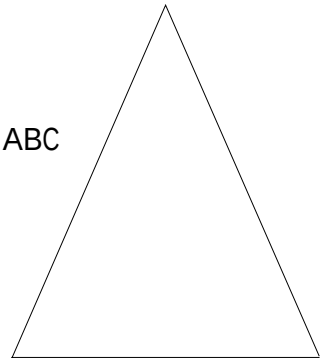
based upon the _____ - _____ - _____ Rule.

If you are uncertain as to whether the two triangles are congruent,
Then measure the three corresponding angles.

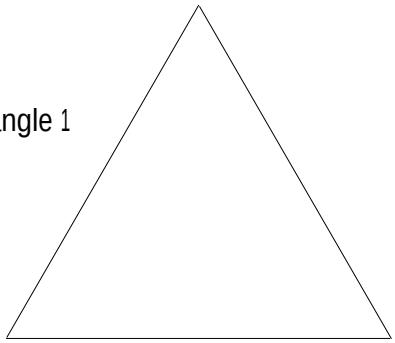
Using the SIDE- SIDE - SIDE Rule determine which triangles below are congruent to △ ABC.

△ ABC is congruent to _____ .

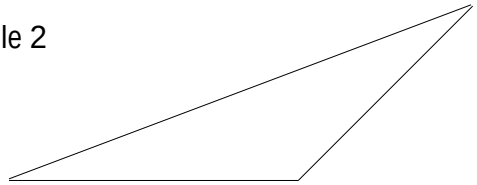
Triangle ABC



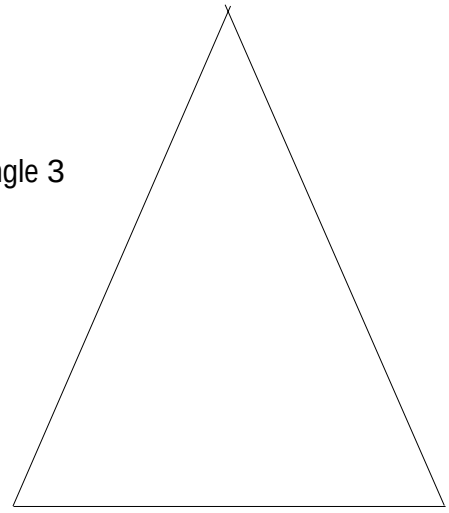
Triangle 1



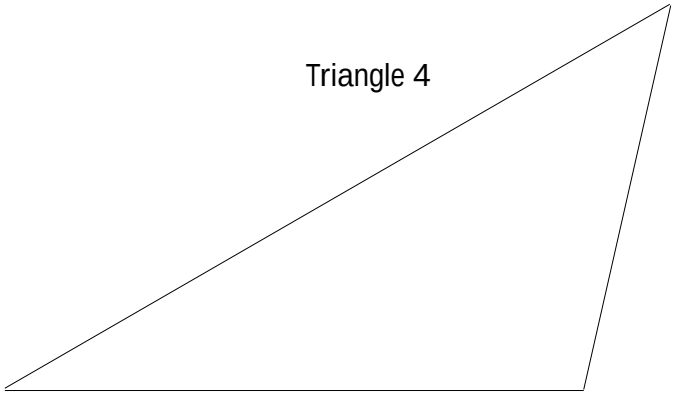
Triangle 2



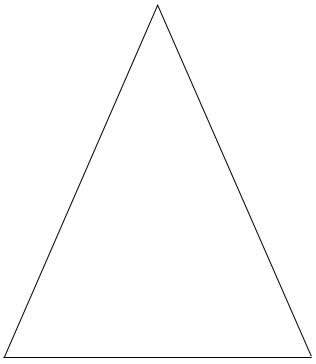
Triangle 3



Triangle 4

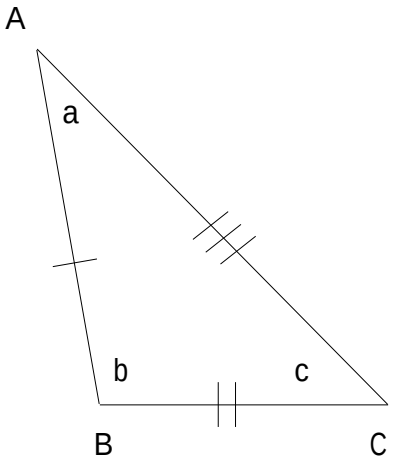


Triangle 5



Measure each side of Triangle 1.
Try to construct a second triangle with the sides equal in length to Triangle 1,
but not congruent to Triangle 1.
In other words, try to construct a second triangle in such a way that the angles are not equal.

Triangle 1



Triangle 2

Are the sides of Triangle 2 equal to the corresponding sides of Triangle 1? YES NO
Measure the angles of Triangle 2 and Triangle 1.
Are the corresponding angles EQUAL or NOT EQUAL?

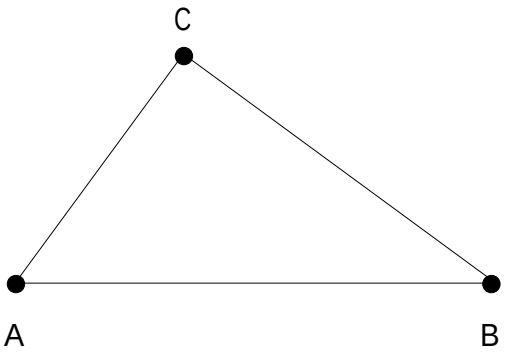
Were you able to construct a second triangle with corresponding sides equal to the first,
but the corresponding angles not equal to the first? YES NO

IF you measure the sides of two triangles, and
IF the corresponding sides are equal,
THEN how are the corresponding angles related? EQUAL NOT EQUAL
THEREFORE the two triangles are _____ .

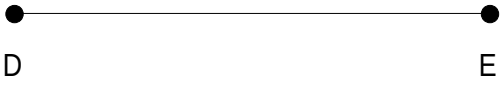
PREDICTION
Is the opposite of this relationship true?
IF each angle of one triangle is congruent to the corresponding angle of another triangle,
THEN would you predict that the triangles are always congruent?

Yes No I Do Not Know

Using your knowledge of how to make a triangle with only a straight edge and a compass, construct a triangle that would be congruent to Triangle 1 below.



Triangle 2



Measure each of the corresponding sides and angles to confirm that these two triangle are indeed congruent- that is, the corresponding sides and corresponding angles are equal.

Now, is it possible to construct a triangle that has exactly the same angles but the sides of the triangle are either larger or smaller than the original triangle?

Yes No I Do Not Know

Construct Triangle 3 in such a way that each side is twice as long as that of Triangle 1.
Measure each corresponding angle. Are the corresponding angles EQUAL or NOT EQUAL?
Is Triangle 3 congruent to Triangle 1? YES NO
Explain your answer:

Triangle 3

If Triangle 1 and Triangle 3 are not congruent, then explain how they are similar.

Expanding the Idea — Part 3

From the previous activity you saw that is it not always necessary to measure all six pairs of corresponding parts of the two triangles to determine if the triangles are congruent.

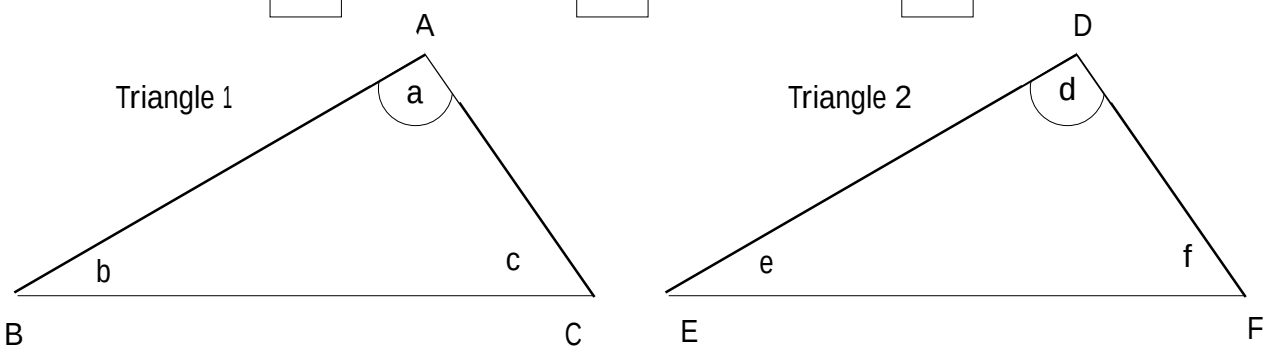
PREDICTION
IF two sides and the angle between them in one triangle are congruent to the corresponding parts in another triangle,
THEN what would you predict regarding the third side and the other two angles?

Equal Not Equal I Do Not Know

MEASURE
Using a ruler measure the sides AB and AC to determine if they are equal to sides DE and DF.

Also, measure ∠ a and ∠ d to see if they are equal. Mark the triangles appropriately.
Put = or ≠ in each box below.

AB ☐ DE AC ☐ DF ∠ a ☐ ∠ d



MEASURE
Using a protractor measure the remaining angles of each triangle and using a ruler measure the length of the other side of each triangle to determine if they are equal.

∠ b ☐ ∠ e ∠ c ☐ ∠ f BC ☐ EF

THEREFORE it can be concluded that △ABC ☐ △ DEF.

What type of triangle is Triangle 1? _____ Triangle 2? _____

What appears to be the relationship in this situation?

_____

_____

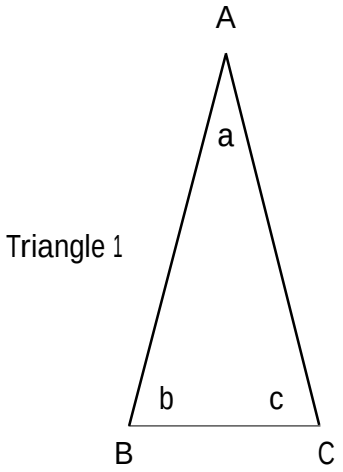
Is this relationship true for the other types of triangles?

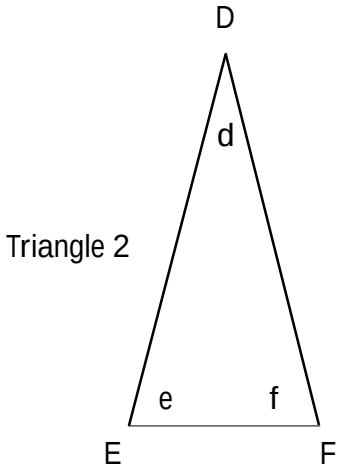
PREDICTION
IF two sides and the angle between them in one triangle are congruent to the corresponding parts in another triangle,
THEN what would you predict regarding the third side and the other two angles?

Equal Not Equal I Do Not Know

MEASURE
Using a ruler measure the sides AB and AC to determine if they are equal to sides DE and DF. Also, measure $\angle$ a and $\angle$ d to see if they are equal. Mark the triangles appropriately.
Put = or $\neq$ in each box below.

AB ☐ DE AC ☐ DF $\angle$ a ☐ $\angle$ d





MEASURE
Using a protractor measure the remaining angles of each triangle and using a ruler measure the length of the other side of each triangle to determine if they are equal.

$\angle$ b ☐ $\angle$ e $\angle$ c ☐ $\angle$ f BC ☐ EF

THEREFORE it can be concluded that $\triangle$ABC ☐ $\triangle$ DEF.

What type of triangle is Triangle 1? _____ Triangle 2? _____

State the rule describing this relationship: _____

TESTING YOUR IDEA

IF two sides and the angle between them in one triangle are congruent to the corresponding parts in another triangle,

THEN what would you predict regarding the third side and the other two angles?

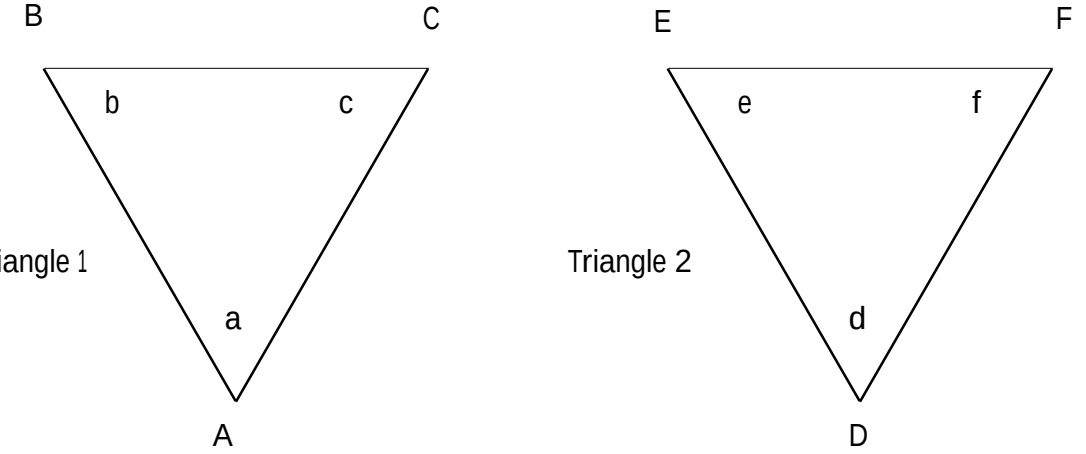
Equal Not Equal Not Possible to Tell

MEASURE

Using a ruler measure the sides AB and AC to determine if they are equal to sides DE and DF. Also, measure $\angle$ a and $\angle$ d to see if they are equal. Mark the triangles appropriately to indicate the equality of corresponding angles and sides.

Put = or $\neq$ in each box below.

AB ☐ DE AC ☐ DF $\angle$ a ☐ $\angle$ d



Triangle 1 Triangle 2

MEASURE

Using a protractor measure the remaining angles of each triangle and using a ruler measure the length of the other side of each triangle to determine if they are equal.

$\angle$ b ☐ $\angle$ e $\angle$ c ☐ $\angle$ f BC ☐ EF

THEREFORE it can be concluded that $\triangle$ABC ☐ $\triangle$ DEF.

THE IDEA

IF two sides and the angle between them in one triangle are congruent to the corresponding parts in another triangle,

THEN the corresponding angles and side are also equal,

THEREFORE, the triangles are CONGRUENT.

This is known as the

SIDE - ANGLE - SIDE RULE.

APPLYING the IDEA

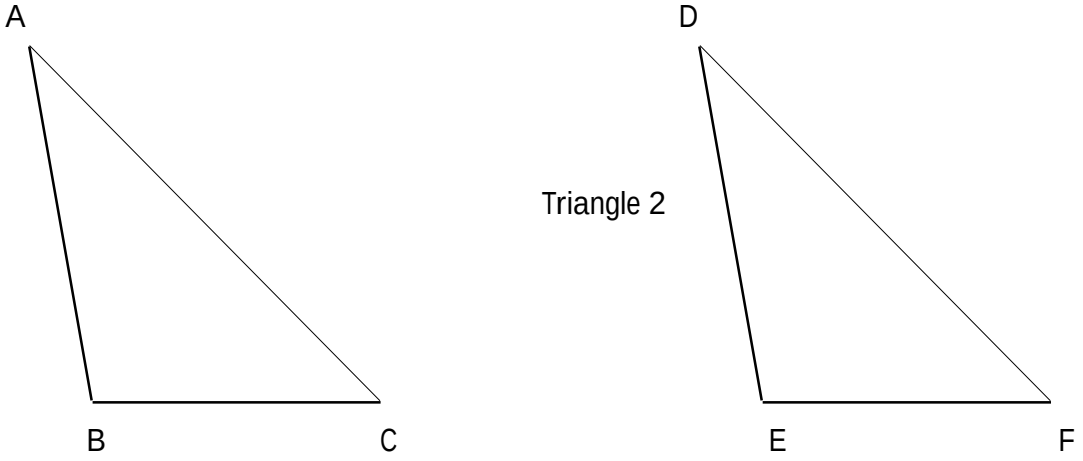
To determine if two triangles are congruent is it always necessary to measure all the sides and all the angles to see if there are equal? Yes No I Do Not Know

IF two sides and the angle between them in one triangle are congruent to the corresponding parts in another triangle,
THEN corresponding angles and corresponding side are

Equal Not Equal I Do Not Know

THEREFOR the two triangles are _____ .

Determine if the two triangles below are congruent by using your knowledge of the SIDE - ANGLE - SIDE RULE.



MEASURE

Measure the sides of each triangle to determine if the corresponding sides are equal.
Put = or ≠ in each box below.

AB ☐ DE BC ☐ EF ∠ ABC ☐ ∠ DEF

Mark the corresponding sides that are equal as on the previous page.

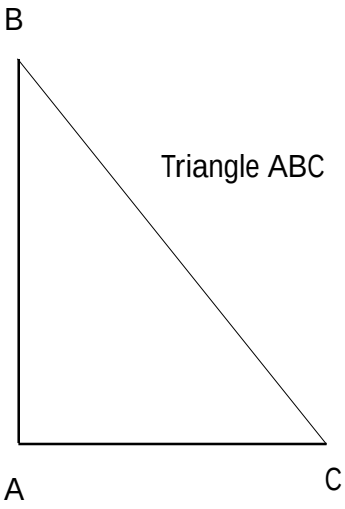
Triangle ABC is _____ to Triangle DEF

based upon the _____ - _____ - _____ Rule.

If you are uncertain as to whether the two triangles are congruent,
Then measure the each corresponding side and each corresponding angle.

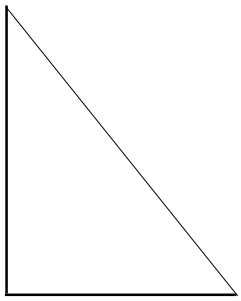
Using the SIDE - ANGLE - SIDE RULE
determine which triangles below are congruent to △ ABC.

△ ABC is congruent to _____ .

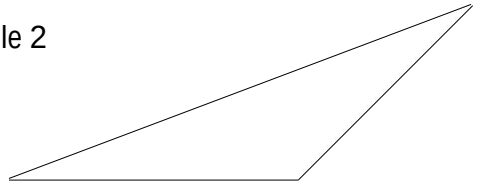


Triangle ABC

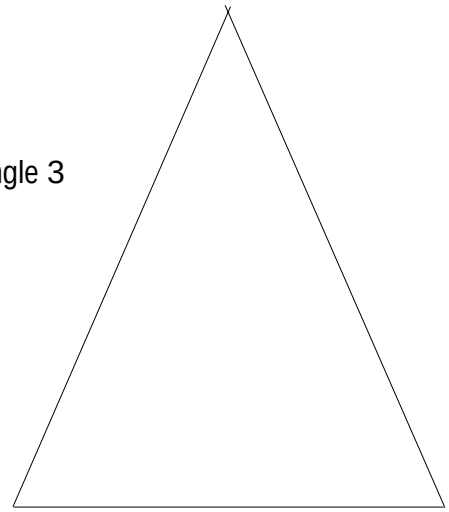
Triangle 1

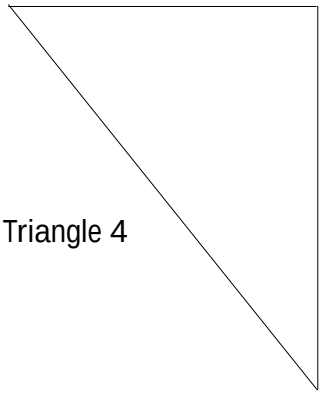


Triangle 2



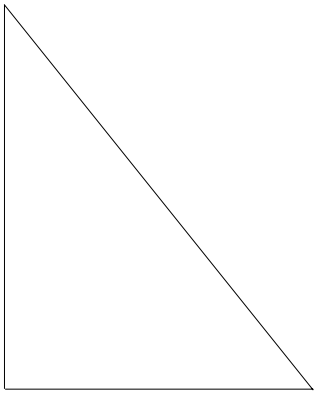
Triangle 3





Triangle 4

Triangle 5

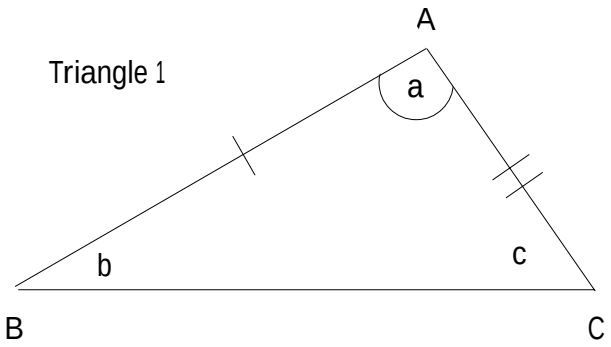


IS IT POSSIBLE?

Try to construct a second triangle with sides congruent to AB and AC and the angle between the two sides equal to $\angle$ a, but the third side and the two other angles not equal to those of Triangle 1.

Measure AB, AC and $\angle$ a of Triangle 1.

Triangle 1



Triangle 2

Were you able to construct a second triangle not congruent to Triangle 1?

YES NO

IF two sides and the angle between them in one triangle are congruent to the corresponding parts in another triangle,
THEN the corresponding angles and side are also equal,
THEREFORE, the triangles are CONGRUENT.

This is known as the

_____ - _____ - _____ RULE.

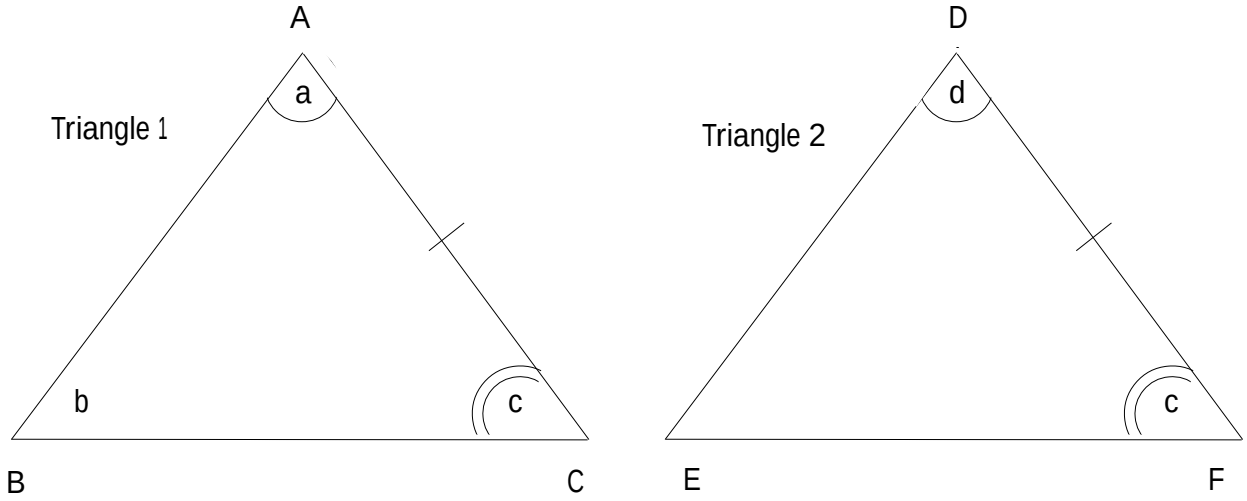
Expanding the Idea — Part 4

Is it always necessary to measure all six pairs of corresponding parts of a triangle to determine if the triangles are congruent?

PREDICTION
IF two angles and the side between them in one triangle are congruent to the corresponding parts in another triangle,
THEN what would you predict regarding the other two sides and the other angle?

Equal Not Equal I Do Not Know

MEASURE
Measure the side AC and ∠ b and ∠ c to determine if they are equal to side DF and ∠ d and ∠ f. If they are equal, mark △ DEF to show that they are equal.



AC ☐ DF ∠ a ☐ ∠ d ∠ c ☐ ∠ f

MEASURE
Using a protractor measure the remaining angle of each triangle and using a ruler measure the length of the other sides of each triangle to determine if they are equal.

∠ b ☐ ∠ e AB ☐ DE BC ☐ EF

THEREFORE it can be concluded that △ABC ☐ △ DEF.

From these two triangles..... you can begin to formulate a relationship....
IF two angles and the side between them in one triangle are congruent to the corresponding parts in another triangle,
THEN _____ .
THEREFORE, the triangles _____ .
This relationship would be described as
ANGLE - _____ - _____ RULE.

From the previous observations you saw that ....

IF two angles and the side between them in one triangle are congruent to the corresponding parts in another triangle,
THEN the remaining sides and angles are also congruent;
THEREFORE, the triangles are congruent.

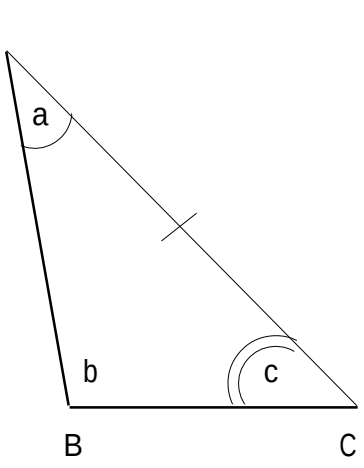
This relationship would be described as

ANGLE - SIDE - ANGLE RULE.

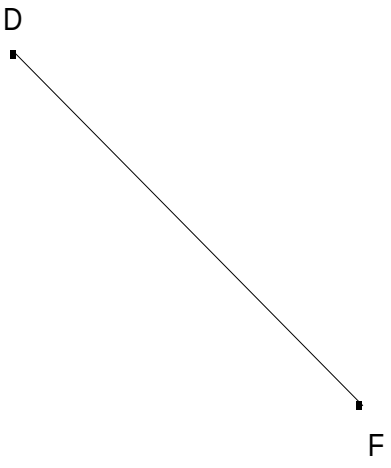
IS IT POSSIBLE?
Using your compass, straight edge and your knowledge of making constructions, is it possible to construct a second triangle (Triangle 2: ▲ DEF) in which two angles and the side between them are congruent to those of Triangle 1, but the remaining angle and sides are not congruent?

YES NO I DON'T KNOW

Triangle 1



Triangle 2



Triangle 1	Triangle 2
∠ a =	∠ d =
AC =	DF =
∠ c =	∠ f =
∠ b =	∠ e =
AB =	DE =
BC =	EF =

In my attempt to construct a second triangle (▲ DEF) in which

∠ d = ∠ a and ∠ f = ∠ c and AC = DF

I found that it was (possible, or not possible) to construct the second triangle in such a way that the two triangles were not congruent. Thought this does not prove the relationship, one can conclude that the ANGLE - SIDE - ANGLE RULE will result in congruent triangles.

Expanding the Idea — Part 5

If a person told you that …

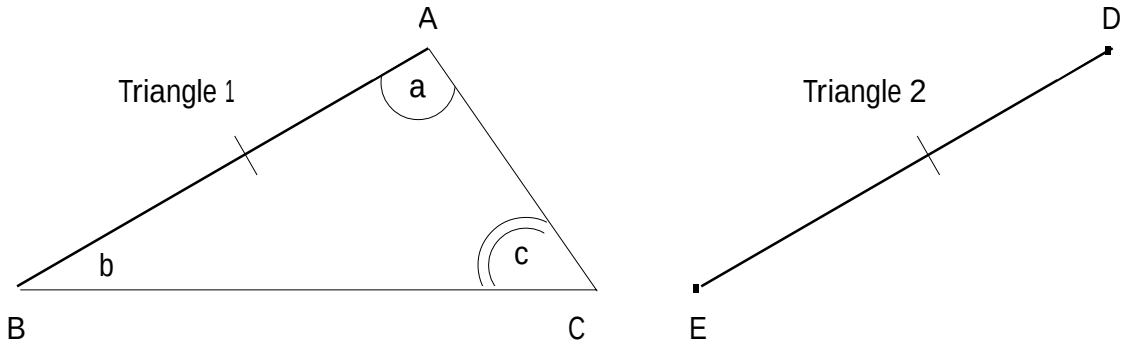
If two angles and a side not between them in one triangle are congruent to the corresponding parts in another triangle, then the triangles are congruent … would you believe him or not?

YES NO I DON'T KNOW

CONSTRUCTION

Using your compass, straight edge and your knowledge of making constructions, make a second triangle (Triangle 2: ▲ DEF) in which two angles and a side not between them are congruent to the first triangle.

$\angle a = \angle d$
$\angle c = \angle f$
AB = DE



Then determine if the two triangles are congruent or not.
Measure the other angles and sides to determine if they are = or ≠ . Write = or ≠ in the box.

$\angle b$ ☐ $\angle e$

AC ☐ DF ▲ ABC is (congruent, not congruent) to ▲ DEF

BC ☐ EF

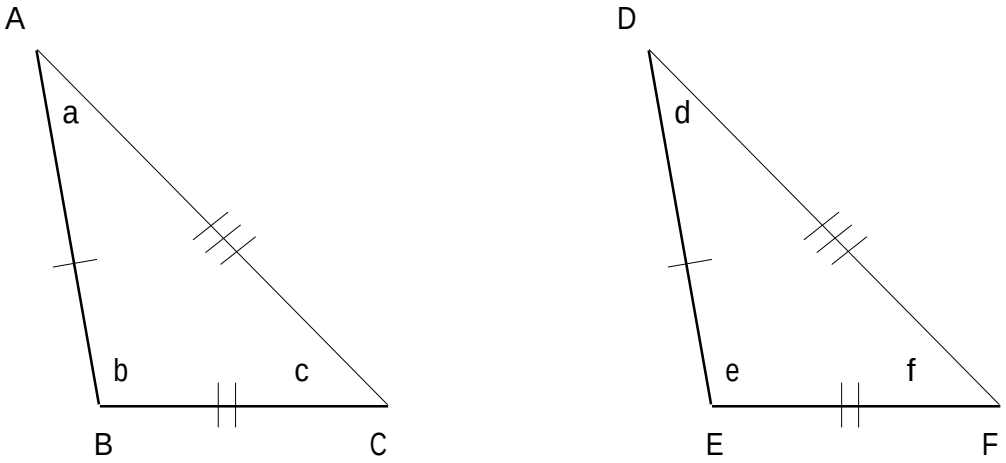
If they are congruent, then attempt to change the second triangle in such a way as to make them not congruent. (However, the original condition - that is, ANGLE - ANGLE - SIDE - must remain congruent.) Is it possible to change the second triangle, to make it not congruent?

YES NO

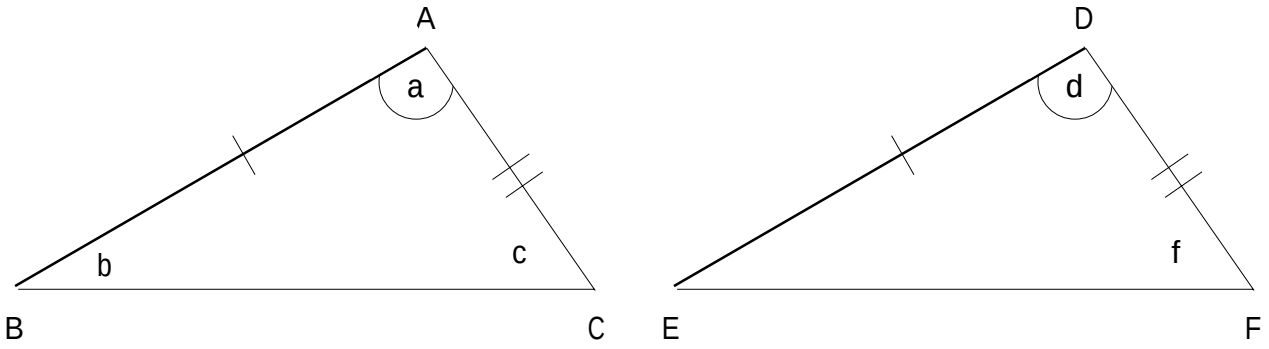
Expanding the Idea — Part 6

From the diagrams below determine why the pairs of triangles are congruent.
State the corresponding rule regarding congruent triangles:

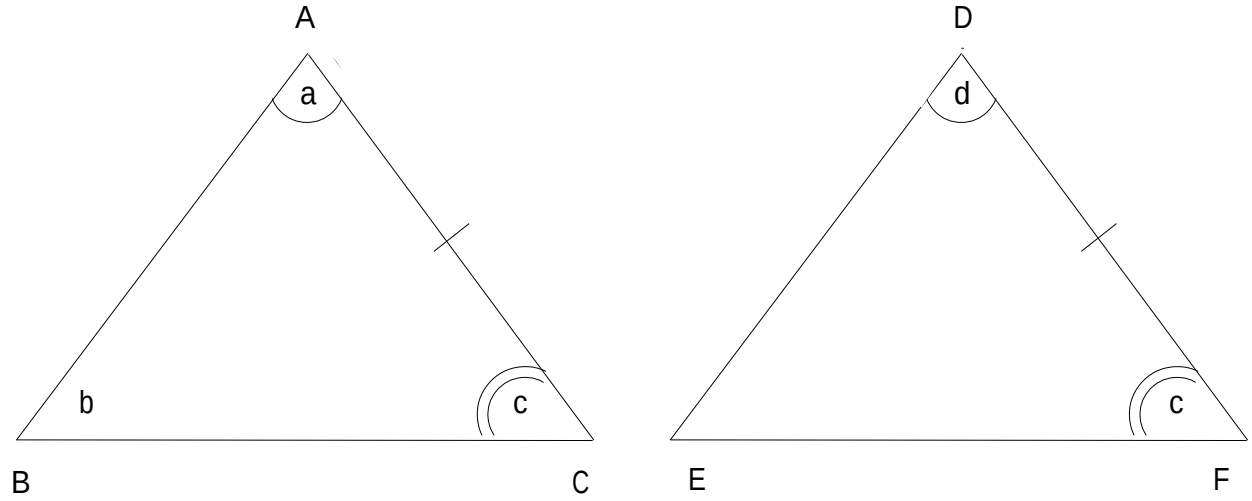
RULE 1: ▲ ABC ≅ ▲ DEF by the SIDE - SIDE - SIDE RULE



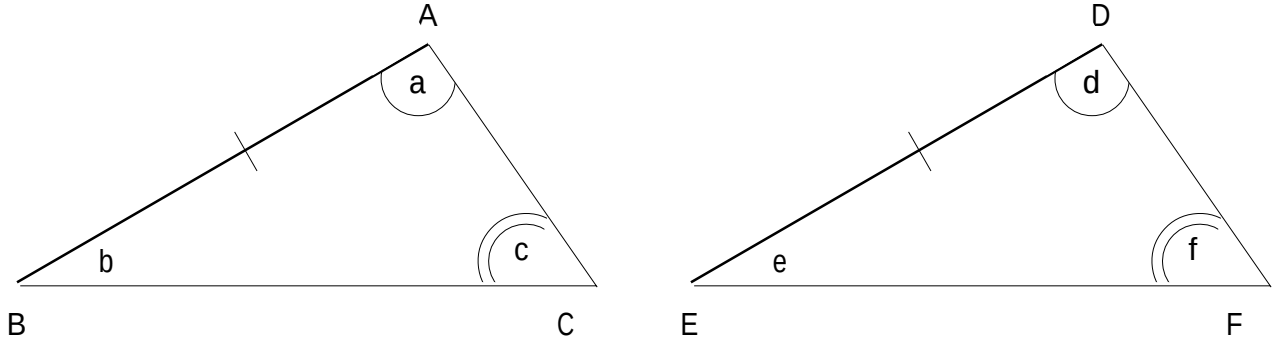
RULE 2: ▲ ABC ≅ ▲ DEF by the_____ - _____ - _____ - RULE



RULE 3: ▲ ABC ≅ ▲ DEF by the_____ - _____ - _____ - RULE



RULE 4: ▲ ABC ≅ ▲ DEF by the_____ - _____ - _____ - RULE



Expanding the Idea — Part 7

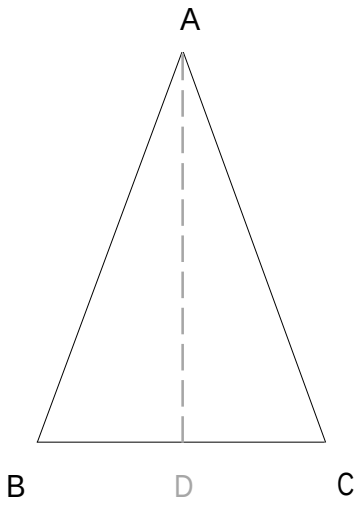
From the diagrams below determine why the pairs of triangles are congruent.
State the corresponding rule regarding congruent triangles:

RULE 1: ▲ ABC ▲ DEF by the SIDE - SIDE - SIDE RULE

RULE 2: ▲ ABC ▲ DEF by the_____ - _____ - _____ - RULE

Expanding the Idea — Part 8

▲ ABC is an isosceles triangle.
Bisect ▲ ABC by drawing a line segment AD such that ∠ BAD = ∠ DAC.



Compare ▲ ADB to ▲ ADC .

RELATIONSHIP _____
Without measuring any angles or distances, would you say that these two triangles are congruent? YES NO

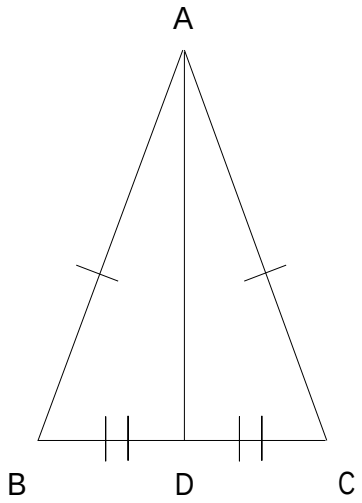
REASON _____
On the basis of what principle do you give your answer?

Measure the legs and angles of each of the triangles.

Write the mathematical relationship

▲ ADB ☐ ▲ ADC .

Two other relationships follow from ▲ ADB being congruent to ▲ ADC .
See if you can see deduce these relationships:

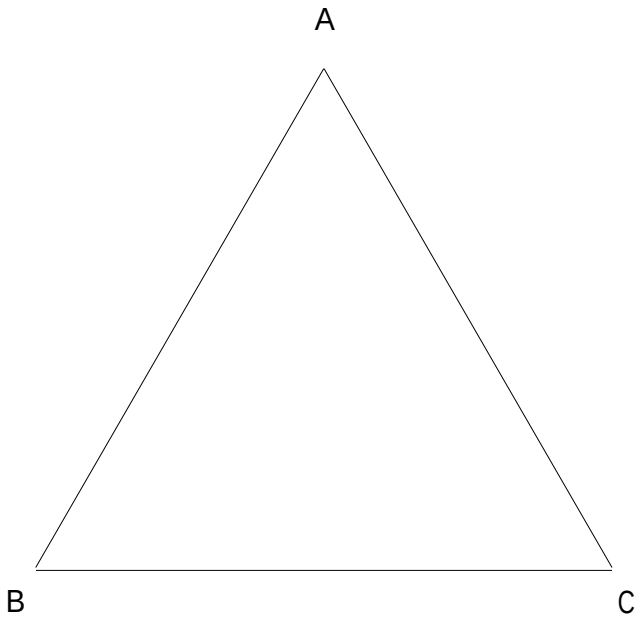


Number 1:
If two sides of a triangle are equal, then how are the angles opposite those sides related.

Number 2:
If two angles of a triangle are equal, then how are the sides opposite these angles related?

Using the above triangle, a rule and protractor determine if you state the above two relationship correctly. If not, restate the proper relationships.

▲ ABC is an equilateral triangle.
Bisect ▲ ABC by drawing a line segment AD such that ∠ BAD = ∠ DAC.



Compare ▲ ADB to ▲ ADC .

RELATIONSHIP _____
Without measuring any angles or distances, would you say that these two triangles are congruent? YES NO

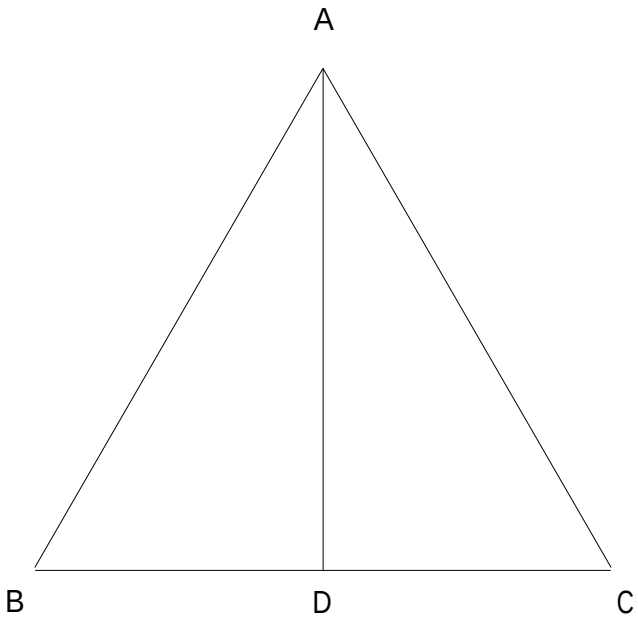
REASON _____
On the basis of what principle do you give your answer?

Measure the legs and angles of each of the triangles.

Write the mathematical relationship

▲ ADB ☐ ▲ ADC .

Two other relationships follow from ▲ ADB being congruent to ▲ ADC .
See if you can see deduce these relationships:



Number 1:
If a triangle is equilateral, then how are the angles related?

Number 2:
If a triangle is equiangular, then how are the sides related?

Using the above triangle, a rule and protractor determine if you state the above two relationship correctly. If not, restate the proper relationships.

Without using a ruler or protractor determine the missing sides and angles of ▲ ABC below:

Since …

$\angle A + \angle B + \angle C =$ _____

and in this triangle

$\angle A = \angle B = \angle C$

Then …

$\angle A =$ ☐

$\angle B =$ ☐

$\angle C =$ ☐

Therefore …

▲ ABC is what type of triangle?

_____

AC = 6 inches

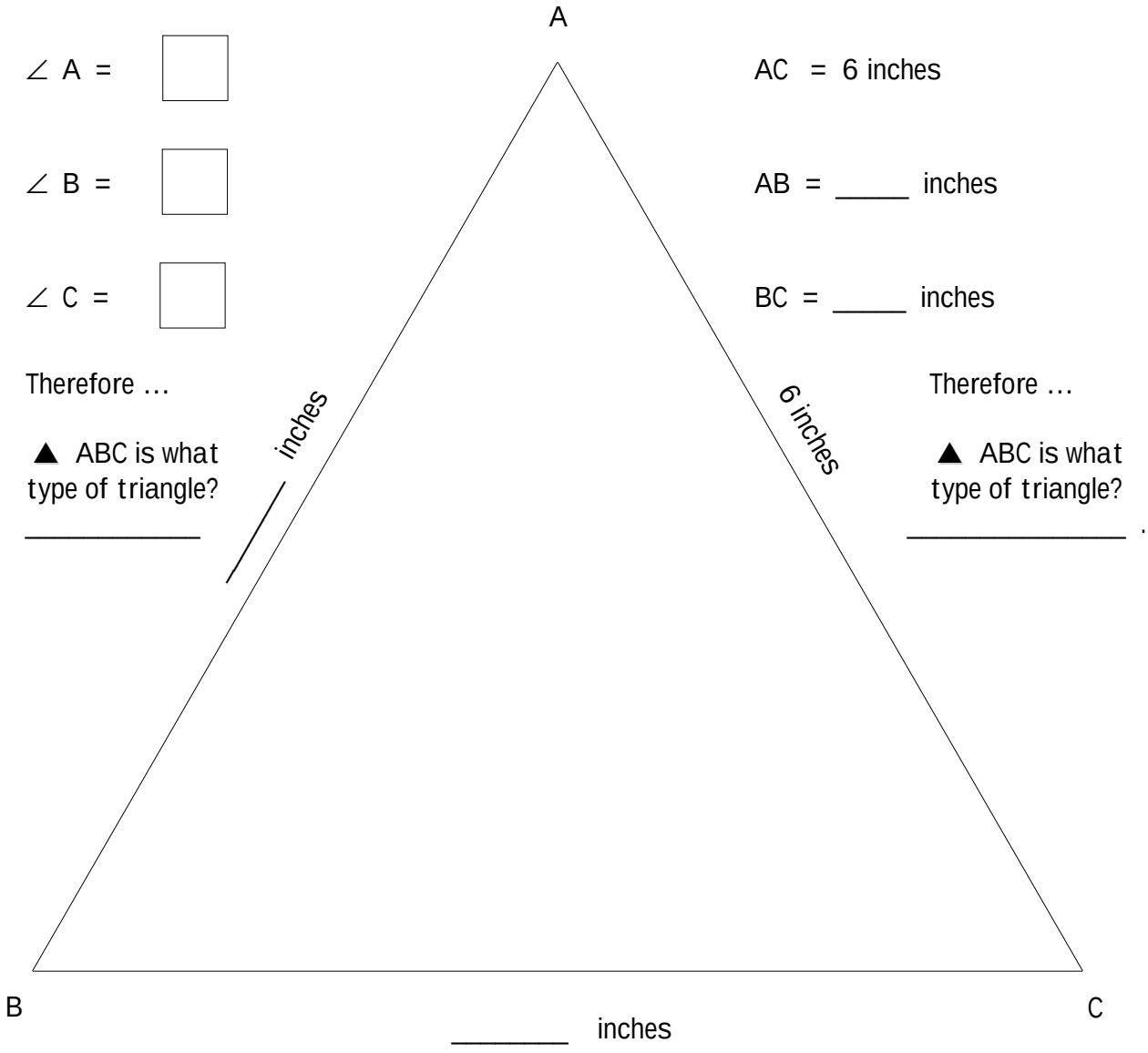
AB = _____ inches

BC = _____ inches

Therefore …

▲ ABC is what type of triangle?

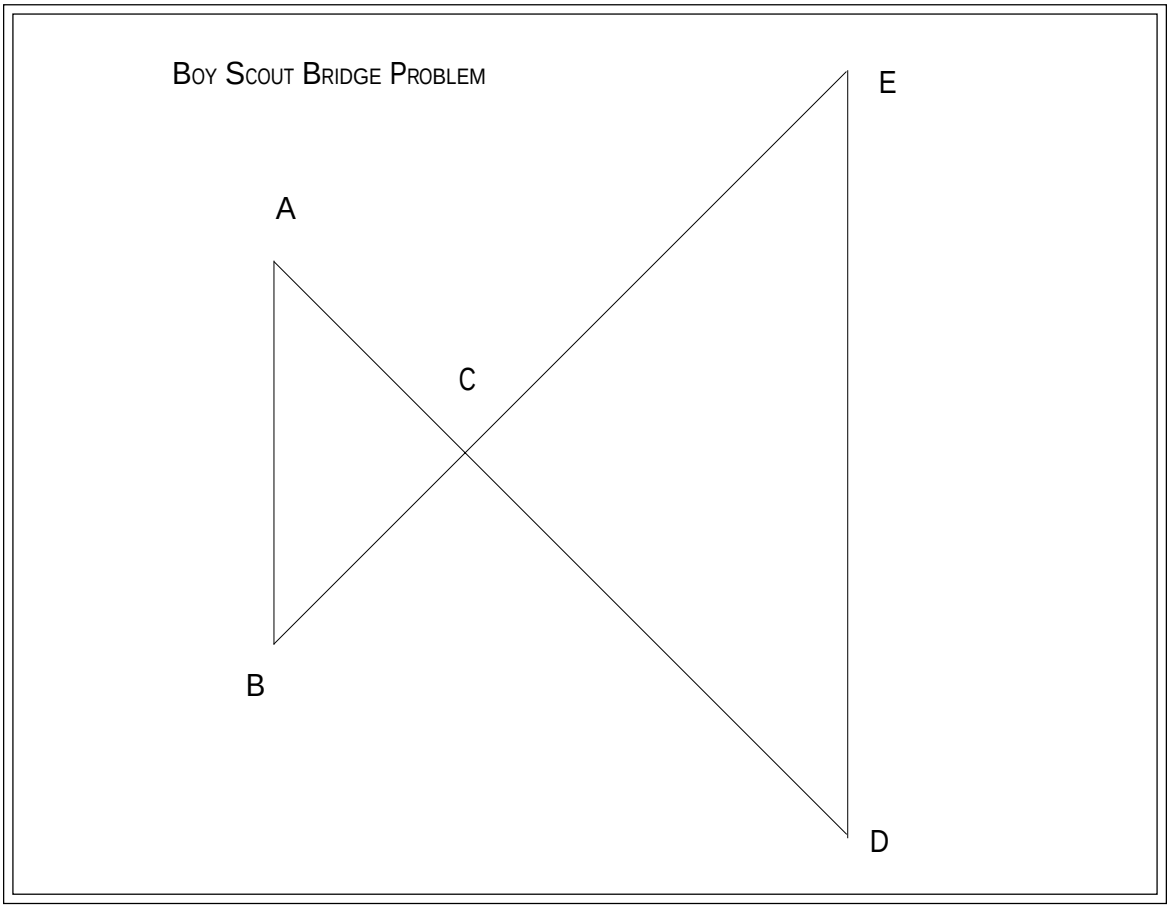
_____ .



Chapter 5: Similar But Not the Same_

Observing the Idea

Look at the diagram below.



Suppose this is a map view overlooking a land and pond area.
Line segment DE represents a bridge that the Boy Scouts need to construct over a small pond. Line segments AB and DE are parallel line segments.

However, the Scouts do not know the distance from D to E.
All the points are on land. Except for the distance from D to E, you have a measuring device to determine all other distances.

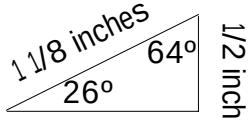
How could the distance for the bridge be calculated?
Give your suggestions:

As you look at the diagram what shapes to you recognize?
How are the shapes alike?

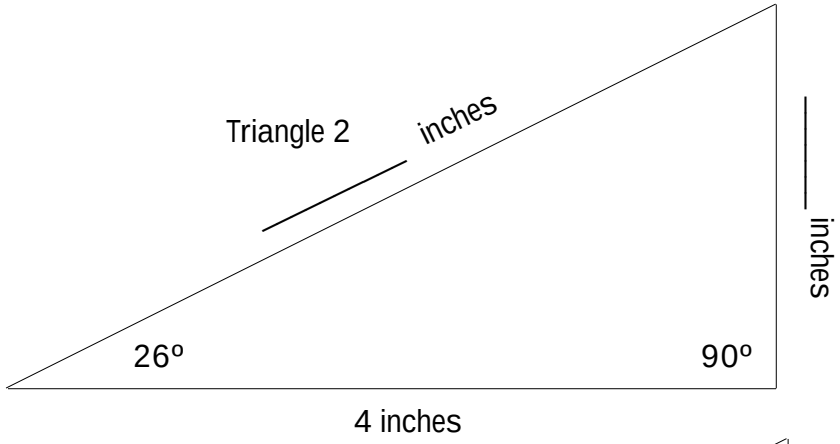
How are the shapes different?

Observe each triangle below.
Using your knowledge of the sum of the angles within a triangle, determine missing angles.
Using a ruler measure the sides of each triangle.
Look for similarities and differences among the four triangles.

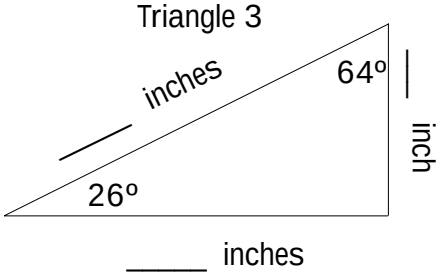
Triangle 1



Triangle 2



Triangle 3



Triangle 4

Describe how the four triangles are alike:

_____

_____

Describe how the four triangles are not alike:

_____

_____

Are these four triangles congruent triangles? YES NO

Naming the Idea

Some triangles are exactly the same size and shape. When the three sides and the three angles of one triangle have the same measurements as the three sides and three angles of a second triangle, the two triangles are called CONGRUENT TRIANGLES.

In the four triangles on the previous page the three angles are the same, however, the sides of the triangles are different. They are similar in shape, but not exactly alike. These triangles are called

SIMILAR TRIANGLES.

The symbol to show that triangles are SIMILAR is ~ .

In the previous problem $\triangle 1 \sim \triangle 2 \sim \triangle 3 \sim \triangle 4$ and is read

"Triangle 1 is SIMILAR to Triangle 2 which is SIMILAR to Triangle 3 which is SIMILAR to Triangle 4."

Expanding the Idea

Look at the sides of each of the four triangles on the previous page.
Record the measurements of the sides in the chart below:

	BASE	SIDE	HYPOTENUSE
Triangle 1	1 inch	1/2 inch	1 1/8 inches
Triangle 2	4 inches		
Triangle 3			
Triangle 4			
Triangle 5			

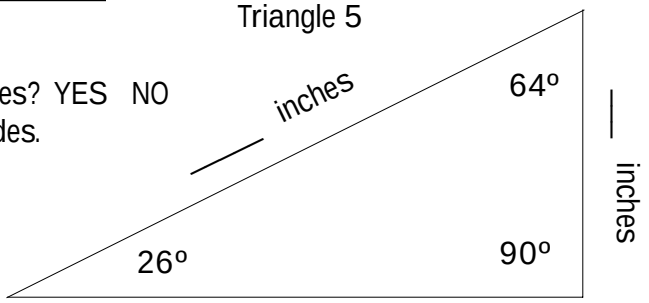
Do you notice any mathematical relationship between the sides of the four triangles?
If so, explain that relationship:

_____

_____

Look at Triangle 5.
Is it similar to the other four triangles? YES NO
Predict the measurements of the sides.
Finally, measure each side.
Was your prediction correct?
YES NO

Triangle 5



3 inches

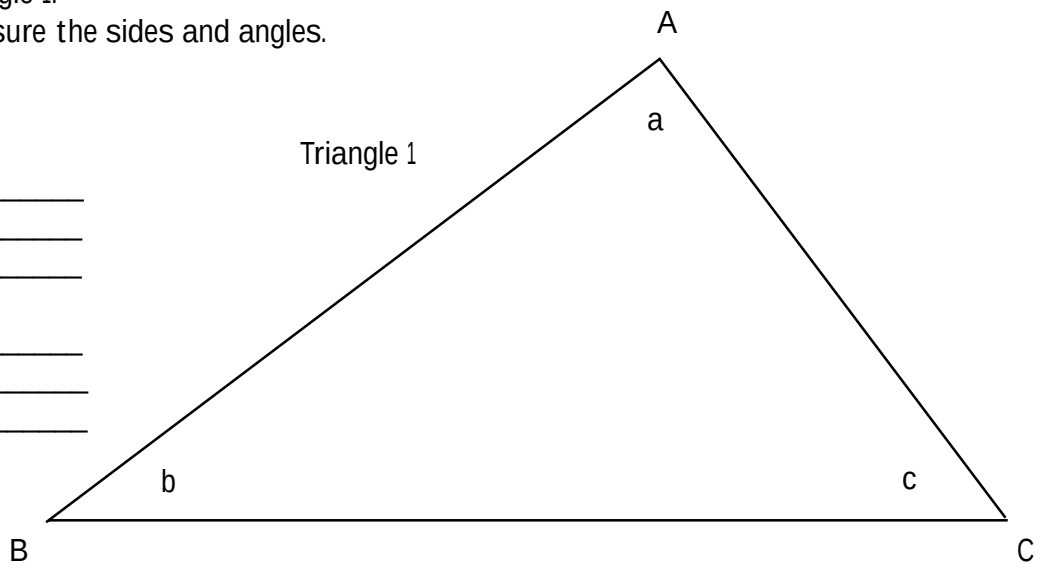
Materials
protractor
ruler

Observe Triangle 1.
Precisely measure the sides and angles.

TRIANGLE 1:

∠ a = _____
∠ b = _____
∠ c = _____

AB = _____
AC = _____
BC = _____

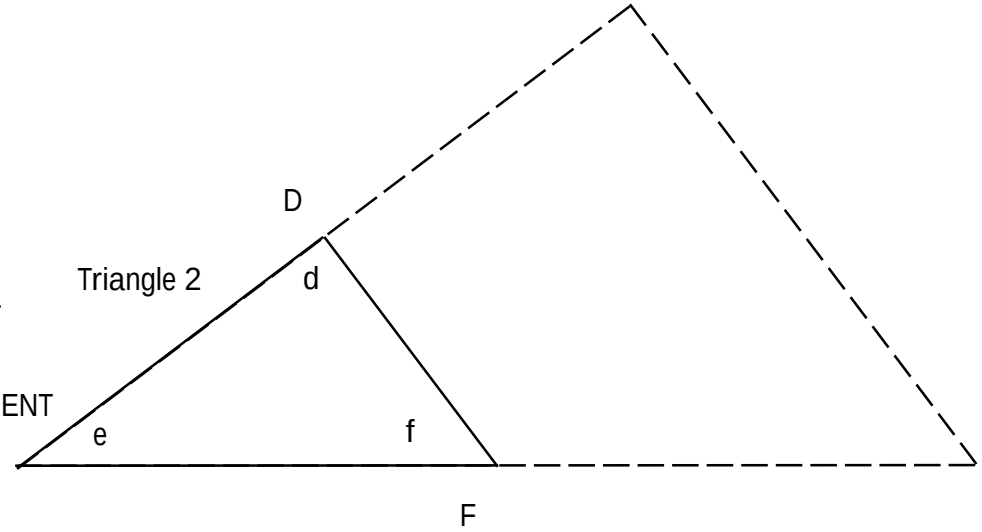


Observe Triangle 2.
The dashed lines indicate Triangle 1. DF is parallel to AC of Triangle 1.

TRIANGLE 2:

∠ d = _____
∠ e = _____
∠ f = _____

DE = _____
DF = _____
EF = _____



Is Triangle 1 CONGRUENT to Triangle 2?
YES NO

Is Triangle 1 SIMILAR to Triangle 2? YES NO

Express the mathematical relationship between Triangle 1 and Triangle 2:

△ 1 ☐ △ 2

Complete the chart on the following page.

Comparing Triangle 1 and Triangle 2 on the Characteristic of Angles:

TRIANGLE 1:		TRIANGLE 2:	
Angle	Measurement	Angle	Measurement
∠ a		∠ d	
∠ b		∠ e	
∠ c		∠ f	

What mathematical relationship exists between the corresponding angles of Triangle 1 compare with Triangle 2?

∠ a ☐ ∠ d ∠ b ☐ ∠ e ∠ c ☐ ∠ f

Comparing Triangle 1 and Triangle 2 on the Characteristic of Line Segments:

TRIANGLE 1:		TRIANGLE 2:	
Line Segment	Measurement	Line Segment	Measurement
AB		DE	
AC		DF	
BC		EF	

What mathematical relationship exists between …

AB and DE? _____ AC and DF? _____ BC and EF? _____

How is Triangle 2 similar to Triangle 1?

How are the triangles not alike?

Which mathematical relationship best describes these two triangles? Circle your answer.

△1 ≅ △2 △1 ~ △2

Perform the following mathematical operations:

$\frac{AB}{DE} = \square$ $\frac{AC}{DF} = \square$ $\frac{BC}{EF} = \square$

Then compare the answers by writing = or ≠ in the box.

$\frac{AB}{DE} \square \frac{AC}{DF} \square \frac{BC}{EF}$

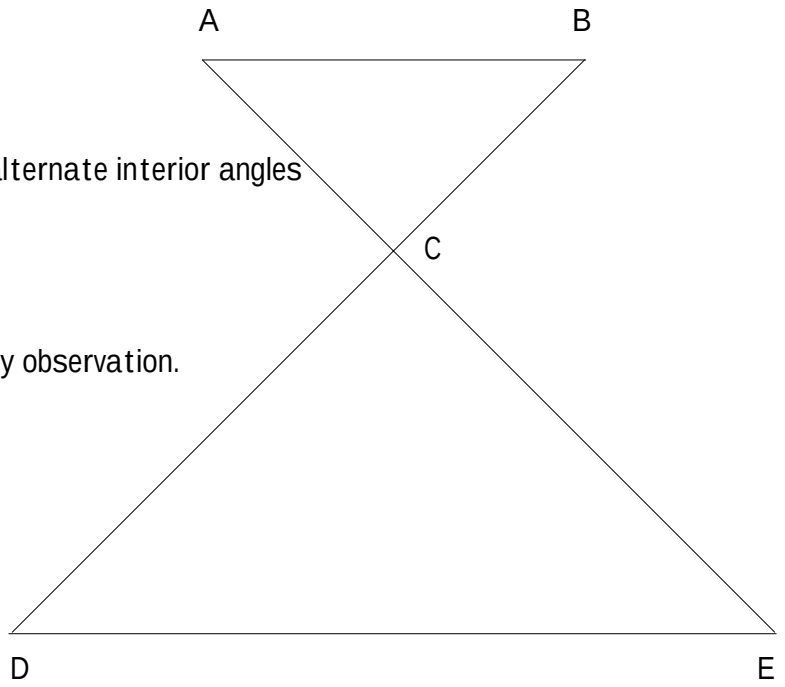
In general, if two triangles are similar, then
1 - all pairs of corresponding angles are equal,
2 - the corresponding sides are not equal, but
2 - all ratios of each pair of corresponding sides are equal.

Look at the diagram below.
How does Triangle ABC compare with Triangle DEC?
AB is parallel to DE.
Use your knowledge of triangles to come to your conclusion. Do not use any measuring devices.

RELATIONSHIP_____ REASON _____

1. ∠ ACB = ∠ DCE •
2. ∠ CAB = ∠ ___ • alternate interior angles
3. ∠ ABC = ∠ ___ •
4. The corresponding sides of the two triangles are not equal. • by observation.
5. Therefore,

▲ ABC ☐ ▲ ECD



Lets return to the Boy Scout Bridge problem.

▲ ABC ~ ▲ ECD

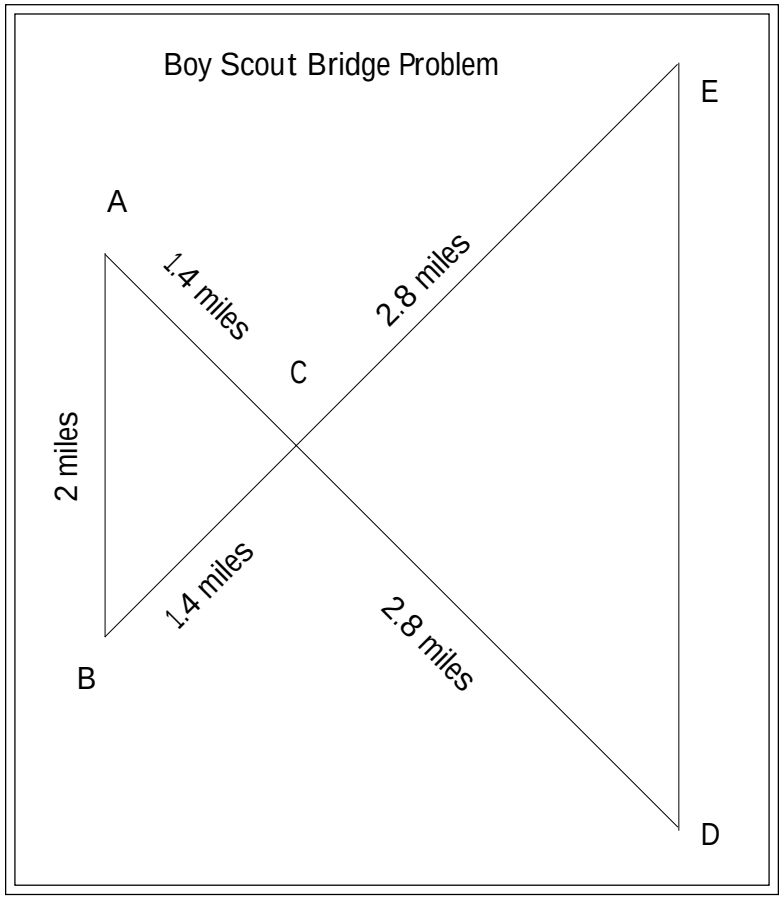
This is a map view overlooking a land and pond area. Line segment DE represents a bridge that the Boy Scouts need to construct over a small pond.

Line segments AB and DE are parallel line segments.

However, the Scouts do not know the distance from D to E. All the points are on land.

All the distances can be measured except the distance from D to E. You have a measuring device to determine distances.

Knowing the relationships between similar triangles, determine the length of the bridge (the line segment DE).



HINT … In Similar Triangles …

How is $\overline{AB}$ related to $\overline{AC}$?

How is $\overline{ED}$ related to $\overline{EC}$?

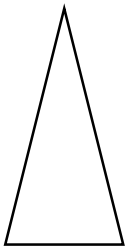
How is the relationship between $\overline{AB}$ and $\overline{AC}$ related to the relationship between $\overline{ED}$ and $\overline{EC}$?

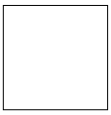
How can this information be used to determine the distance between Point D and Point E?

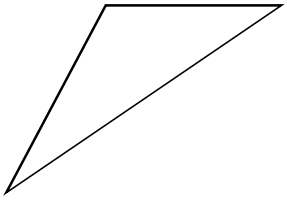
CHAPTER 6: Shapes with Funny Names

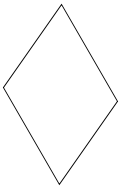
EXPLORING THE IDEA

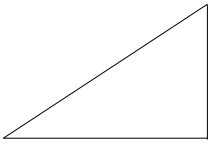
1. Describe each of the following shapes.
Give three to four characteristics of each shape.

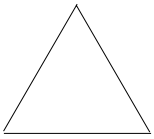




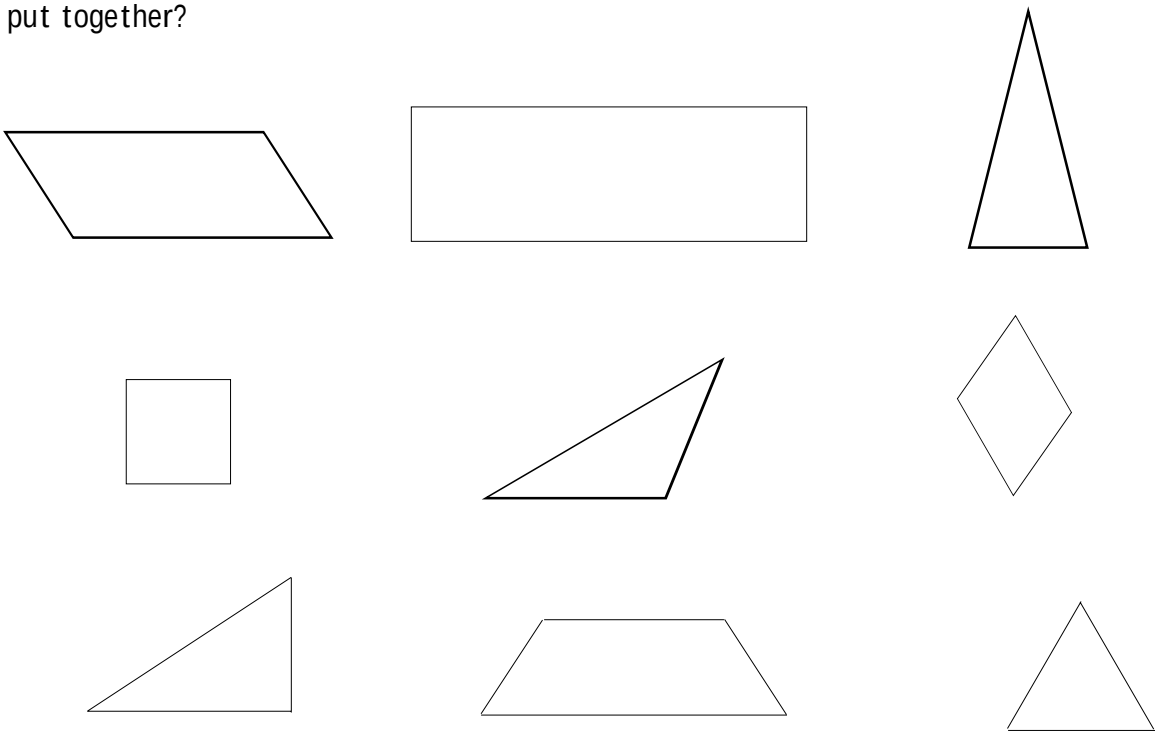








Which shapes have the same characteristics? How are they alike? How are these shapes difference from the others? Could you separate the shapes into two major groupings? If so which shapes would you put together?



List the Common Characteristic.

Make a sketch of the shapes with the same characteristics.

The shapes you have been observing connect on all sides.
Shapes of this type are called

CLOSED SHAPES.

In this case the shapes had either _____ or _____ sides.
They also had either _____ or _____ angles.

Line Segments with three or more sides are called

POLYGONS

Three - Sided Polygon are called

TRIANGLES

You will remember from a previous section that triangles could be grouped into categories based upon either the relation of the sides or by the measure of the angles.

Fill in the blanks below. If you need help in remembering refer back to the section on triangles.

CLASSIFICATION BASED ON SIDES:

3 SIDES EQUAL: _____ Triangle

2 SIDES EQUAL: _____ Triangle

NO SIDES EQUAL: _____ Triangle

CLASSIFICATION BASED ON ANGLES:

ALL ANGLES LESS THAN 90º: _____ Triangle

1 ANGLE EQUAL to 90º: _____ Triangle

1 ANGLE GREATER THAN 90º: _____ Triangle

Four - Sided Polygon are called

QUADRILATERALS

Quadrilaterals can be grouped into categories based upon the relation of the sides.

How can quadrilaterals be classified?

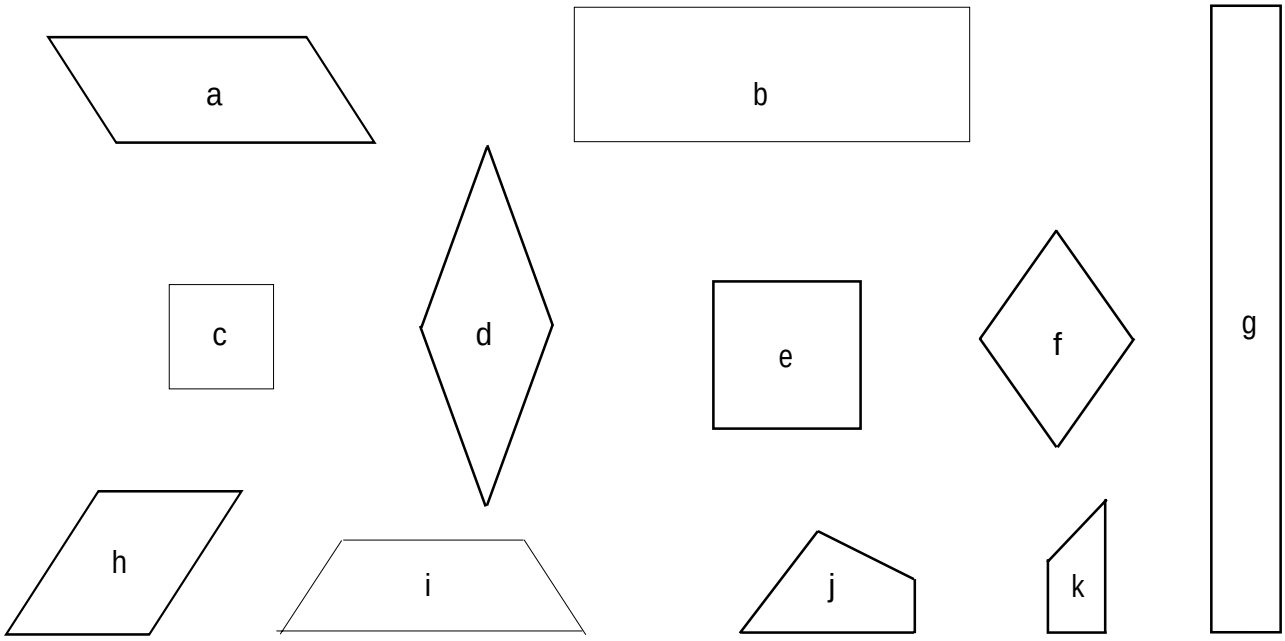
Look closely at the relationship of the sides of the quadrilaterals. Describe any pattern that you see....

_____

_____

Look at the drawings on the next page. Group them by common characteristics.

Which shapes have the same characteristics? How are they alike? How are these shapes difference from the others? Could you separate the shapes into two major groupings? If so which shapes would you put together? One shape is very different from the other ten. Use your ruler and protractor to as you make the comparisons Circle the shape that is unique.



Complete the diagram on the following page by separating the shapes by specific characteristics.

Which of the shapes seemed completely different from all the others? _____ .

The unique shape above (shape j) that you circled will be placed on the left side of the diagram by itself. This quadrilateral has no parallel sides. It is an example of an IRREGULAR QUADRILATERAL.

Next, separate the remaining quadrilaterals based upon the relationship of the opposite sides.

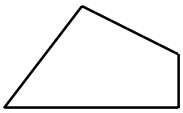
Which shapes above have only one pair of opposite sides that are parallel?
Draw those shapes below:

Which shapes above have both pairs of opposite sides parallel? Draw those shapes below:

Complete the diagram below by separating the shapes by specific characteristics. The one unique shape above that you circled will be placed on the left side by itself. Next, separate the remaining quadrilaterals based upon the relationship of the opposite sides.

QUADRILATERALS
A simple classification of the four sided shapes

A quadrilateral with no sides parallel is called
IRREGULAR QUADRILATERAL.



REGULAR QUADRILATERALS

Only one pair of opposite sides are parallel.
TRAPEZOIDS
Sketch that shape below:

Both pairs of opposite sides are parallel.
PARALLELOGRAM

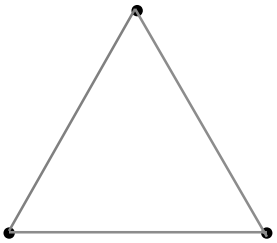
All sides equal
RHOMBUS

Only opposite sides equal
four 90º angles
RECTANGLE

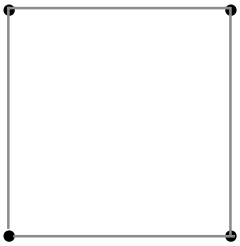
All sides equal and
4- 90º angels
SQUARE

EXPANDING THE IDEA

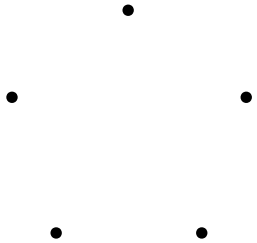
For each problem, connect the dots to make one closed shape, a polygon.
State the number of sides of each polygon.

1. 

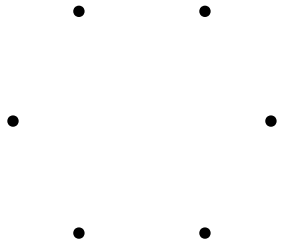
number of sides _____

2. 

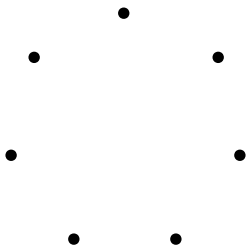
number of sides _____

3. 

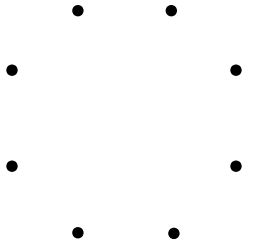
number of sides _____

4. 

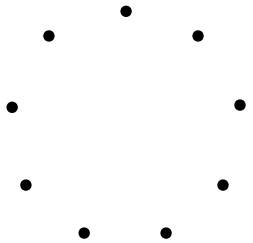
number of sides _____

5. 

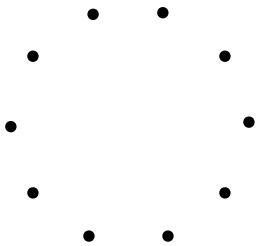
number of sides _____

6. 

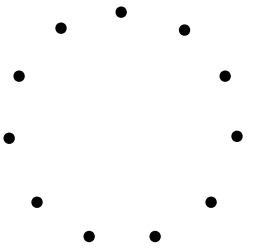
number of sides _____

7. 

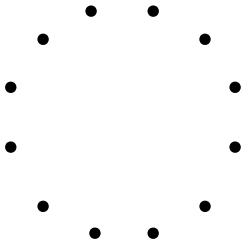
number of sides _____

8. 

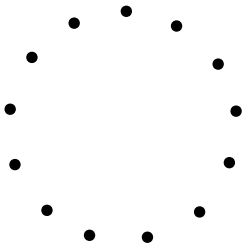
number of sides _____

9. 

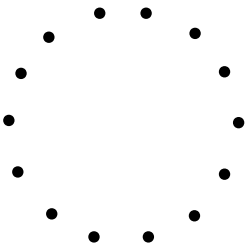
number of sides _____

10. 

number of sides _____

11. 

number of sides _____

12. 

number of sides _____

As the number of sides increases what affect does it have upon the overall shape of the object? In other words, describe the change in shape from the first drawing to the last on the previous page.

_____

_____ .

If the number of sides continued to increase, what shape would result?

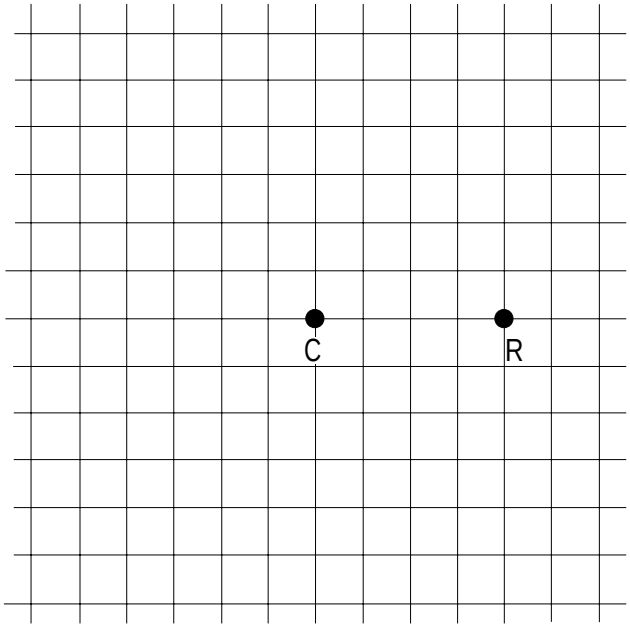
_____

_____ .

Materials:
- Compass
- Pencil
- Ruler

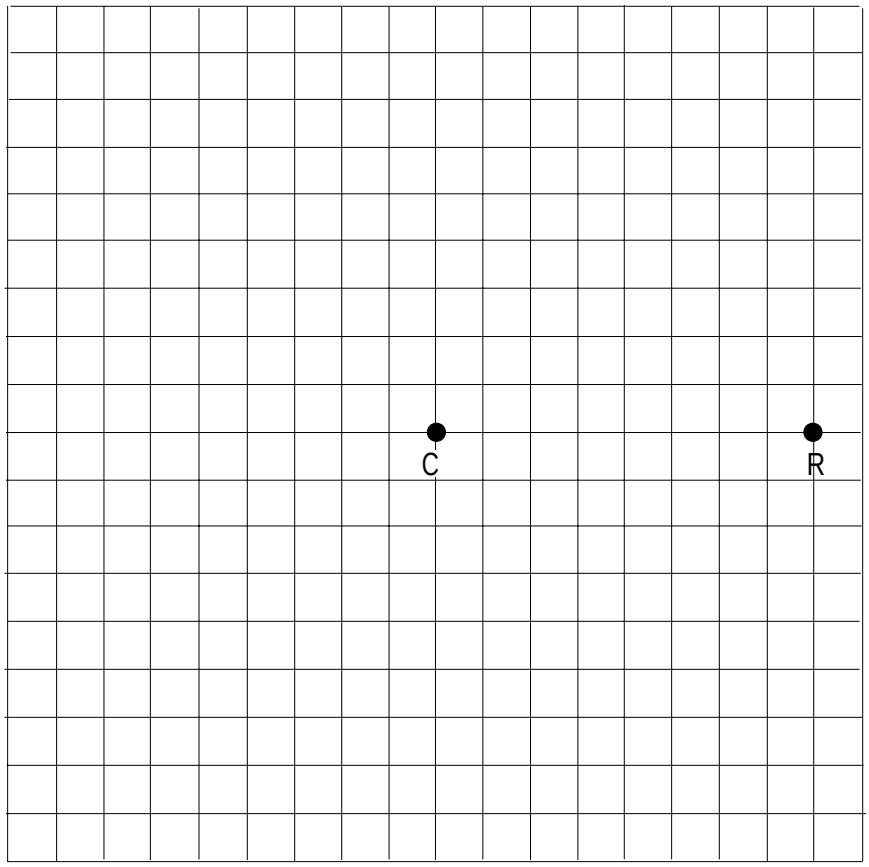
Set the compass point at point C.
Set the pencil point of the compass at Point R.

Turn the pencil around Point C until the pencil returns to Point R.



Set the compass point at Point C.
Set the pencil point of the compass at Point R.

Turn the pencil around Point C until the pencil returns to Point C.



How is this shape like the first shape you drew?

_____
_____
_____

How is this shape different from the first shape you drew?

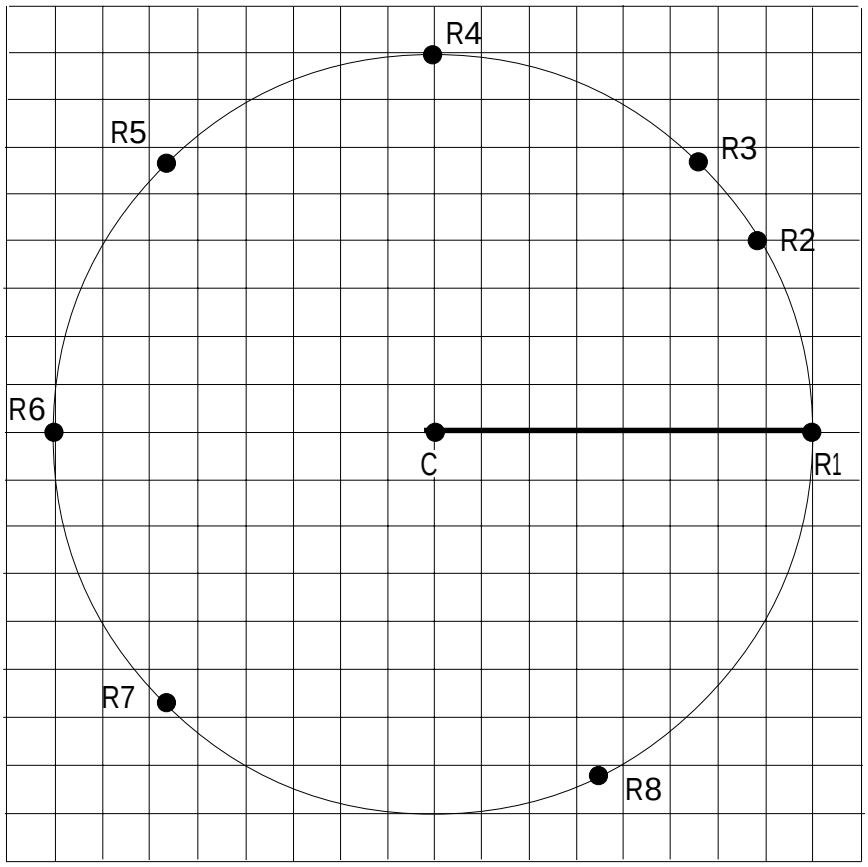
_____
_____
_____

State several characteristics of this shape....

_____
_____
_____
_____

Make line segments from Point C to each of the points along this shape.

Measure each line segment (to the nearest inch) and record your data in the chart below.



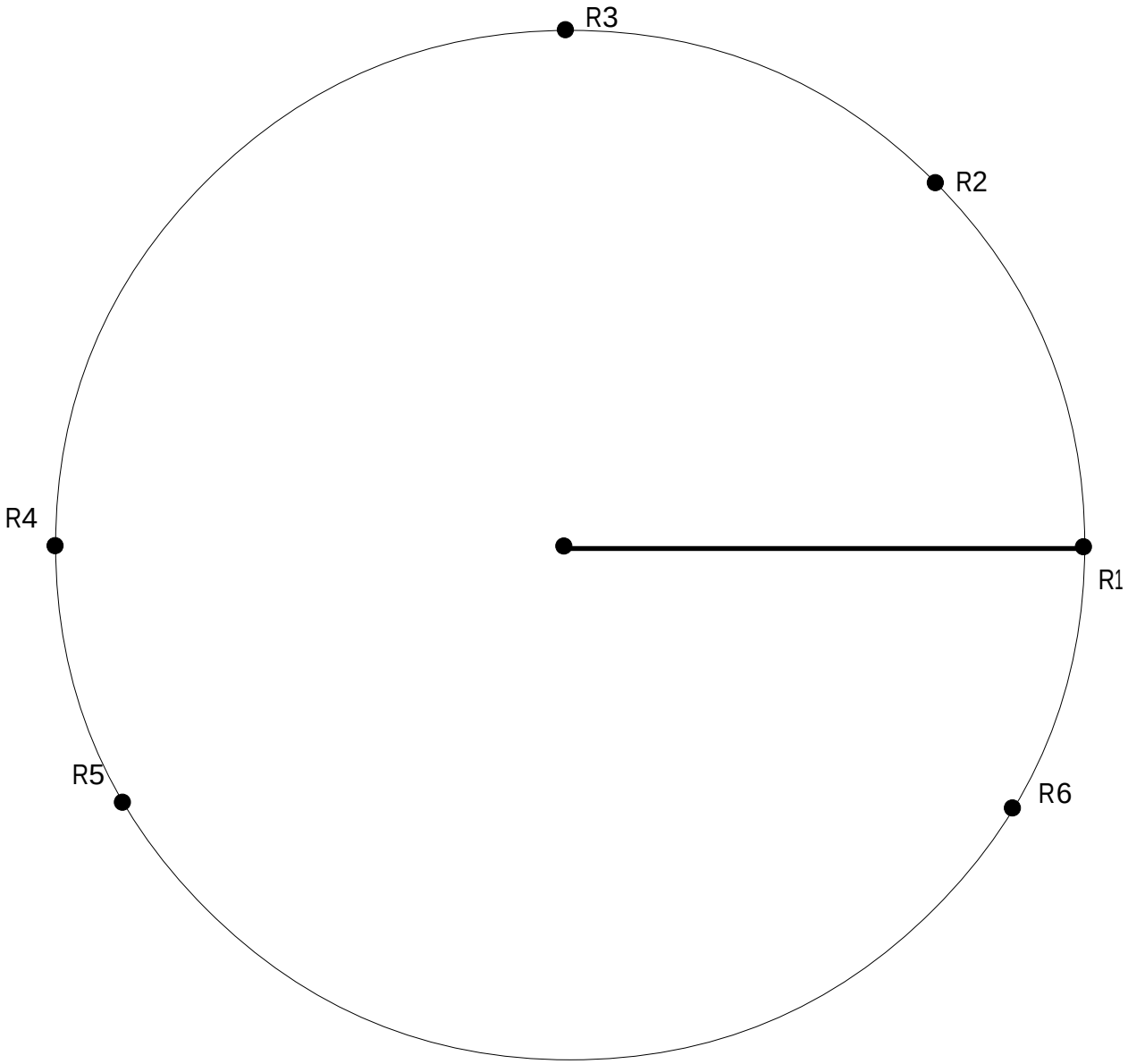
LINE SEGMENT	DISTANCE IN INCHES
$\overline{CR1}$	
$\overline{CR2}$	
$\overline{CR2}$	
$\overline{CR4}$	
$\overline{CR5}$	
$\overline{CR6}$	
$\overline{CR7}$	
$\overline{CR8}$	

Compare the length of each line segment.

Are they EQUAL or NOT EQUAL?

Make line segments from Point C to each of the points along the shape.

Measure each line segment (to the nearest inch) and record your data in the chart below.



LINE SEGMENT	DISTANCE IN INCHES
$\overline{CR1}$	
$\overline{CR2}$	
$\overline{CR2}$	
$\overline{CR4}$	
$\overline{CR5}$	
$\overline{CR6}$	

Compare the length of each line segment of this circle..

Are they EQUAL or NOT EQUAL?

What does this information tell you about the nature of this shape?

NAMING THE IDEA

A closed figure with all points the same distance from the center point is called a

CIRCLE.

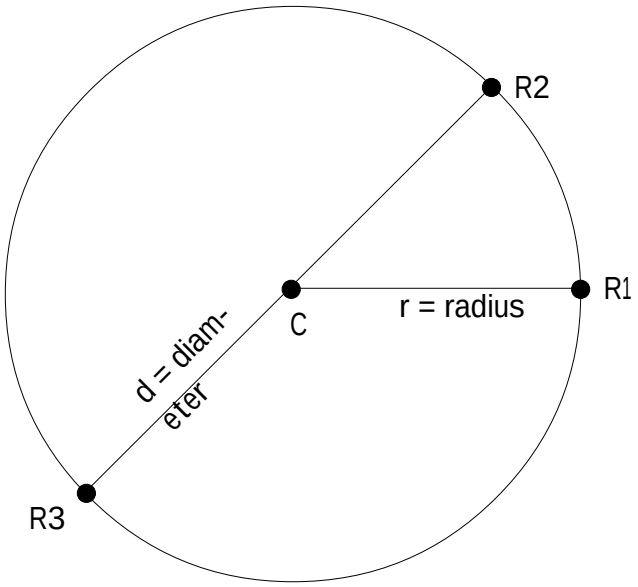
Any line segment that goes from the center of the circle to a point on the circle is called

the RADIUS of the CIRCLE.

Any line segment which passes through the center having both endpoints on the circle is called

the DIAMETER of the CIRCLE.

The diameter is twice as long as the radius.



EXPANDING THE IDEA - \Part 1

Find the RADIUS, and DIAMETER for each circle below.
Make your measurement to the nearest one-half inch.

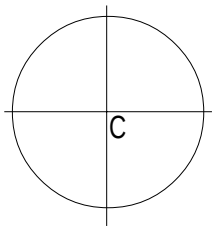


FIGURE 1

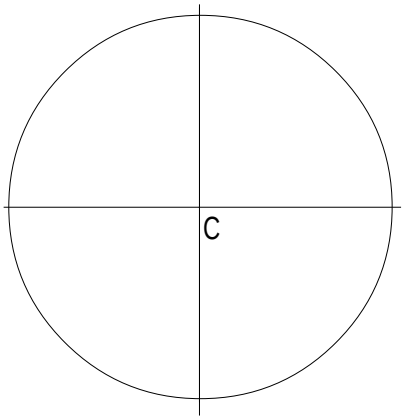


FIGURE 2

	RADIUS	DIAMETER
FIGURE 1		
FIGURE 2		
FIGURE 3		

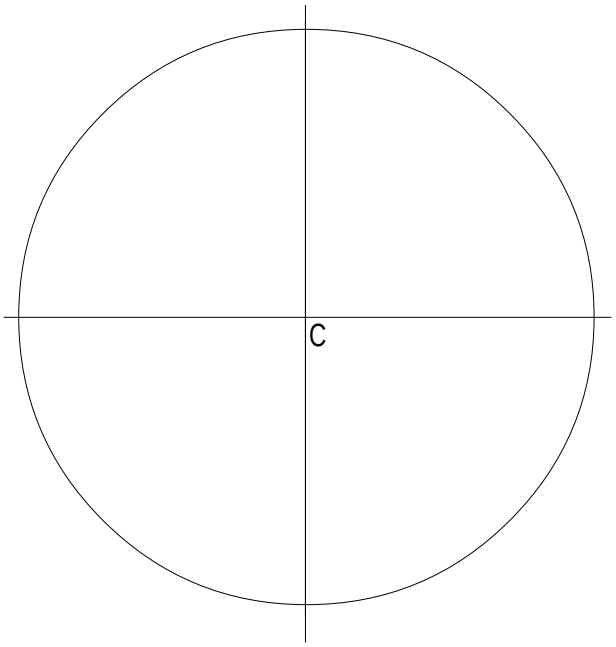
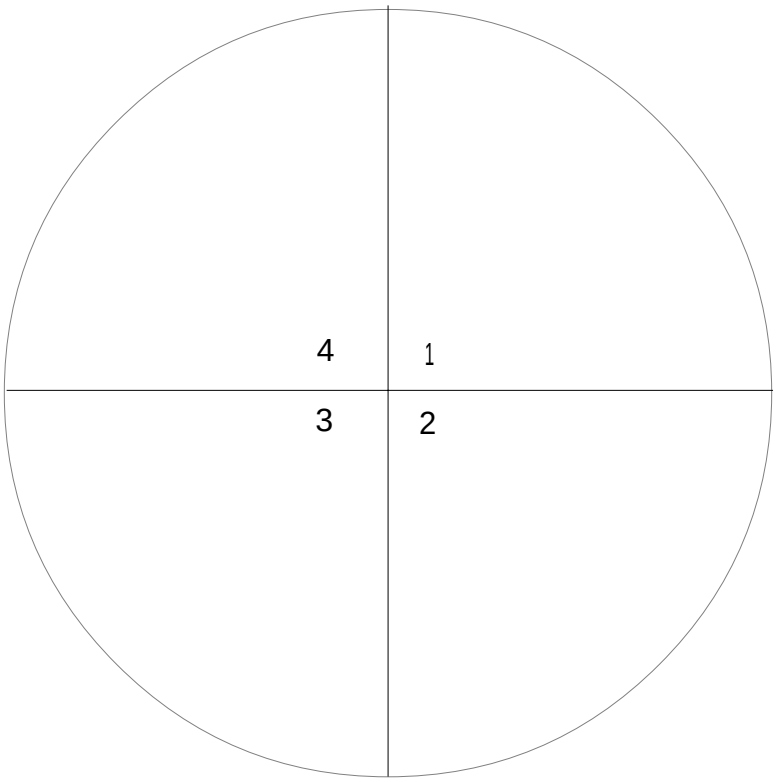


FIGURE 3

Explain the relationship between the radius and diameter of a circle.

EXPANDING THE IDEA - Part 2

Measure ∠ 1 _____ .



Predict

∠ 2 = _____

∠ 3 = _____

∠ 4 = _____

Measure angles 2, 3, and 4.

∠ 2 = _____

∠ 3 = _____

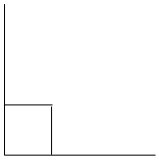
∠ 4 = _____

Did your predictions match your measurements?
If not, why not?

A 90º angle is called a

RIGHT ANGLE.

A RIGHT ANGLE is designated as follows:



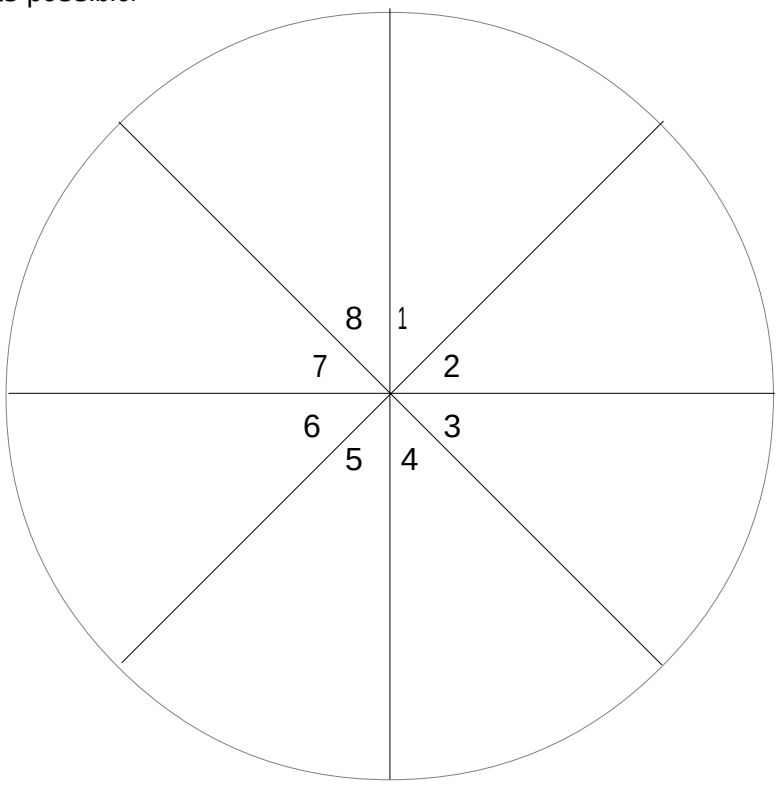
The two lines are said to be

PERPENDICULAR.

Complete the chart below:

ANGLE	PREDICTION	ACTUAL MEASUREMENT
∠ 1 + ∠ 2 =		
∠ 2 + ∠ 3 =		
∠ 3 + ∠ 4 =		
∠ 4 + ∠ 1 =		
∠ 1 + ∠ 2 + ∠ 3 + ∠ 4 =		

Measure each angle as precisely as possible.



∠ 1 = _____

∠ 2 = _____

∠ 3 = _____

∠ 4 = _____

∠ 5 = _____

∠ 6 = _____

∠ 7 = _____

∠ 8 = _____

PREDICT the sum of …

∠ 1 + ∠ 2 + ∠ 3 + ∠ 4 = _____

ADD the sum of …

∠ 1 + ∠ 2 + ∠ 3 + ∠ 4 = _____

Did your predictions match your measurements? If not, why not?

PREDICT the sum of …

∠ 1 + ∠ 2 + ∠ 3 + ∠ 4 ∠ 5 + ∠ 6 + ∠ 7 + ∠ 8 = _____

ADD the sum of …

∠ 1 + ∠ 2 + ∠ 3 + ∠ 4 ∠ 5 + ∠ 6 + ∠ 7 + ∠ 8 = _____

Did your predictions match your measurements? If not, why not?

How many degrees are there in a complete circle? _____

Using your compass:

Place the compass point at Point C.

Place the pencil at Point R.

Rotate the pencil around Point C one complete turn.

What shape did you make? _____

How many degrees did your pencil travel? _____

Draw several line segments that pass through Point C and have endpoints on the circle.

Measure the angles that are formed.

Add all the angles together.

The sum of the angles is equal to how many degrees? _____

The sum should have been 360º.

If your measurements are not precisely 360º, at least they should have been very, very close.

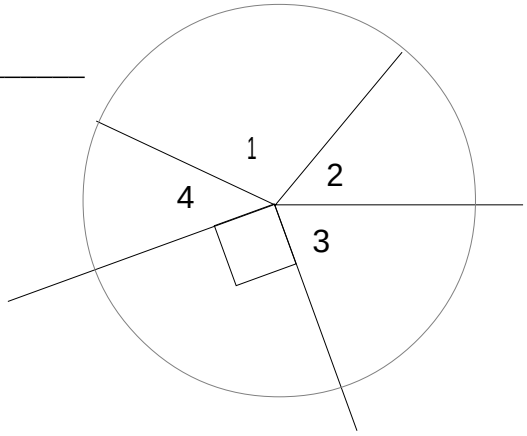
The total number of degrees in a circle is

360º

Using your knowledge of the number of degrees in a circle, and using your knowledge of the symbol for a right angle,

PREDICT the sum of …

$\angle 1 + \angle 2 + \angle 3 + \angle 4 =$ _____



Explain your reasoning:

_____
_____
_____
_____
_____
_____

Measure each angle:

$\angle 1 =$ _____

$\angle 2 =$ _____

$\angle 3 =$ _____

$\angle 4 =$ _____

CALCULATE the sum of …

$\angle 1 + \angle 2 + \angle 3 + \angle 4 =$ _____

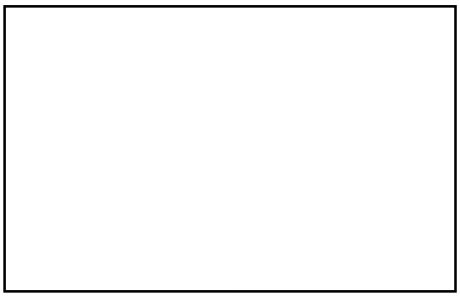
Did your predictions match your measurements? If not, why not?

_____
_____
_____
_____

CHAPTER 7: How Far Around and Just How Big

EXPLORING THE IDEA

There are two special characteristics that are important to know about the shapes you have been studying: 1) The distance around; and 2) the amount of space the shape takes up. Look at the various shapes below. How do you think the distance around each object would be determined? And how could you determine the amount of space within the closed shape. Give you thoughts to these two questions for each shape in the space provided.

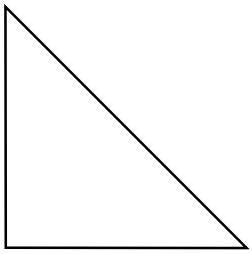


Distance Around:_____
_____
_____

Space within:_____
_____
_____

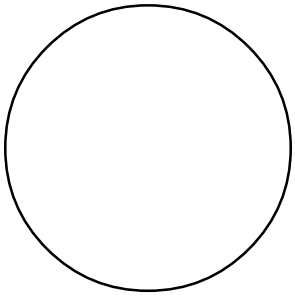
Distance Around:_____
_____
_____

Space within:_____
_____
_____



Distance Around:_____
_____
_____

Space within:_____
_____
_____

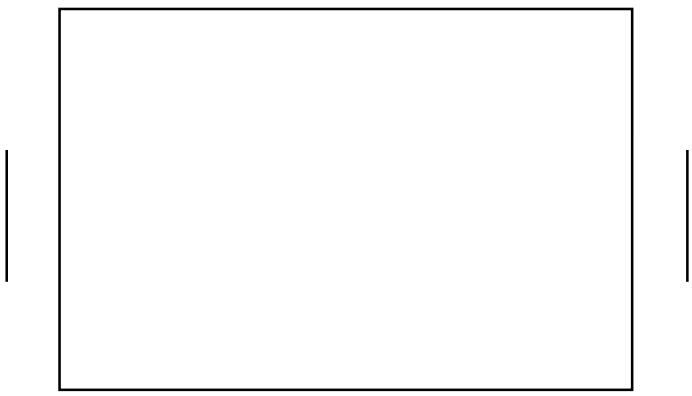


Distance Around:_____
_____
_____

Space within:_____
_____
_____

Measure the four sides of the rectangle below and record the distance of each line segment.

_____

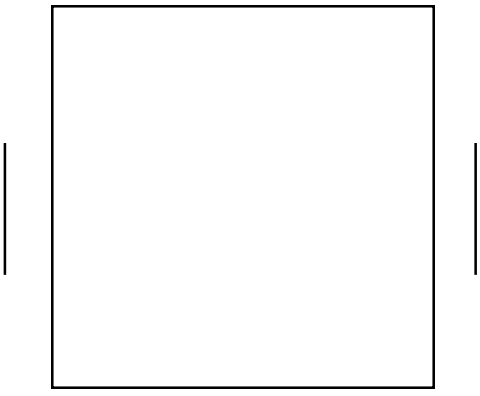


_____

If you started in the upper left hand corner and moved clockwise around the rectangle how far would you travel before coming back to the upper left hand corner?

Measure the four sides of the square below and record the distance on each line segment.

_____



_____

If you started in the upper left hand corner and moved clockwise around the rectangle how far would you travel before coming back to the upper left hand corner?

Explain how you calculated the distance. _____
_____ .

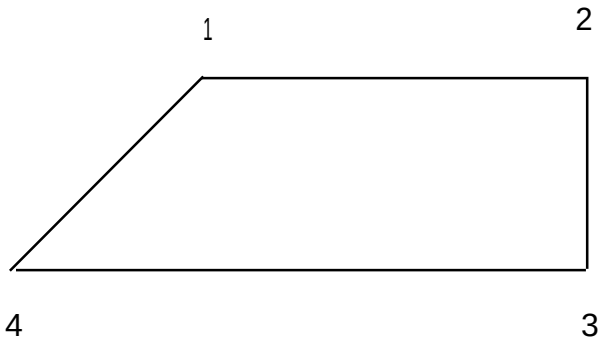
EXPLAINING THE IDEA

The distance around any enclosed object is called the

PERIMETER, often designate by the letter P

EXPLAINING THE IDEA

Look at the shape below.



The distance from Point 1 to Point 2 is 'a'.
The distance from Point 2 to Point 3 is 'b".
The distance from Point 3 to Point 4 is 'c'.
The distance from Point 4 to Point 1 is 'd'.

How would you determine the perimeter of this shape?

Which mathematical operation do you use to calculate the perimeter?

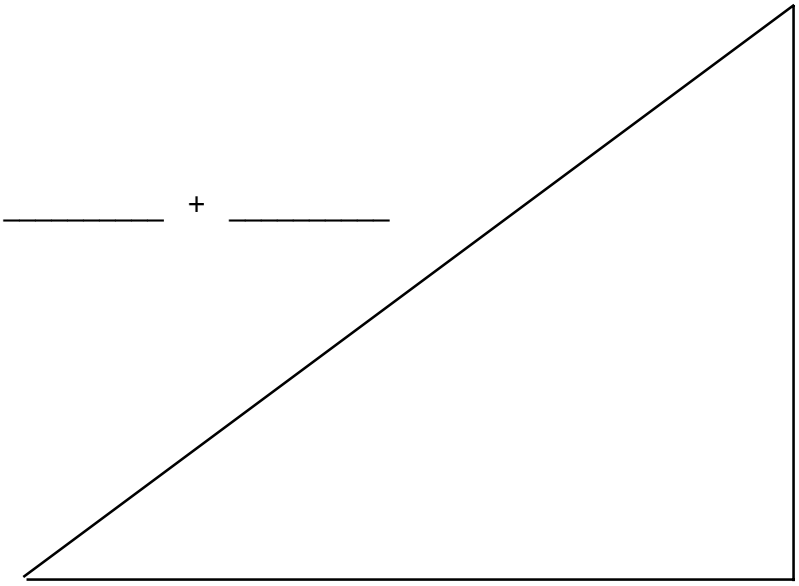
Addition Subtraction Multiplication Division

Using the letters 'a', 'b'. 'c', and 'd', write a math equation for the perimeter of the shape above.

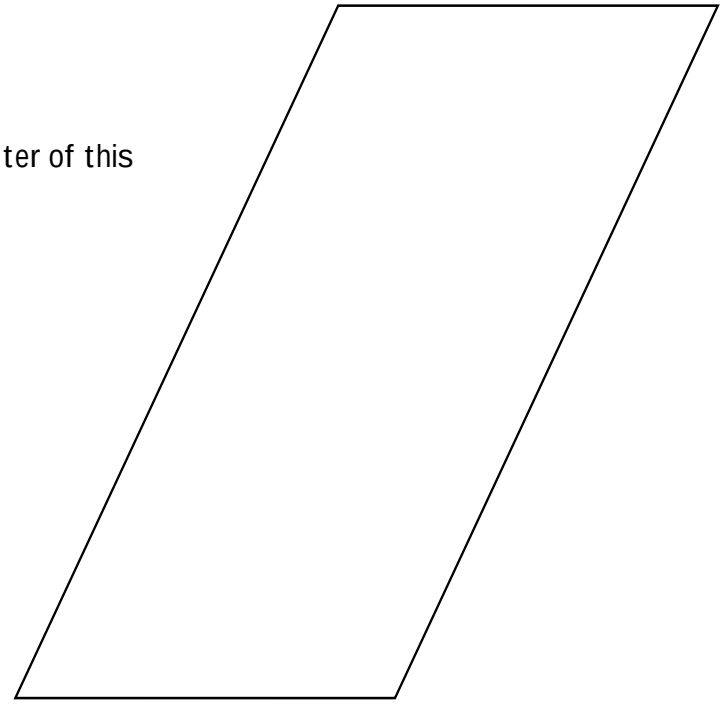
P =

Look at the triangle below. Using a ruler determine its perimeter.

P = _____ + _____ + _____



Using a ruler determine the perimeter of this parallelogram.



Explain how to determine the PERIMETER of any closed shape.

_____

_____

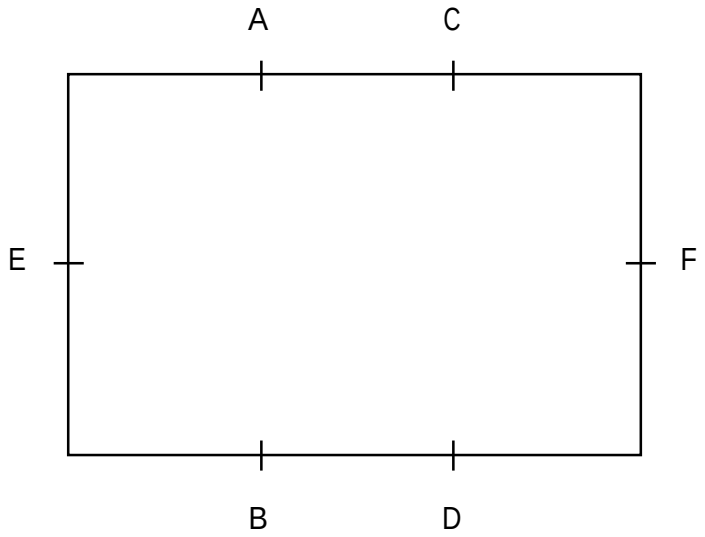
_____

_____ .

EXPLORING THE IDEA

Is it possible to determine the amount of space within the boundaries of the closed shape? This characteristic is a little more complicated to determine.

For the rectangle below, using a ruler measure each side.



The small marks indicate each inch.
The longest side of the rectangle — the LENGTH — is 3 inches.
The shortest side of the rectangle — the WIDTH — is 2 inches.

Connect Point A to Point B.
Connect Point C to Point D.
Connect Point E to Point F.

How many small squares are contained within this rectangle? _____

What is the length of each side of each small square? _____

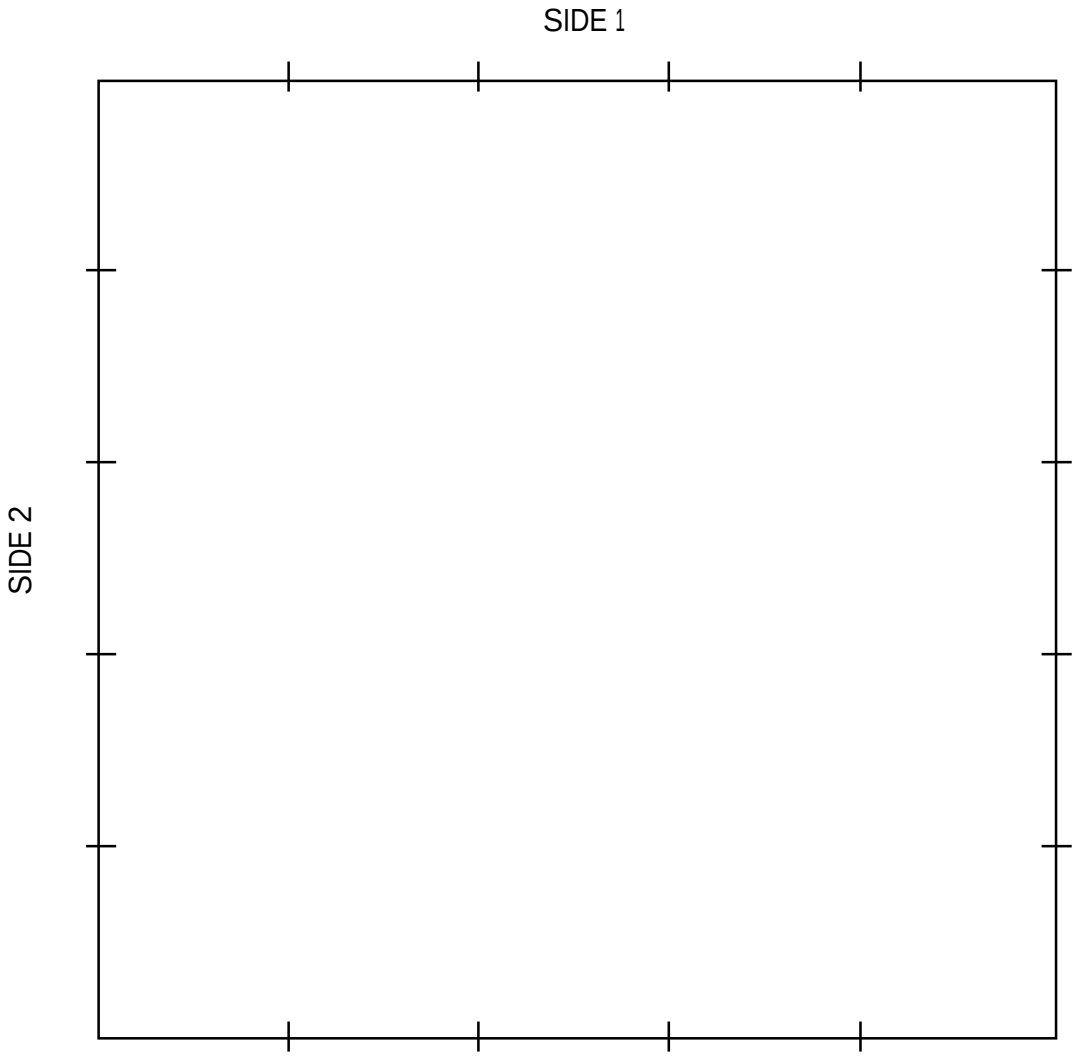
Each small square is one inch on each side.

This rectangle is made up of six one-inch squares.

LENGTH	WIDTH	NUMBER OF SQUARES
3	2	6

What mathematical relationship exists between
the LENGTH, the WIDTH, and the NUMBER OF SQUARES within the rectangle?

A square is a special case of the rectangle. For the square below, measure each side.



The LENGTH of each side of a square is EQUAL.

The small marks indicate each inch.
Connect the small marks that are opposite to one another.

How many small squares are contained within the large square? _____
What is the length of each side of each small square? _____
Each small square is one inch on each side.
The large square is made up of _____ one-inch squares.

LENGTH of Side 1	LENGTH of Side 2	NUMBER OF SQUARES
5 inches	5 inches	_____

What mathematical relationship exists between the
LENGTH of SIDE 1, LENGTH of SIDE 2 and the NUMBER OF SQUARES within the square?

EXPLAINING THE IDEA

The space within a closed polygon is called the

AREA, often designate by the letter A.

The AREA of a RECTANGLE is found by multiplying the LENGTH times the WIDTH.
The mathematical relationship is

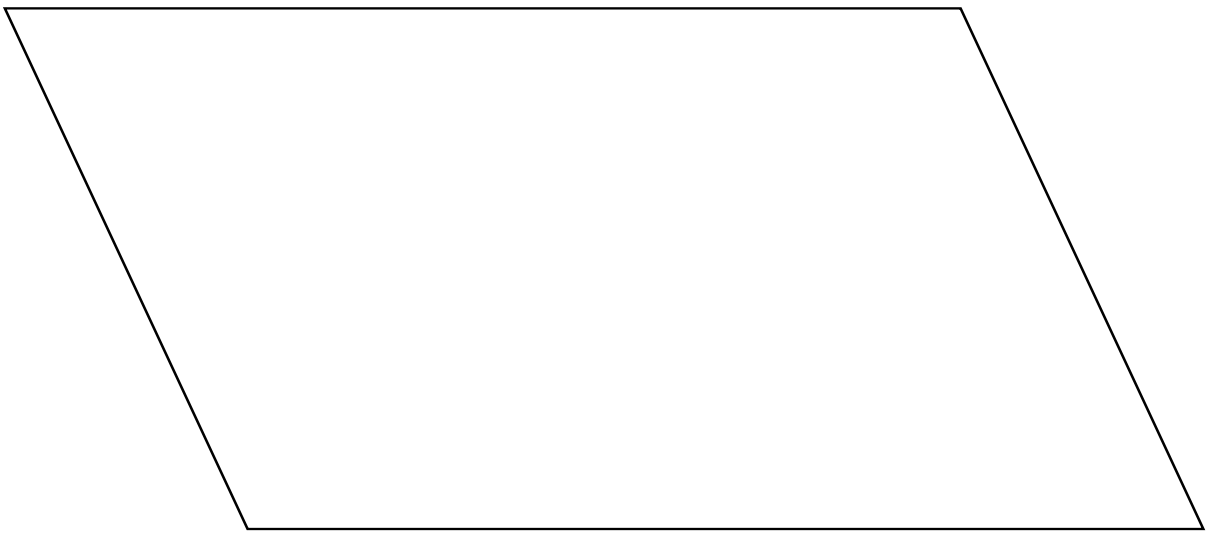
AREA of a Rectangle = LENGTH X WIDTH

The mathematical relationship explaining how the sides of a square are related to its AREA is

AREA of a Square = SIDE 1 X SIDE 2

EXPANDING THE IDEA - PART 1

Look at the parallelogram below. Measure each side.



Explain how would you find the area of the parallelogram above? _____

_____

_____ .

Determining the area of a parallelogram …

Cut out the parallelogram below.

Next, cut along the dashed line.

Place the cut out triangle on the opposite end of the parallelogram.

What is the resulting shape? _____

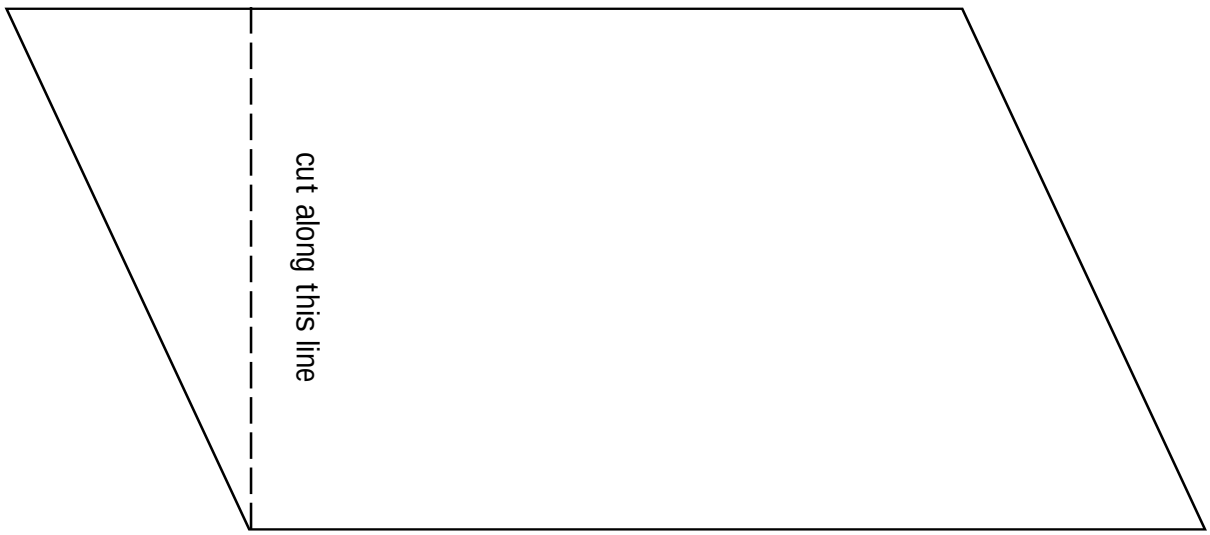
Next, measure the sides of the new shape.

Finally, calculate the area.

What two measurements are needed in order to calculate the area of a parallelogram?

---

cut out the parallelogram



Determining the area of a parallelogram …

What two measurements are needed in order to calculate the area of a parallelogram?

Cut out the parallelogram below.

Next, cut along the dashed line.

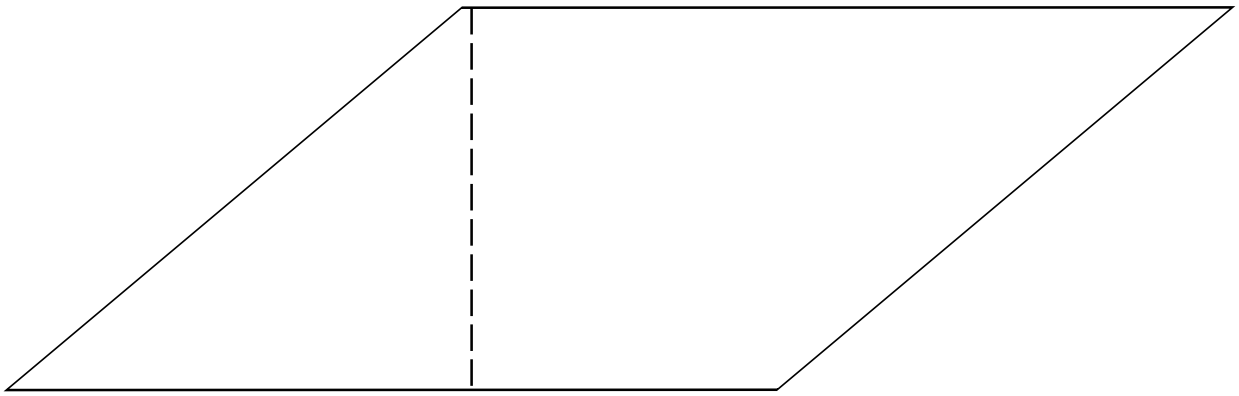
Place the cut out triangle on the other end of the parallelogram.

What is the resulting shape? _____

Using your knowledge of determining the area of a rectangle,
calculate the area of the parallelogram.

---

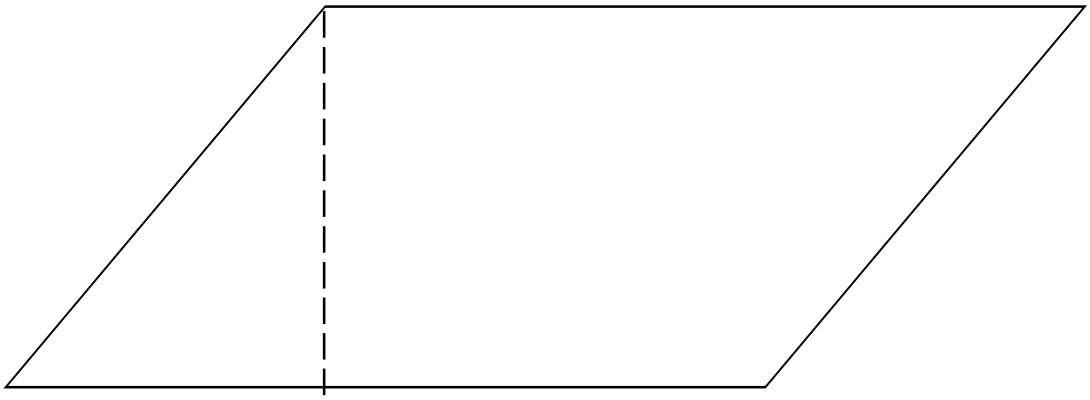
cut out the parallelogram



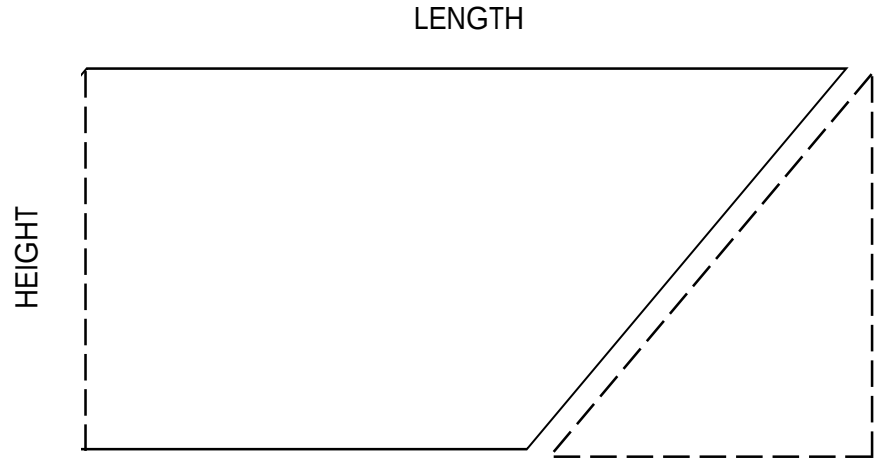
Determining the area of a parallelogram …

Only two measurements are needed in order to calculate the area of a parallelogram.

When the triangle from one end of the parallelogram is moved to the other end, the parallelogram becomes a rectangle.



The area of the rectangle is determined by multiplying the length by the width.



In order to calculate the area of a parallelogram….

First, measure the length of one of the longest sides.

Second, measure the height of the parallelogram.

Finally, multiply the LENGTH by the Height.

AREA of the Parallelogram = LENGTH X HEIGHT

EXPANDING THE IDEA - PART 2

How is the area of the square and the area of a triangle related? _____

_____

_____

Cut out the two squares. Are these two squares the same size?

YES NO

Fold the bottom square along the dashed line. What new shape was made?

_____

Compare the size of the triangle with the size of the square.

_____

AREA of TRIANGLE = what fractional part of the AREA of the SQUARE?

_____ .

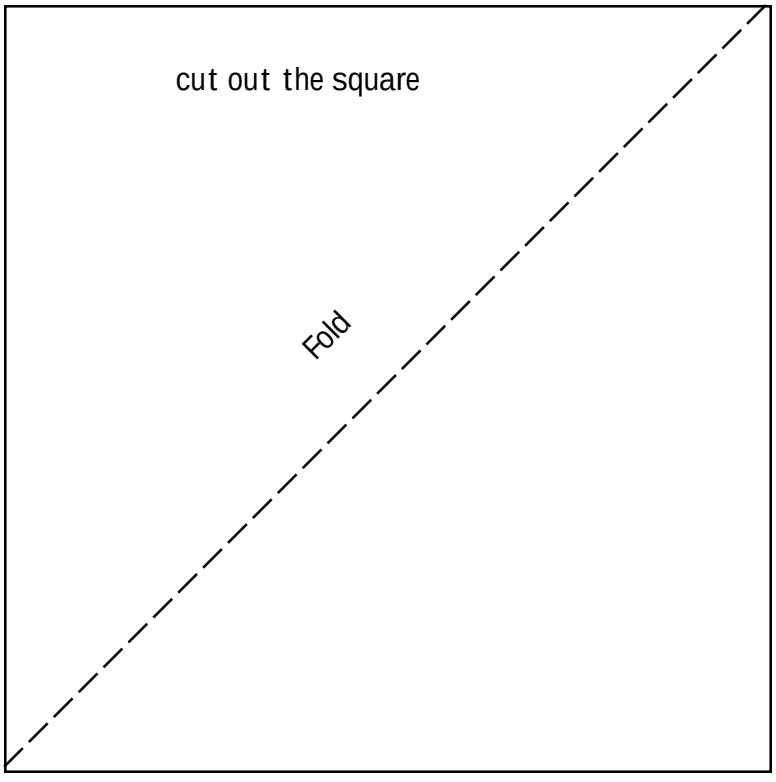
What is the equation for the area of a square?

_____ .

What do you think is the equation for the area of a triangle?

_____ .

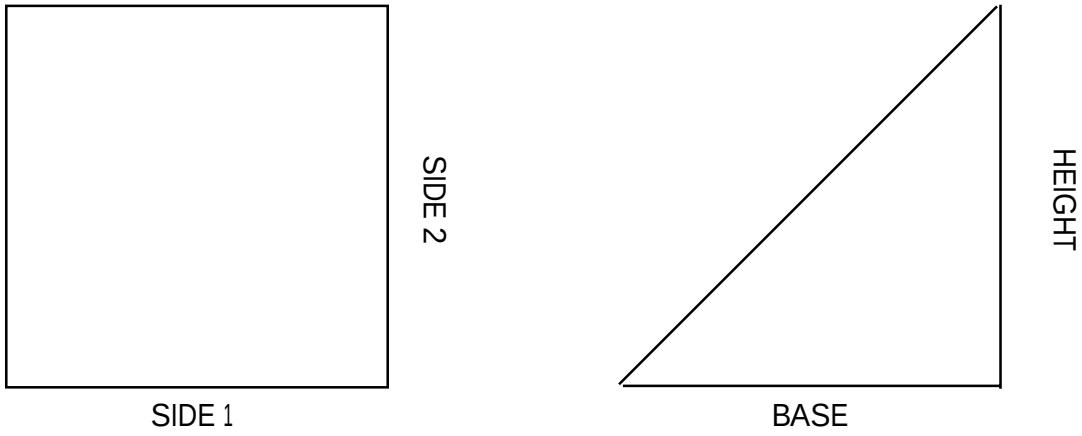
cut out the square



cut out the square

How is the area of the square and the area of a triangle related?

Measure SIDE 1 and SIDE 2 of the square
Measure the BASE and HEIGHT of the triangle. Fill in the chart below.



SQUARE RIGHT TRIANGLE

To determine the relationship cut out the square and the right triangle above.

When the two shapes are cut out, place the triangle on top of the square. Shade

How is the area of the triangle related to the area of the square?
What fractional part of the square is the triangle? _____

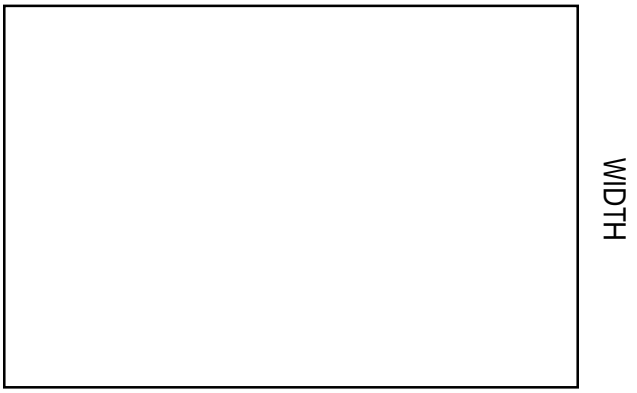
_____

_____ .

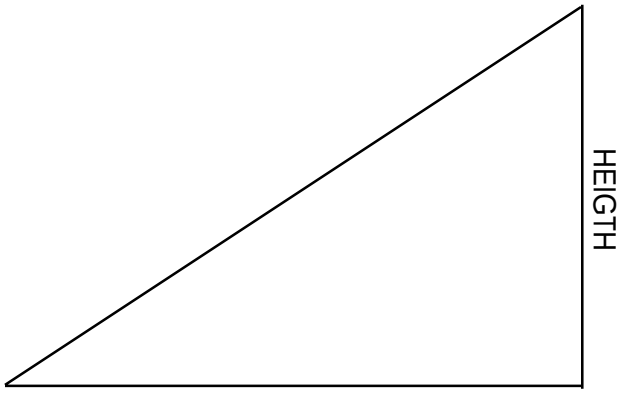
SQUARE	TRIANGLE
SIDE 1: _____	BASE: _____
SIDE 2 : _____	HEIGHT: _____
AREA = SIDE 1 X SIDE 2	AREA = _____

How is the area of the rectangle and the area of a triangle related?

Measure the length and width of the rectangle.
Measure the BASE and HEIGHT of the triangle.
Fill in the chart at the bottom of the page.



RECTANGLE



RIGHT TRIANGLE

To determine the relationship cut out the rectangle and the right triangle above.

When the two shapes are cut out, place the triangle on top of the rectangle.
Trace the shape of triangle onto the rectangle. Shade the triangle.

How is the area of the triangle related to the area of the rectangle?
What fractional part of the rectangle is the triangle? _____

_____

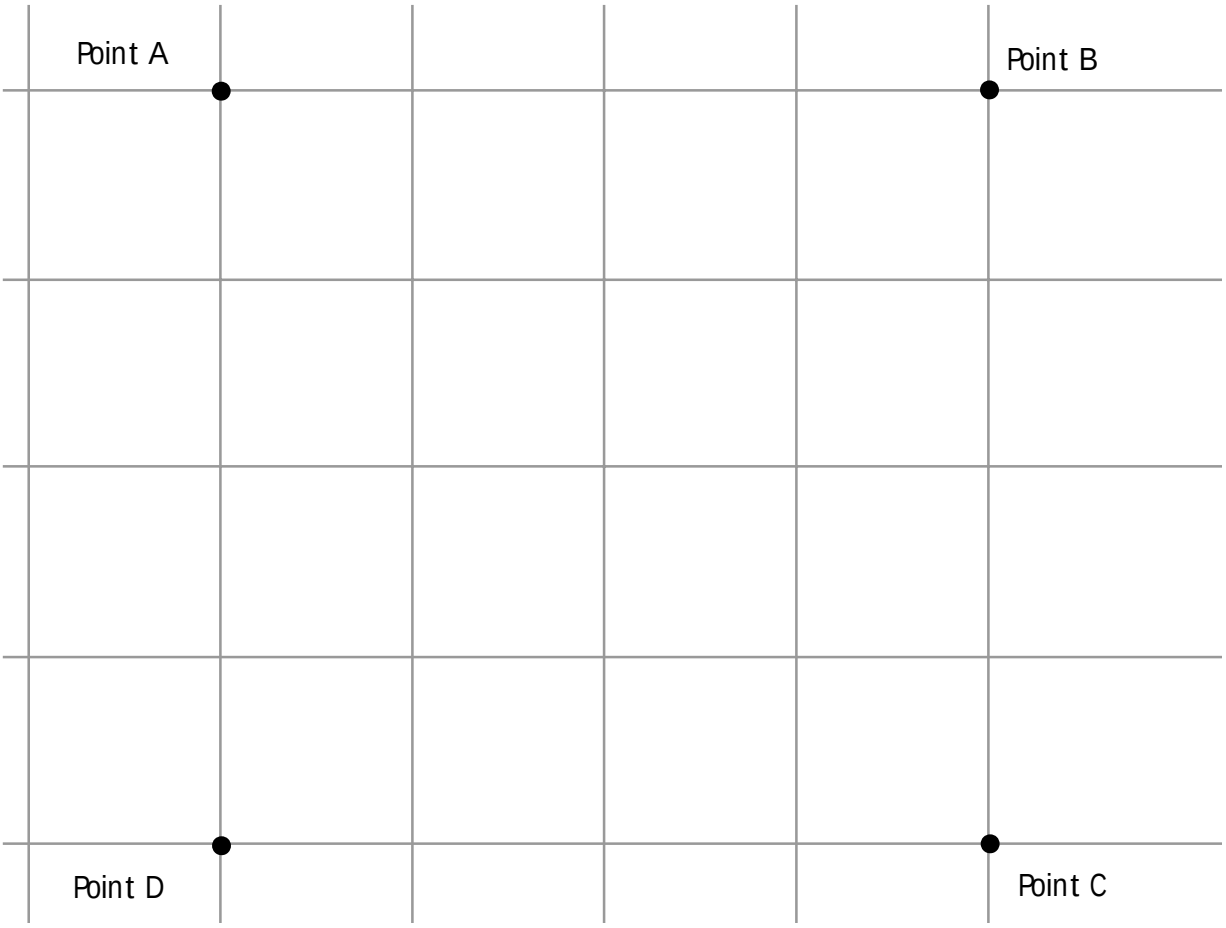
_____ .

RECTANGLE	TRIANGLE
LENGTH: _____	BASE: _____
WIDTH: _____	HEIGTH: _____
AREA = LENGTH X WIDTH	AREA = _____

Connect Point A to Point B; connect Point B to Point C; connect Point C to Point D; and finally, connect Point D to Point A. What shape did you make? _____

What is the math relationship to determine the area of the square?

Area of the Square = _____



The area of Square ABCD is _____ square inches.

If you were to connect Point A to Point C, what shape would you make? _____

Shade Triangle ADC with your pencil.

If the area of the square is 16, predict the area of the triangle.... _____

Now count the number of small squares in the shaded triangle area.
You should have counted 6 complete squares plus 4 halves which is equal to 8.

If the side containing Points A and D is called H,
and the side containing Points D and C is called B,
write an equation expressing the area of Triangle: ADC AREA = _____

The area of the triangle is one-half the area of the rectangle.

Since the area of the rectangle is found by multiplying the length times the height,

the area of the triangle would be one-half the length times the height.

In a triangle the length is referred to as the BASE

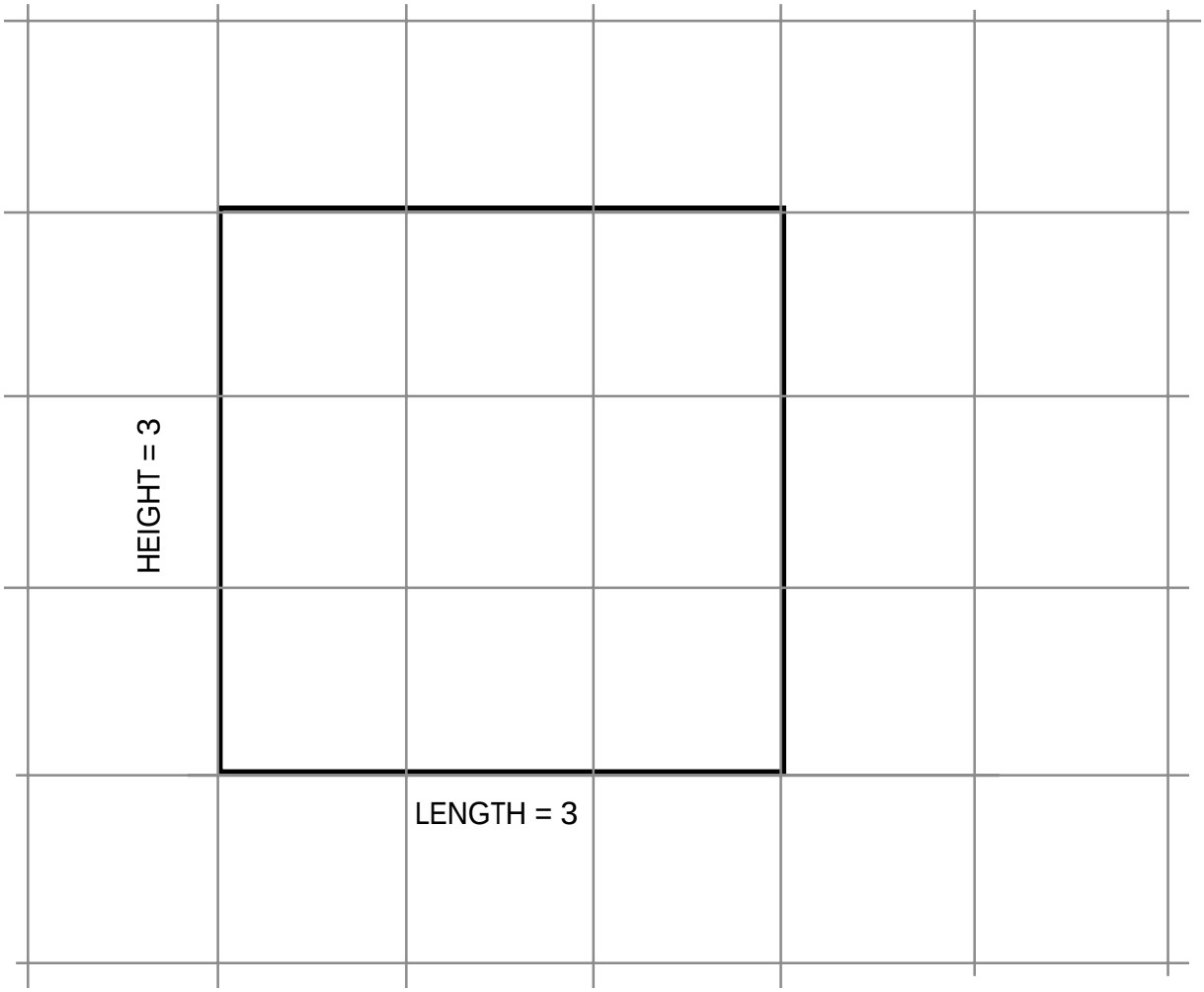
and the width is known as the HEIGHT.

Therefore, the area of the triangle is found by the following relationship:

$$\text{Area of Triangle} = \frac{1}{2} \text{ b x h}$$

Up to this point the investigation of triangles has been limited to right triangles. Does the area relationship stated above apply to equilateral, acute, and isosceles triangles or is there some other relationship? The fundamental question is whether or not the above relationship is true for all shapes of triangles?

To begin to understand the answer to these important questions, first calculate the area of the square below. Lightly shade the area of the square in the drawing below.



How are the LENGTH, HEIGHT and AREA of a square (rectangle) related? _____

What is the LENGTH of the square on the proceeding page? _____

What is the HEIGHT of the square on the proceeding page? _____

What is the area of the square on the proceeding page? _____ .

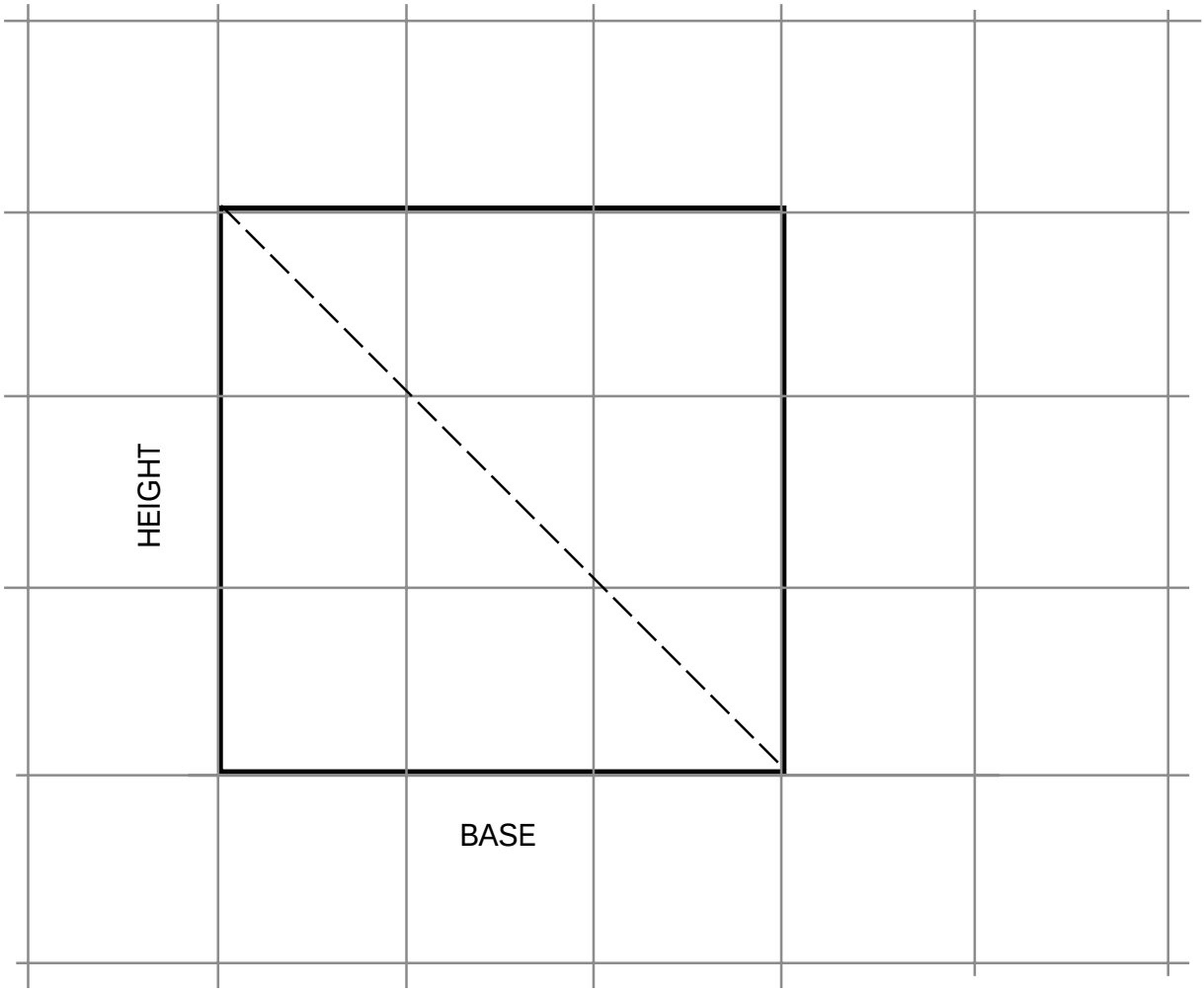
If you were to make a right triangle by drawing a line from the upper left corner to the lower right corner, what do you predict the area of the triangle to be? _____ .

Using your pencil draw a line from the upper left corner to the lower right corner of the square. Next, shade the triangle in the lower portion of the square.
Count the squares in the lower triangle. Was your prediction correct? YES NO

Finally, using the formula for the area of a triangle calculate the area.

AREA of the TRIANGLE = _____ X _____ X _____

Is the area of the triangle half the area of the square? YES NO



Evaluate the triangle below. What type of triangle is this?

ISOSCELES SCALENE EQUILATERAL

The BASE of this triangle is the same as the base of the proceeding problem.

BASE = 3

What is the HEIGHT of this triangle?

HEIGHT = _____

Is the area of the triangle in the diagram below still half the area of the square? YES NO

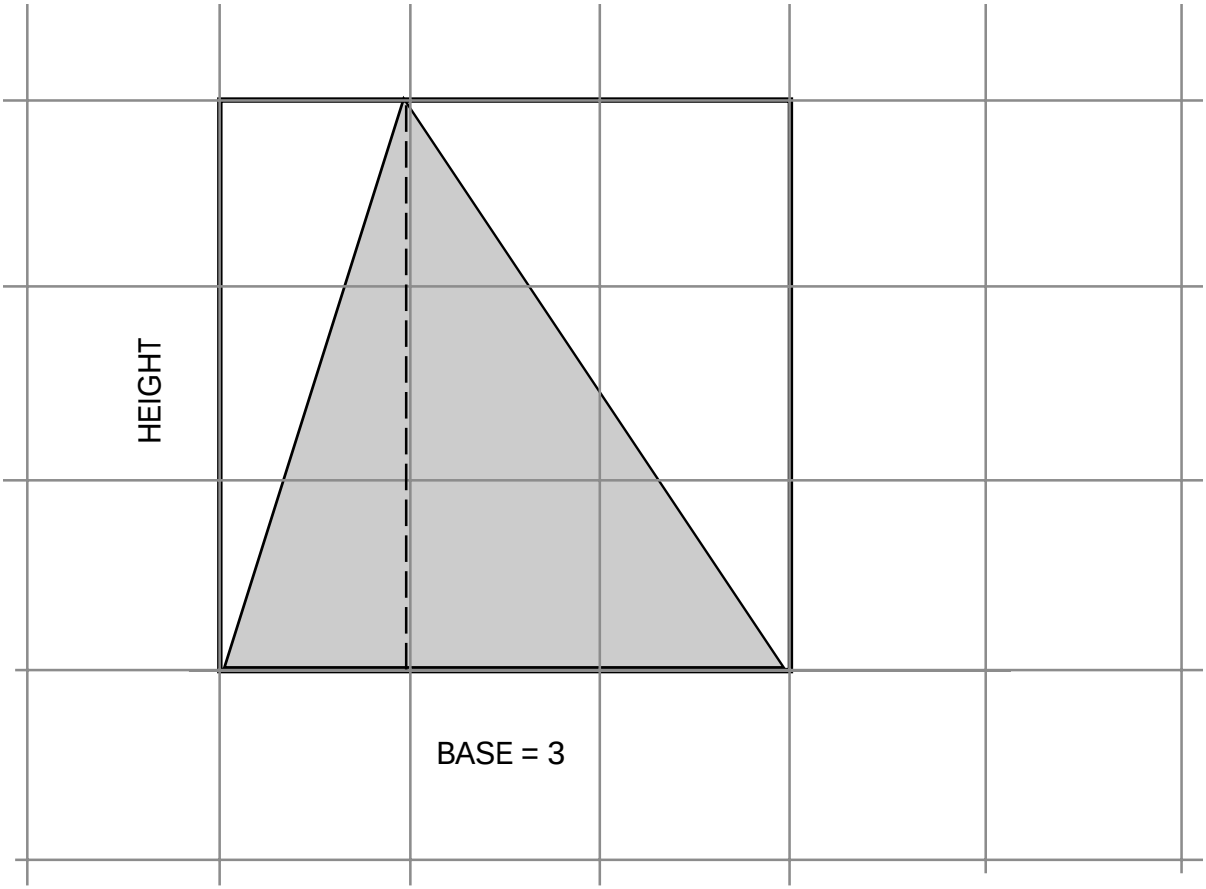
How can you tell for sure?

Darken the vertical dashed line. Next, trace the two triangles onto another sheet of paper And cut them out.

Can the two triangles be arranged onto the diagram below to completely fill in the square?

What does this demonstrate? _____ .

If this SCALENE TRIANGLE is one half the area of the square, then the formula for the area of the scalene triangle is the same as for the right triangle.



Evaluate the triangle below. What type of triangle is this?

ISOSCELES SCALENE EQUILATERAL

Measure the base and height of this triangle.

BASE = _____ HEIGHT = _____

Is the area of this isosceles triangle half the area of the square? YES NO

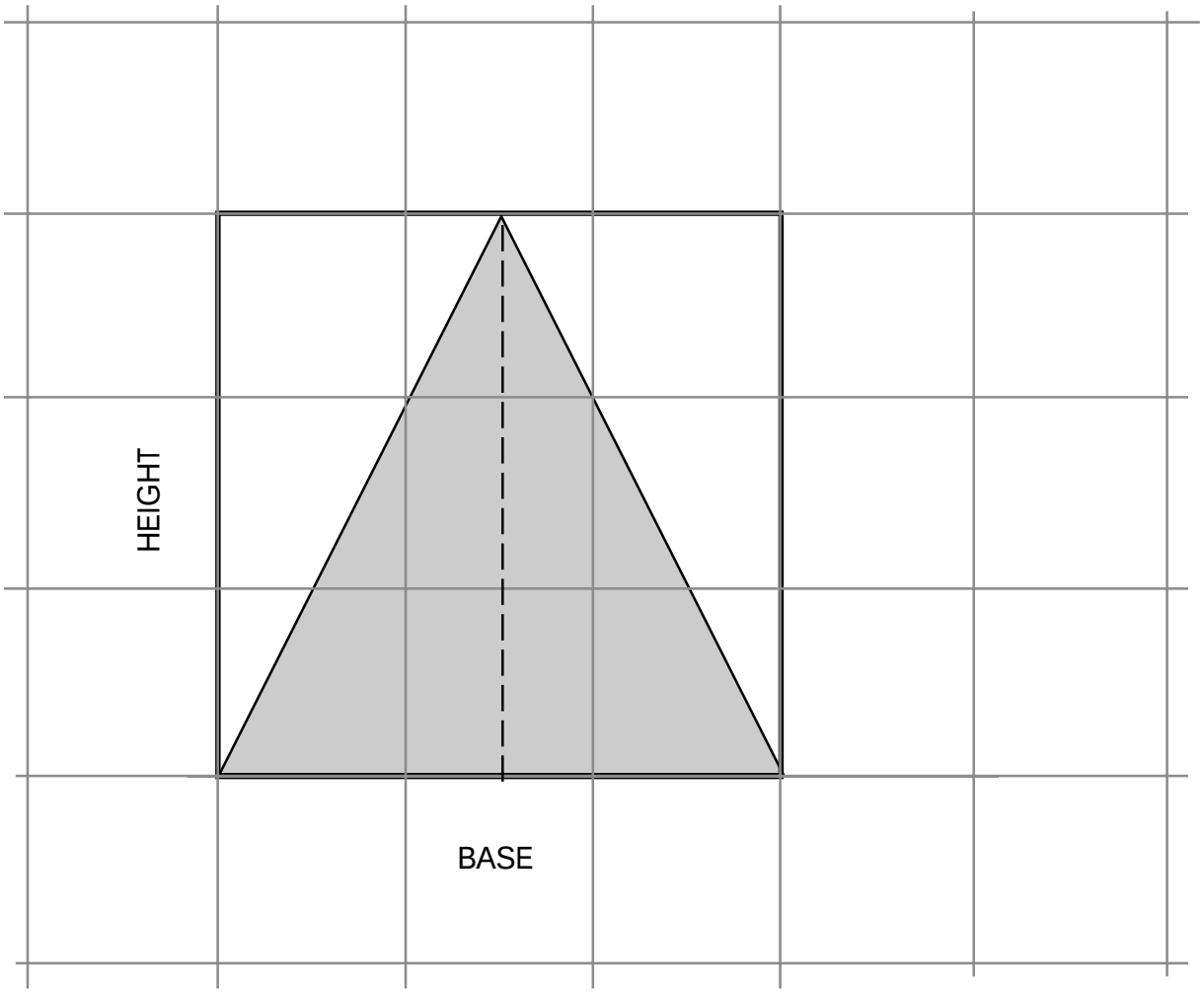
How can you tell for sure?

Darken the vertical dashed line. Next, trace the two triangles onto another sheet of paper And cut them out.

Can the two triangles be arranged onto the diagram below to completely fill in the square?

What does this demonstrate? _____ .

If this ISOSCELES TRIANGLE is one half the area of the square, then the formula for the area of the isosceles triangle is the same as for the right triangle.



Evaluate the triangle below. What type of triangle is this?

ISOSCELES SCALENE EQUILATERAL

Measure the base and height of this triangle.

BASE = _____ HEIGHT = _____

Is the area of this scalene triangle half the area of the square? YES NO

How can you tell for sure?

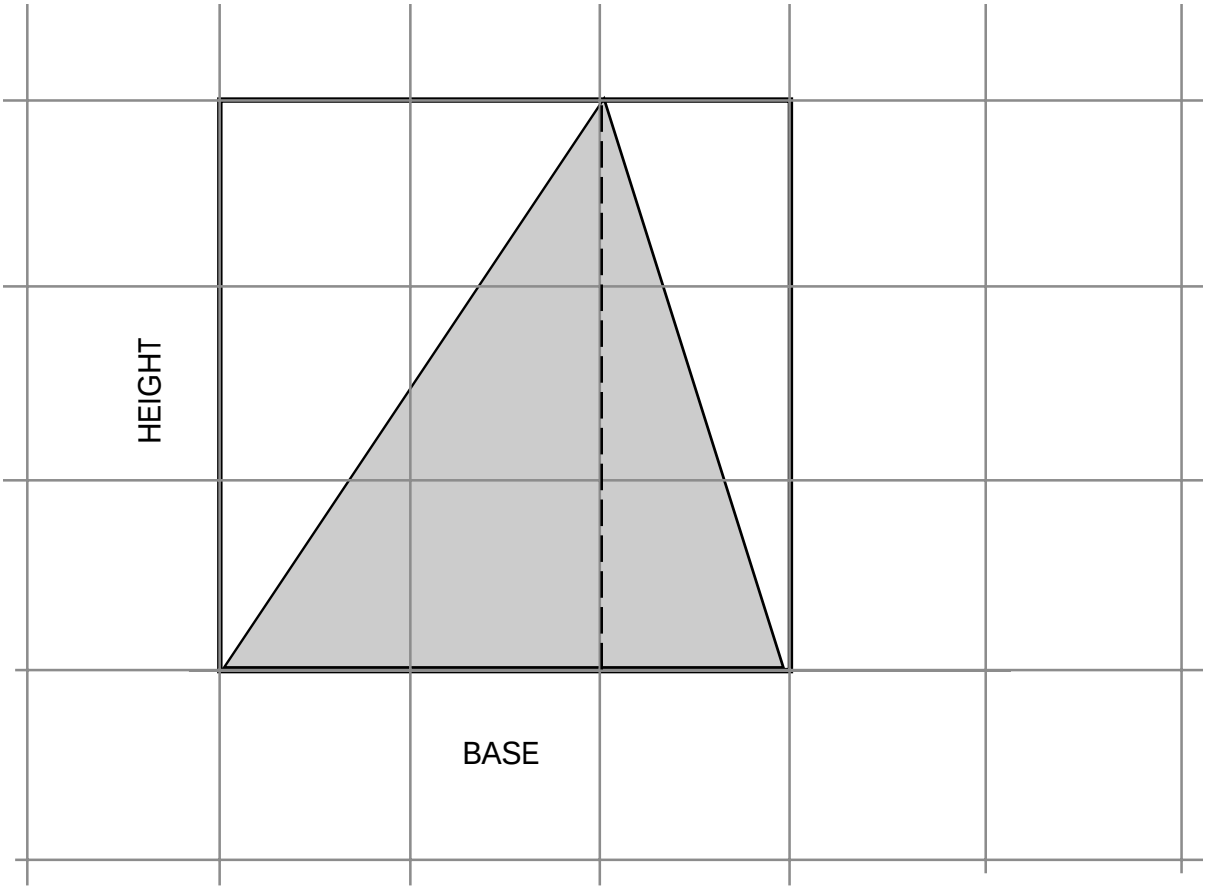
Darken the vertical dashed line. Next, trace the two triangles onto another sheet of paper And cut them out.

Can the two triangles be arranged onto the diagram below to completely fill in the square?

What does this demonstrate? _____ .

If this SCALENE TRIANGLE is one half the area of the square, then the formula for the area of the SCALENE TRIANGLE is the same as for the right triangle. Write the relationship below:

AREA of TRIANGLE = _____ X _____ X _____



Evaluate the triangle below. What type of triangle is this?

ISOSCELES SCALENE EQUALATERIAL

Measure the base and height of this triangle.

BASE = _____ HEIGHT = _____

Is the area of this ISOSCELES TRIANGLE half the area of the square? YES NO

How can you tell for sure?

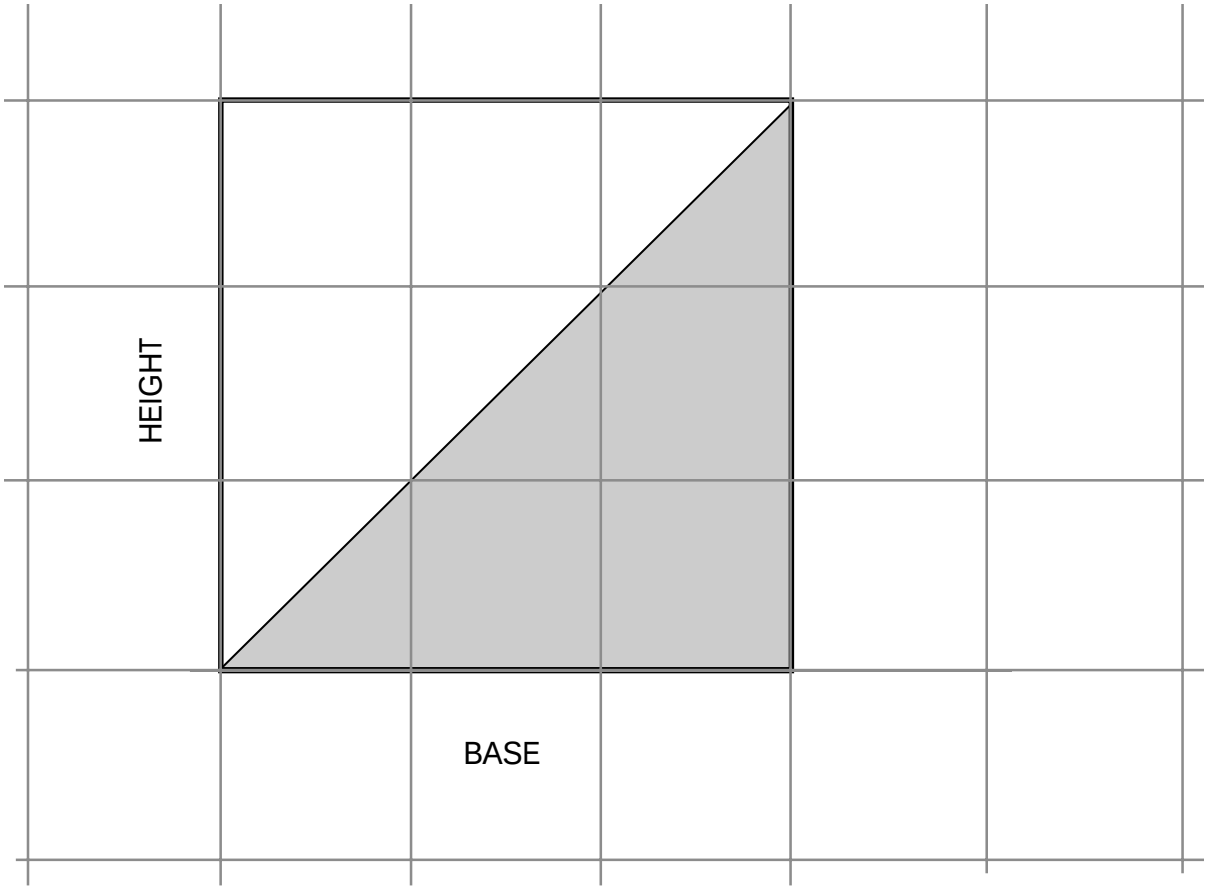
Darken the vertical dashed line. Next, trace the two triangles onto another sheet of paper And cut them out.

Can the two triangles be arranged onto the diagram below to completely fill in the square?

What does this demonstrate? _____ .

Write the mathematical relationship between the area, base and height of a triangle.

AREA of TRIANGLE = _____ X _____ X _____



Evaluate the triangle below. What type of triangle is this?

ISOSCELES SCALENE EQUALATERIAL

Measure the base and height of the rectangle.

BASE = _____ HEIGHT = _____

AREA of RECTANGLE = _____

Is the area of this triangle half the area of the square? YES NO

How can you tell for sure?

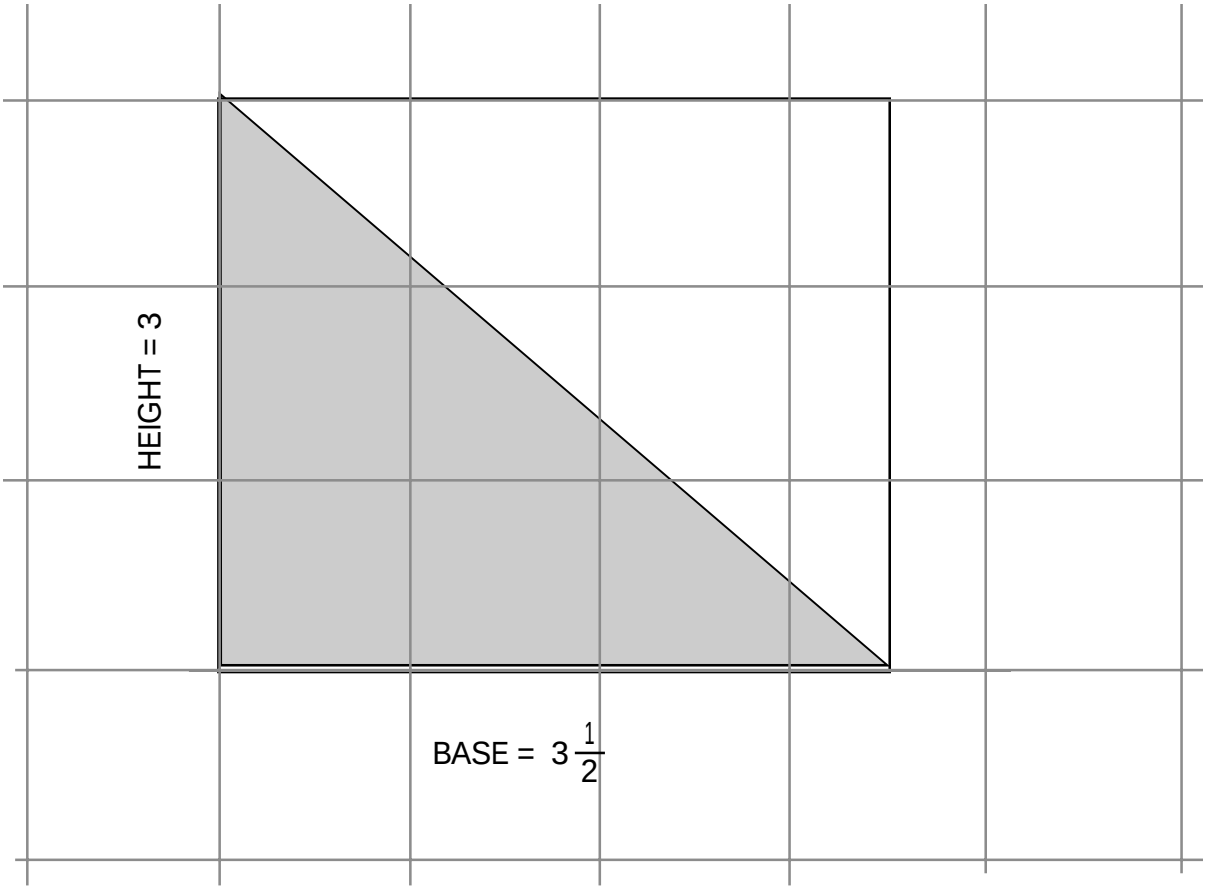
Next, trace the triangle onto another sheet of paper and cut it out.

Can this triangle be set onto the diagram below to completely fill in the square?

What does this demonstrate? _____ .

State the area of the triangle in relationship to the base and height of the rectangle.

AREA of TRIANGLE = _____ X _____ X _____



Evaluate the triangle below. What type of triangle is this?

ISOSCELES SCALENE EQUALATERIAL

Measure the base and height of the rectangle.

BASE = _____ HEIGHT = _____

AREA of RECTANGLE = _____

Is the area of this triangle half the area of the square? YES NO

How can you tell for sure?

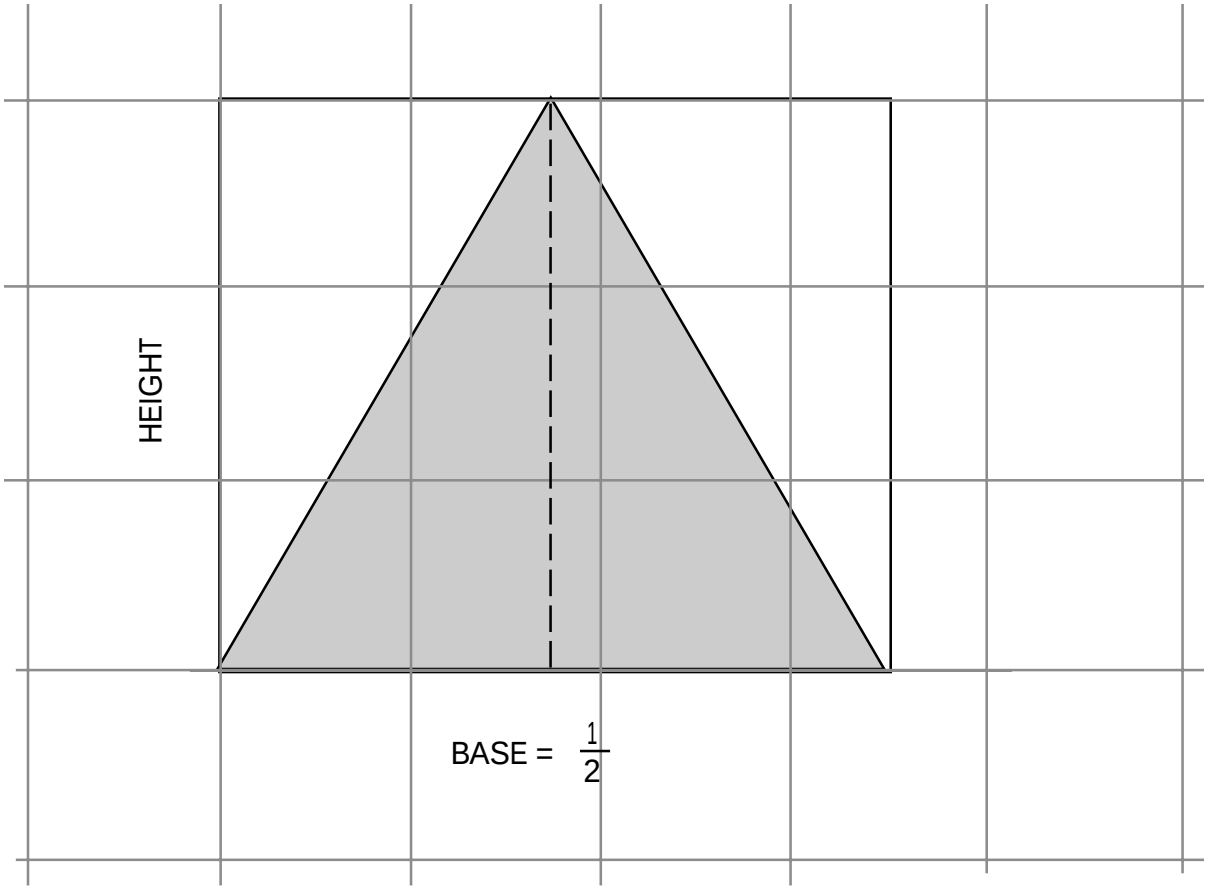
Next, trace the triangle onto another sheet of paper and cut it out.

Can this triangle be set onto the diagram below to completely fill in the square?

What does this demonstrate? _____ .

State the area of the triangle in relationship to the base and height of the rectangle.

AREA of TRIANGLE = _____ X _____ X _____



The area of any triangle is one-half the area of the rectangle.

Since the area of the rectangle is found by multiplying the length times the height,

the area of any triangle would be one-half the length times the height.

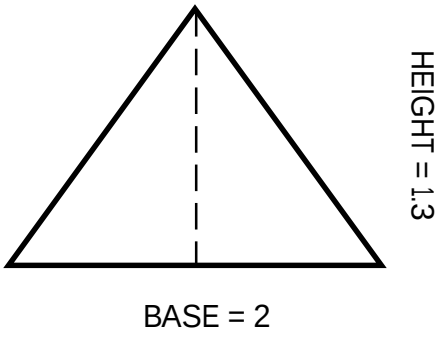
Since the length of a triangle is referred to as the BASE, b

and the width is known as the HEIGHT, h.

Then the area of any triangle is found by the following relationship:

$$\text{Area of Triangle} = \frac{1}{2} b \times h$$

EXPLORING THE IDEA



Make five copies of the triangle by tracing it onto another piece of paper.

Cut out each new triangle.

Place the triangles together in such a way to form an closed polygon.

How many sides does this polygon have? _____

A polygon with five sides is called a

PENTAGON.

Make a simple sketch of a pentagon…

What is the perimeter of this pentagon? _____

How do you calculate the perimeter of a pentagon?

Did you … ADD SUBTRACT MULTIPLY DIVIDE circle your answer

Let the letter 'P' represent the perimeter of the pentagon …
and let the letter 's' represent the length of one side of the pentagon …
write an equation expressing the perimeter of the pentagon….

P = _____

If the length of each side of a pentagon is equal, then the PERIMETER of the pentagon is equal to five times the length of one side.

Calculate the area of one triangle that makes up the pentagon.

AREA of one triangle = _____

If you know the area of one of the triangles,
how can you use this information to determine the area of the pentagon?
Explain how you would calculate the area of a pentagon? _____
_____
_____ .

The area of the pentagon is determined by multiplying the area of one of the triangles by five.

Calculate the area of the pentagon you formed. _____ .

Make six copies of the triangle by tracing it onto another piece of paper.

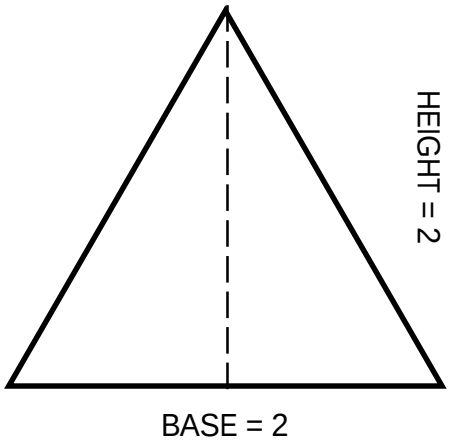
Cut out each new triangle.

Place the triangles together in such a way to form an closed polygon.

How many sides does this polygon have? _____

A polygon with six sides is called a

HEXAGON.



Make a simple sketch of a hexagon….

What is the perimeter of this hexagon? _____

How do you calculate the perimeter of a hexagon?

Did you … ADD SUBTRACT MULTIPLY DIVIDE circle your answer

Let the letter 'P' represent the perimeter of the hexagon …
and let the letter 's' represent the length of one side of the hexagon …
write an equation expressing the perimeter of the hexagon….

P = _____

If the length of each side of a hexagon is equal, then the PERIMETER of the hexagon is equal to six times the length of one side.

Calculate the area of one triangle that makes up the hexagon.

AREA of one triangle = _____

If you know the area of one of the triangles,
how can you use this information to determine the area of the hexagon?
Explain how you would calculate the area of a hexagon? _____
_____
_____ .

The area of the hexagon is determined by multiplying the area of one of the triangles by six.

Calculate the area of the hexagon you formed. _____ .

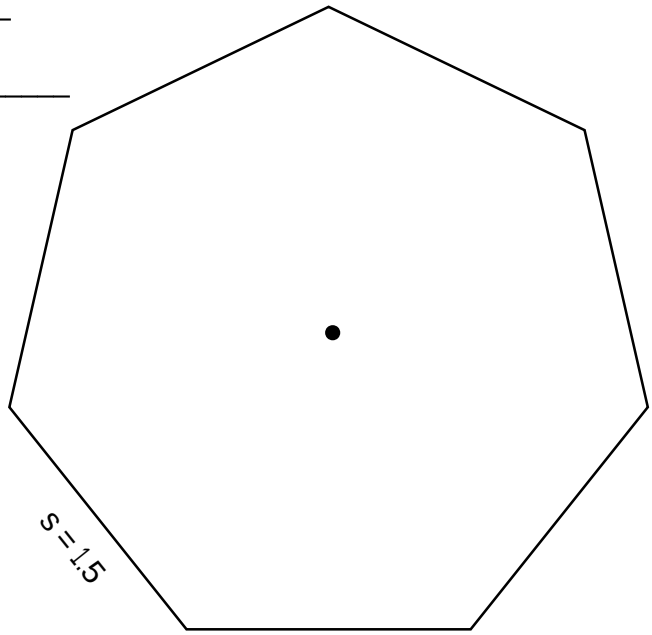
Calculate the perimeter and area of the following polygons....

How many sides does this polygon have? _____

A polygon with seven sides is called a SEPTIGON.

What is the length of each side? _____

What is the perimeter of a septigon? _____



SEPTIGON

From the center point draw a line segment to each vertex. The first two are done for you..

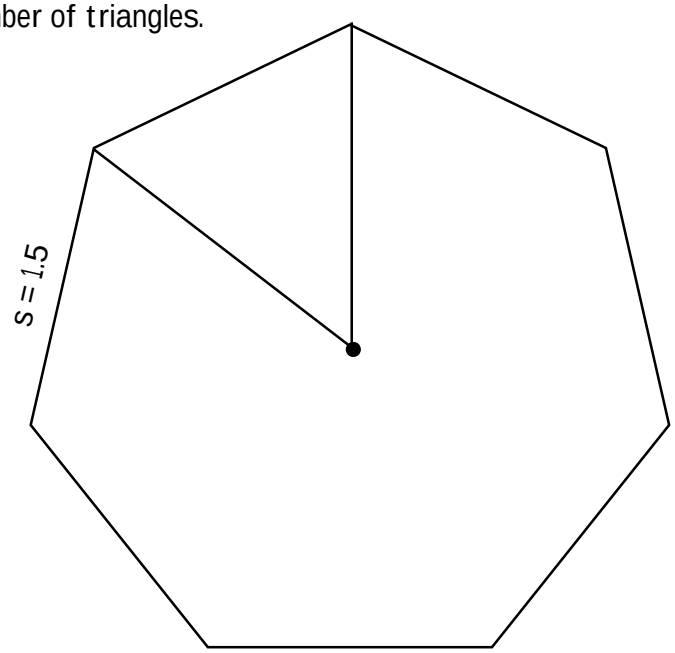
How many triangles make up a septigon? _____

To calculate the area of the septigon, multiply the area of the triangle by the number of triangles.

To determine the area of one triangle
1 - find the point halfway between one side.

2- Draw a line from that point to the center of the septigon. Measure that line segment. This is the height of the triangle.

3 - Determine the Area of one triangle.



Determine the area of the septigon....
• multiply the area of one triangle by the number of triangles

AREA of Septigon = _____

How many sides does this polygon have? _____

A polygon with eight sides is called a OCTAGON.

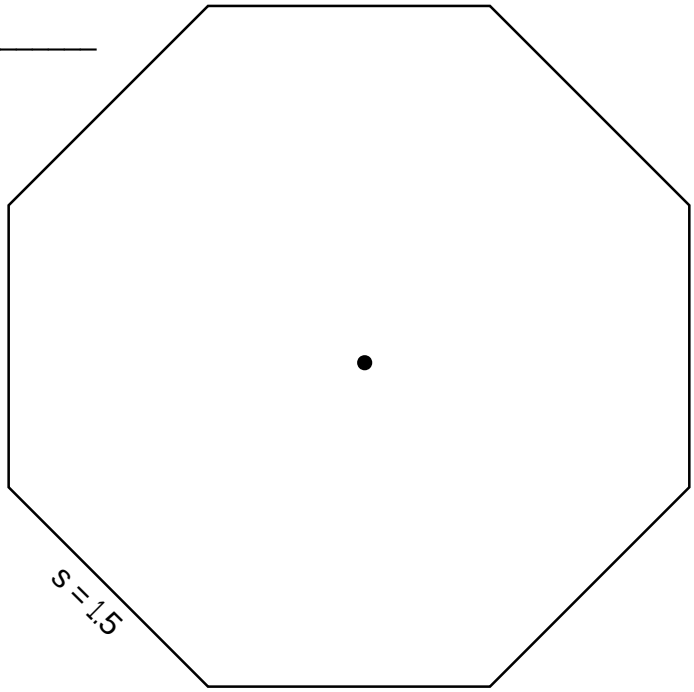
What is the length of each side? _____

What is the perimeter of a octagon? _____

Explain how to calculate the perimeter of any polygon.... _____

_____

_____ .



OCTAGON

From the center point draw a line segment to each vertex. The first two are done for you.. The dashed line represents the height of the octagon.

How many triangles make up a octagon? _____

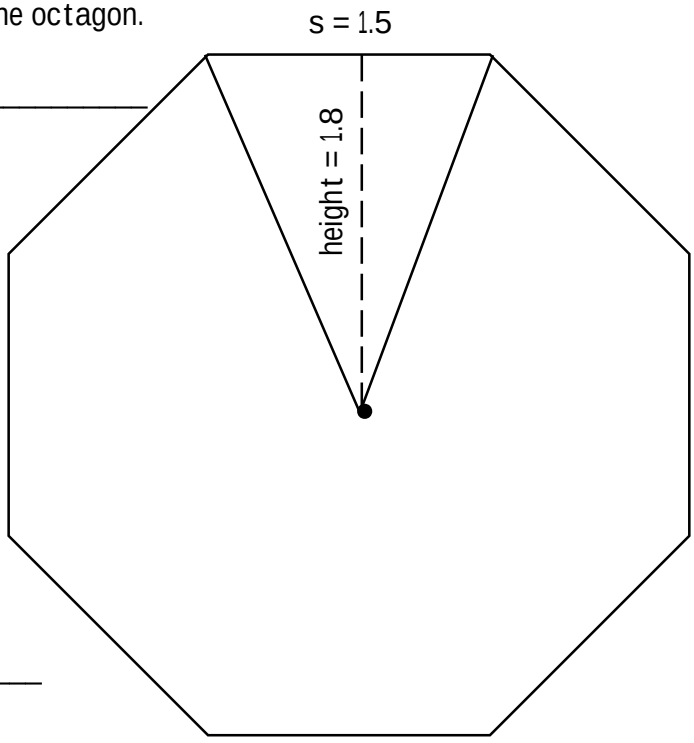
To calculate the area of the octagon, multiply the area of the triangle by the number of triangles.

What is the area of one of the triangles?

AREA of Octagon = _____

Explain how to calculate the area of any polygon.... _____

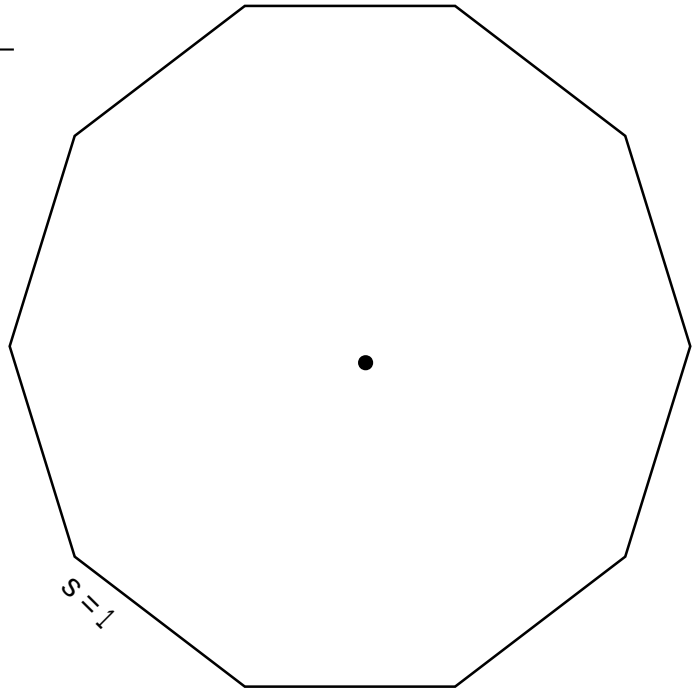
_____ .



OCTAGON

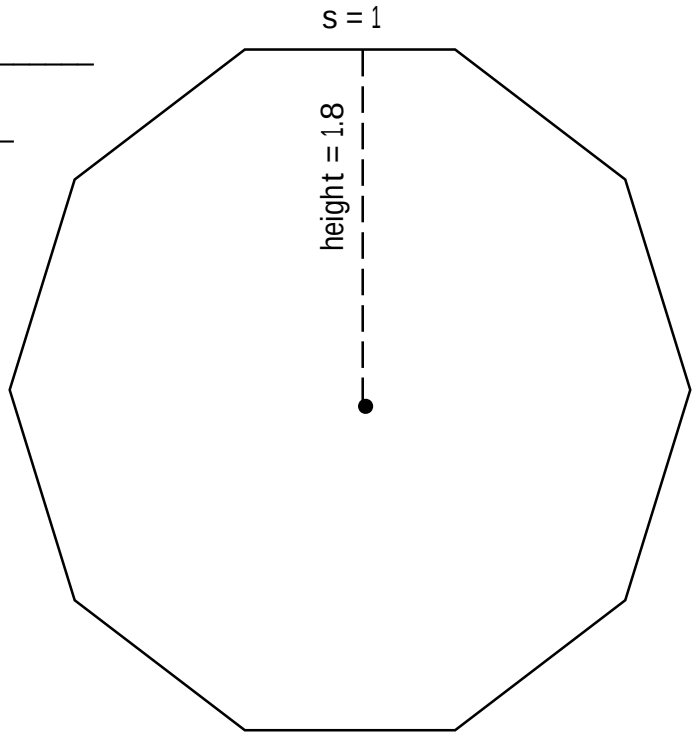
A polygon with ten sides is called a DECAGON.

Calculate the perimeter of this decagon....



DECAGON

Calculate the area of this decagon....



DECAGON

EXPLAINING THE IDEA

The perimeter of a polygon is the distance around the polygon.

The perimeter is determined by either adding the length of each of the sides, or

multiplying the length of on side by the number of sides of the polygon.

Perimeter = length of side 1 + length of side 2 + length of side 3 + ... + length of last side

Perimeter = (number of sides) (length of one side)

The area of a polygon is determined by multiplying the number of triangles within the polygon times the area of one of the triangles.

Area = (number of triangles) (area of one triangle)

EXPANDING THE IDEA - PART 1

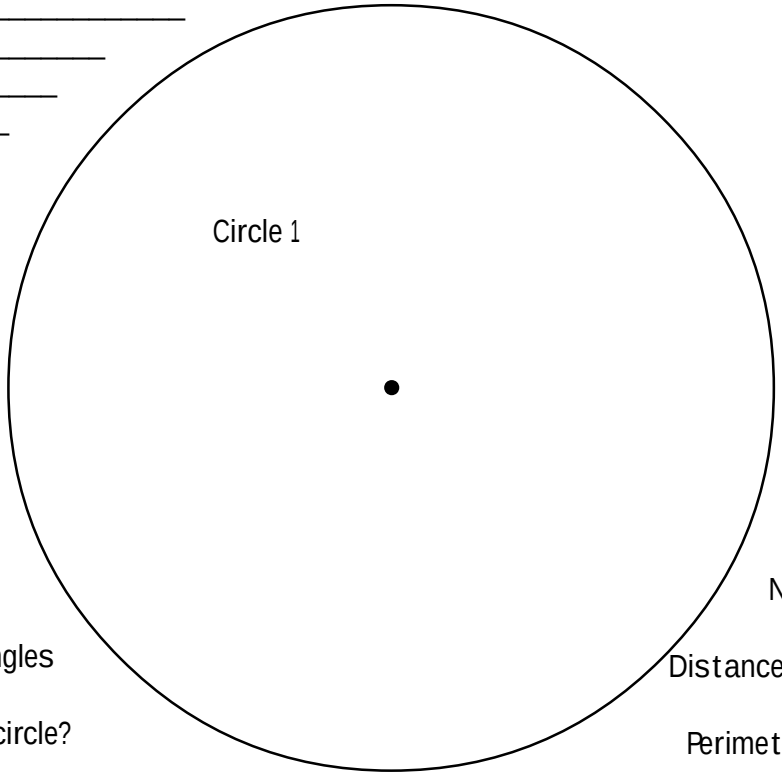
How is it possible to determine the distance around a shape that does not have any sides but is one continuous curve? And how is possible to determine the amount of space contained within that closed shape. The circle presents just such a difficulty. What are your thoughts on how this could be done? _____
_____
_____
_____
_____
_____ .

Using a string measuring around the circle.

Next, using a ruler measure the length of the string.

Is it possible to use your knowledge of the perimeter of triangles to help determine the distance around the circle?
_____

_____ .



_____

Number of Sides: 3

Distance around Circle: $12\frac{1}{2}$ inches

Perimeter of Polygon: _____

Difference between lengths: _____

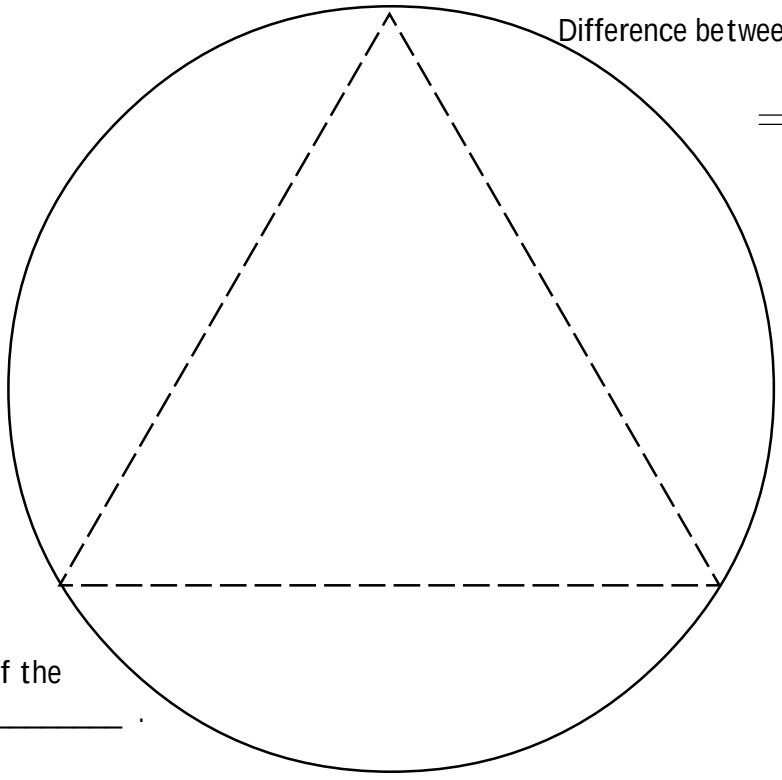
_____

Using another string, measure around the perimeter of the triangle.

Compare the two strings.

Are they very close in length? _____

What would happen if the number of sides of the polygon increases? _____ .



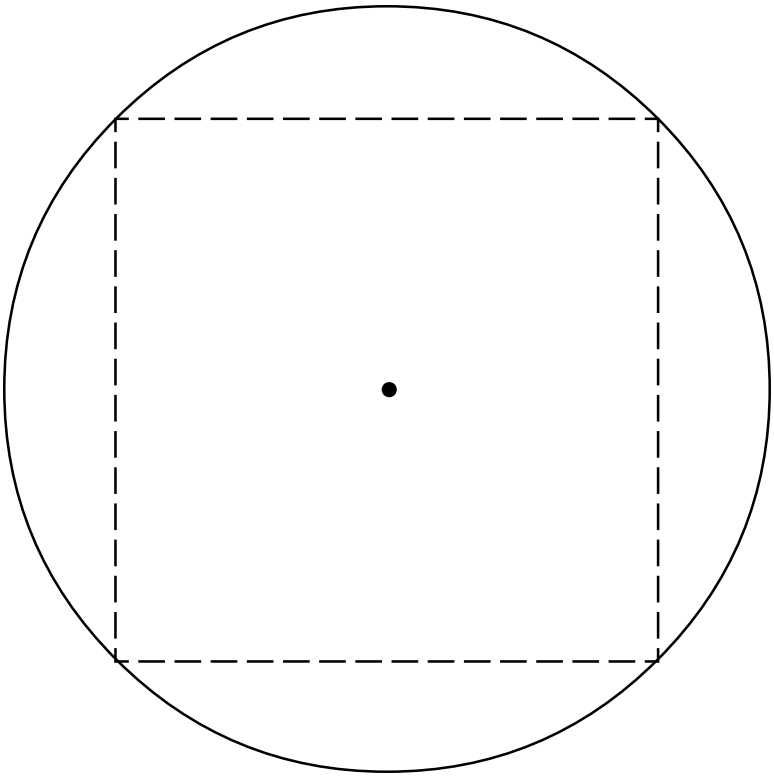
Measure the distance around the polygon that is within the circle.

Number of Sides: 4: Square

Distance around Circle: $12\frac{1}{2}$ inches

Perimeter of Polygon: _____

Difference between lengths: _____

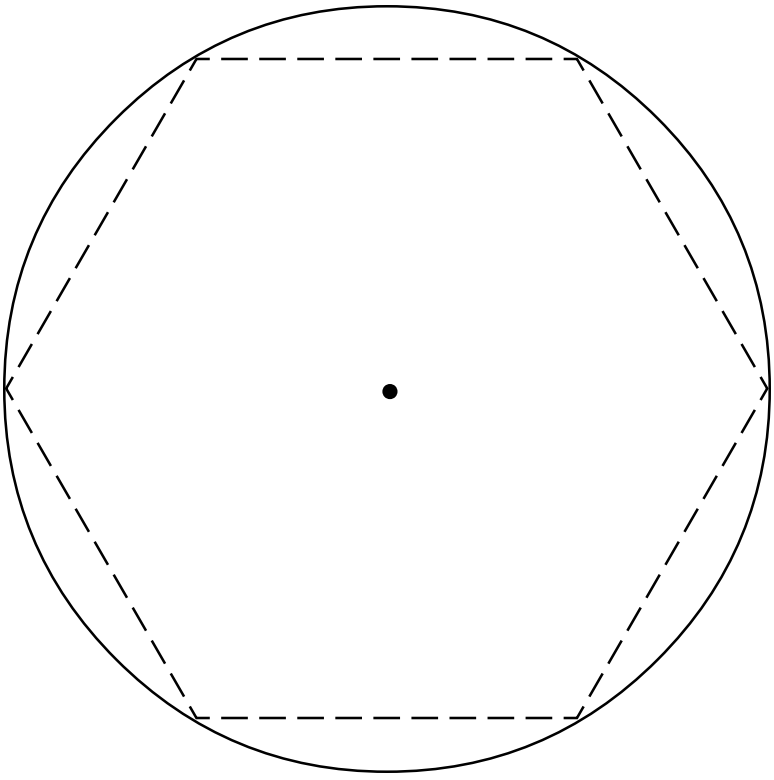


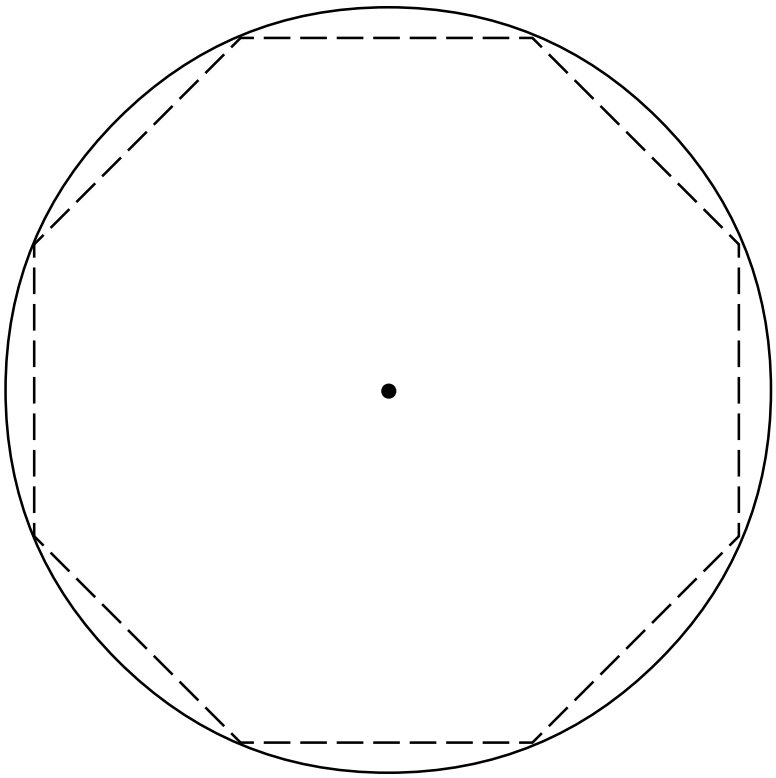
Number of Sides: 6: Hexagon

Distance around Circle: $12\frac{1}{2}$ inches

Perimeter of Polygon: _____

Difference between lengths: _____



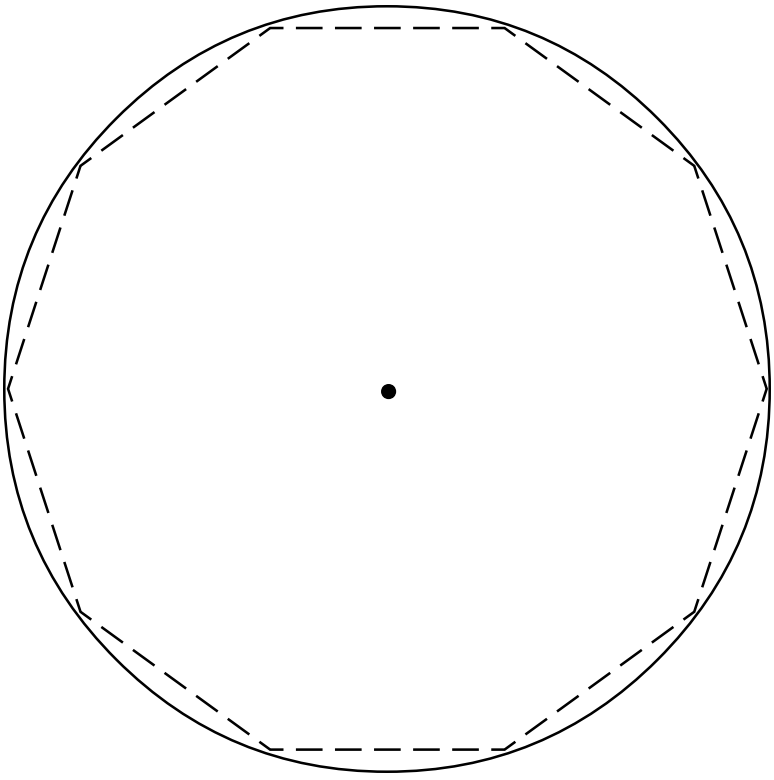


Number of Sides: 8: Octagon

Distance around Circle: $12\frac{1}{2}$ inches

Perimeter of Polygon: _____

Difference between lengths: _____



Number of Sides: 10: Decagon

Distance around Circle: $12\frac{1}{2}$ inches

Perimeter of Polygon: _____

Difference between lengths: _____

Complete the chart below....

Number of Sides of the Polygon	Distance Around the Circle	Perimeter of Polygon
3	12 1/2 inches	_____
4	12 1/2 inches	_____
6	12 1/2 inches	_____
8	12 1/2 inches	_____
10	12 1/2 inches	_____

As the sides of the polygon increase, the perimeter approaches the distance around the circle.

The distance around the circle is called the

CIRCUMFERENCE.

If the number of sides of the polygon continues to increase, what do you predict the perimeter of the polygon will be?

_____
_____ .

Determining the circumference using this method is not only difficult, it is not very precise.

Is there a relationship between the diameter of the circle and the circumference of a circle?

In this problem ...

- the circumference of the circle is _____ inches.

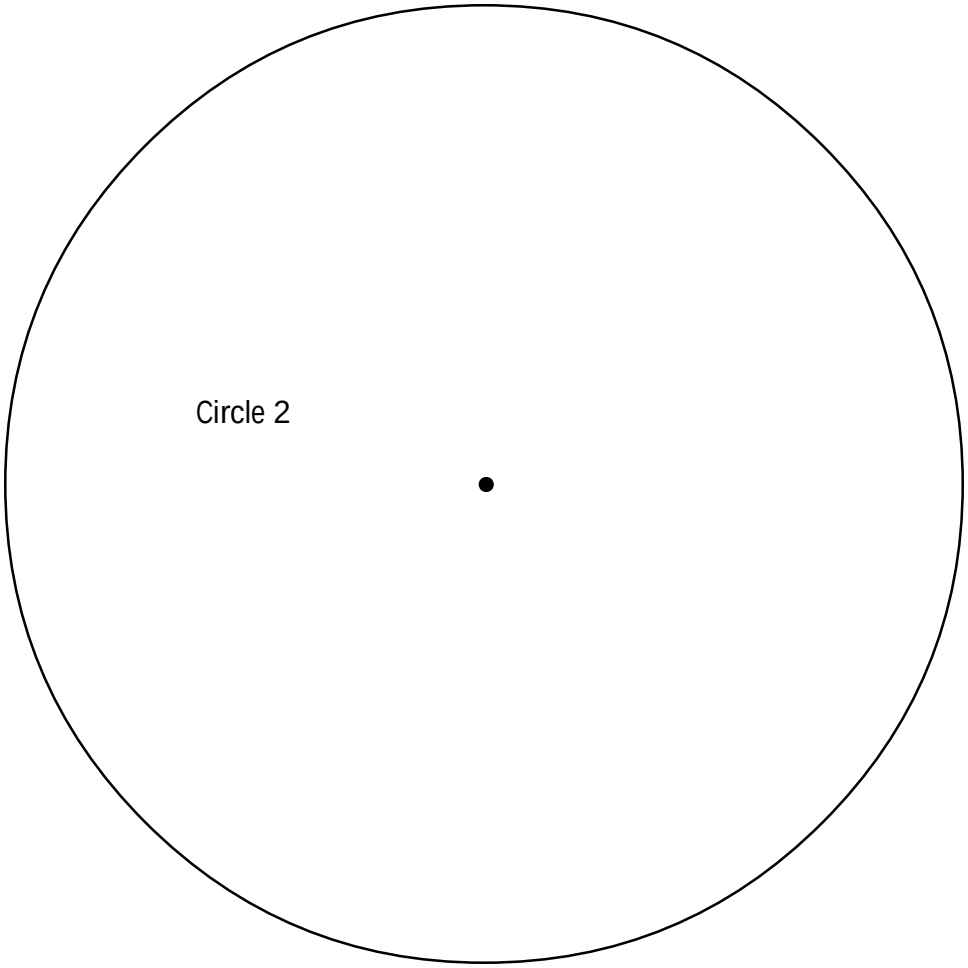
- the diameter of the circle is equal to _____ .

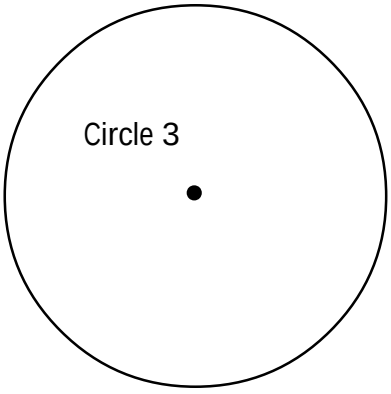
In order to see if there is a relationship or pattern, you will need to observe several more circles.

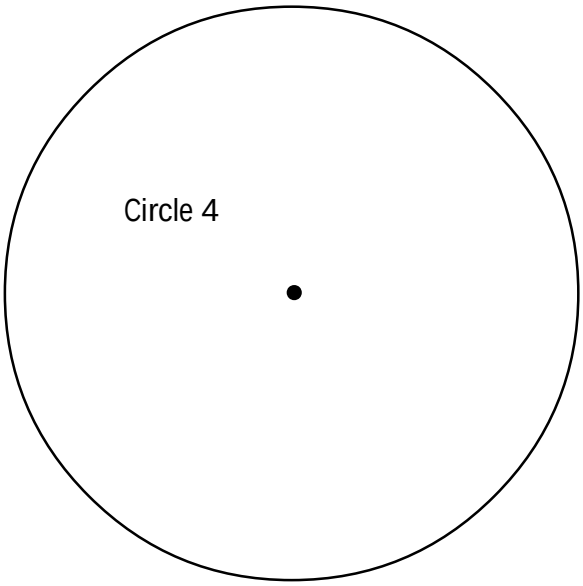
For each of the three circles that follow....

- Measure the circumference by using a piece of string and a ruler.
- Measure the diameter of the circle.

Record the measurements in the chart on the next page.







	Circumference (to the nearest tenth of an inch)	Diameter	Mathematical Relationship
Circle 1	12.5	4	
Circle 2			
Circle 3			
Circle 4			

Is there a mathematical relationship between the circumference of the circle and the diameter?

For each of the four circles studied … look for a mathematical relationship:

If you add the circumference to the diameter, is there a pattern? YES NO

If you subtract the diameter from the circumference, is there a pattern? YES NO

If you multiply the circumference by the diameter, is there a pattern? YES NO

If you divide the circumference by the diameter, is there a pattern? YES NO

	circumference ÷ diameter = □
Circle 1	
Circle 2	
Circle 3	
Circle 4	

There does exist a mathematical relationship between the diameter and the circumference of the circle. Dividing the circumference of a circle by its diameter will always give the same number. This number is not a whole number. In fact, the number never has an end. The number is often written in its abbreviated form as 3.14159. This number is given a special name. It is call

pi and is represented by the symbol π.

How does this relationship help in determining the circumference of a circle?

The Circumference (c) of a circle is equal to the product of the diameter (d) times pi (π).

$c = d\,\pi$ or since $d = 2\,r$ then $c = 2\,r\,\pi$

To test this relationship look again at the four circles....

	Diameter	times	π	equals	Circumference	Circumference Measured
Circle 1	4	x	3.14159	=	_____	_____
Cirlc e 2	5	x	3.14159	=	_____	_____
Circle 3	2	x	3.14159	=	_____	_____
Circle 4	3	x	3.14159	=	_____	_____

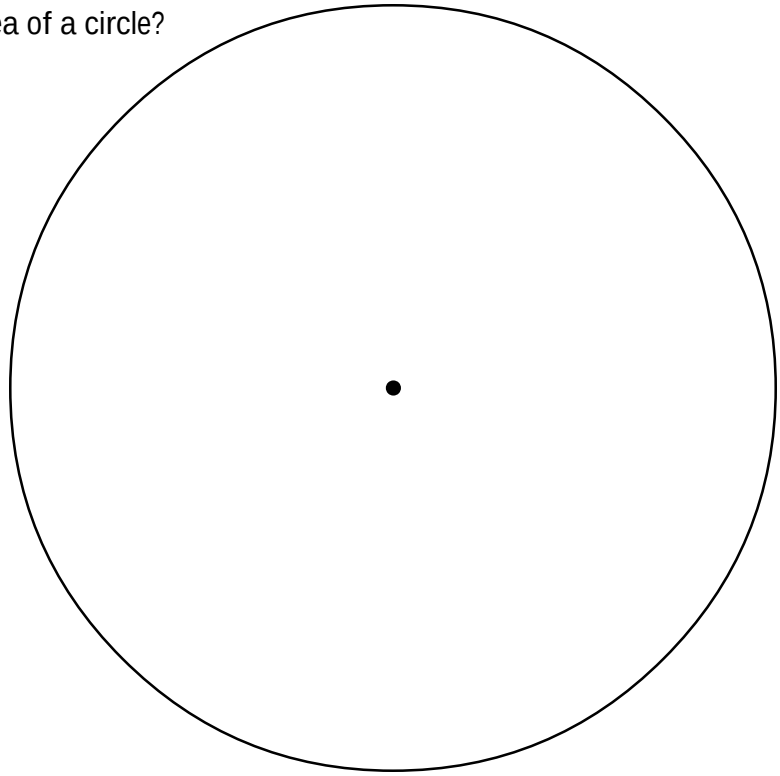
This mathematical relationship allows the circumference of a circle to be determined by simply measuring the diameter of the c circle and multiplying the diameter by 3.14159.

This relationship answers the first of the difficult problems involving the circle - how is it possible to determine the distance around a shape that does not have any sides but is one continuous curve?

And now for the second difficulty - how is possible to determine the amount of space contained within a circle?

EXPANDING THE IDEA - PART 2

Using your pencil shade the area within the circle le.
How much space does it contain?
Is there a way to determine the area of a circle?



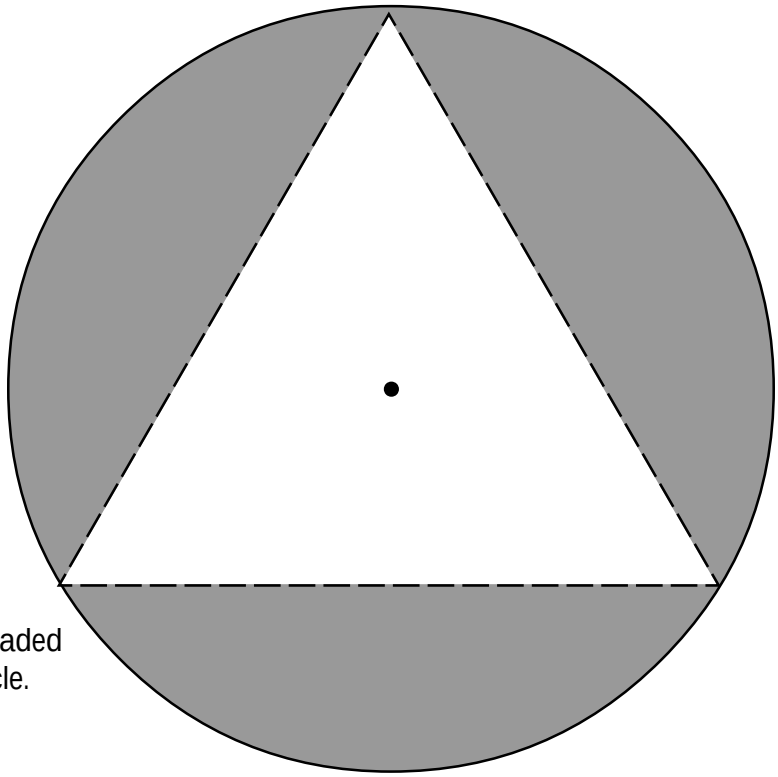
Is it possible to calculate the area of the triangle within the circle? YES NO

Measure the base and height of the triangle.

Base: _____

Height: _____

Calculate the area of the triangle: _____



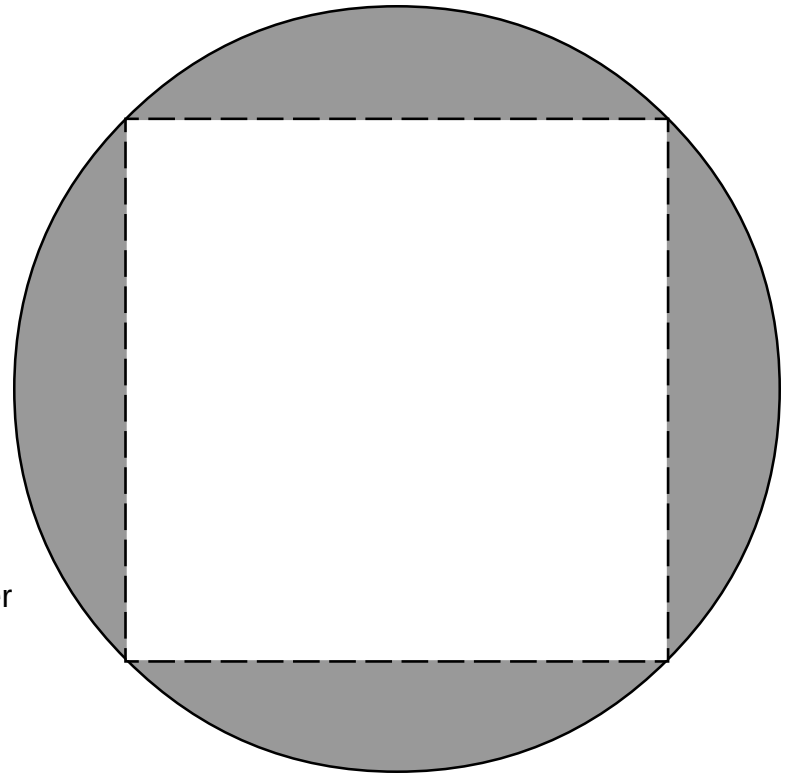
The space between the triangle and the edge of the circle has been shaded. The area of the triangle plus the shaded area is equal to the area of the circle.

Is there a way to determine the area of the shaded region?

Is it possible to calculate the
area of the square within the circle? YES NO

Measure the side of the square and then
calculate the area of the square.

SIDE: _____



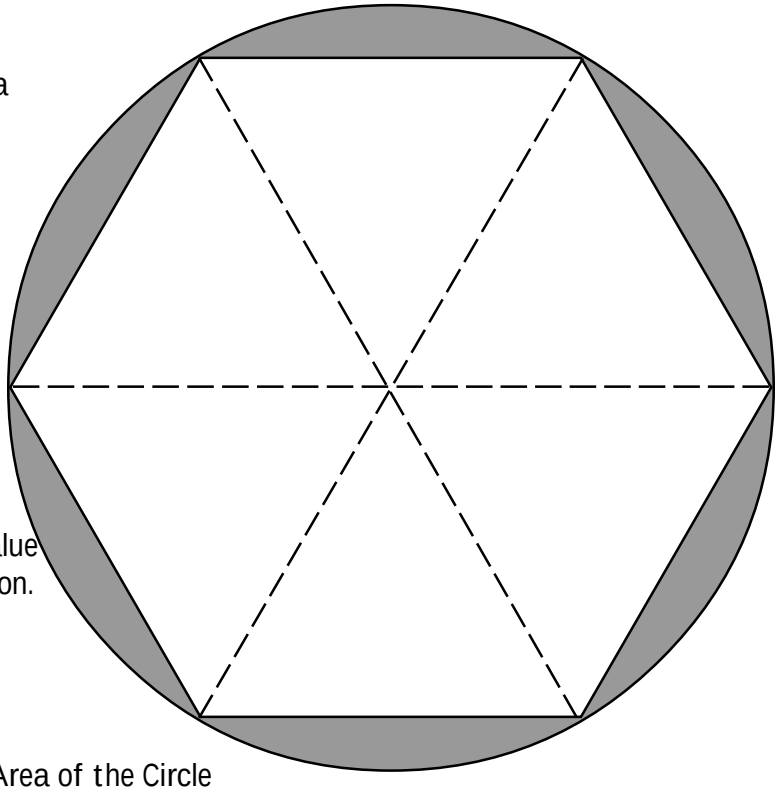
Calculate the
area of the square: _____

The space between the square and the
edge of the circle has been shaded.
The area of the square plus the shaded
area is equal to the area of the circle.

Does the area of the square give a closer
approximation of the area of the circle
than the triangle?

YES NO

If a six sided polygon called a hexagon)
is placed within the circle, and if the area
of this polygon can be calculated, will
this value be even closer to the area
of the circle? YES NO



Shade the area between the hexagon
and the circle. Is the shaded area
becoming less and less?
YES NO

The hexagon is separated into 6
congruent triangles. Calculate the area
of one triangle and then multiply that value
by 6 to determine the area of the hexagon.

Area of Hexagon + Shaded Area = the Area of the Circle

Is it possible to calculate the
area of the eight sided polygon (octagon) within the circle? YES NO

Find the area of one of the triangles within the circle.

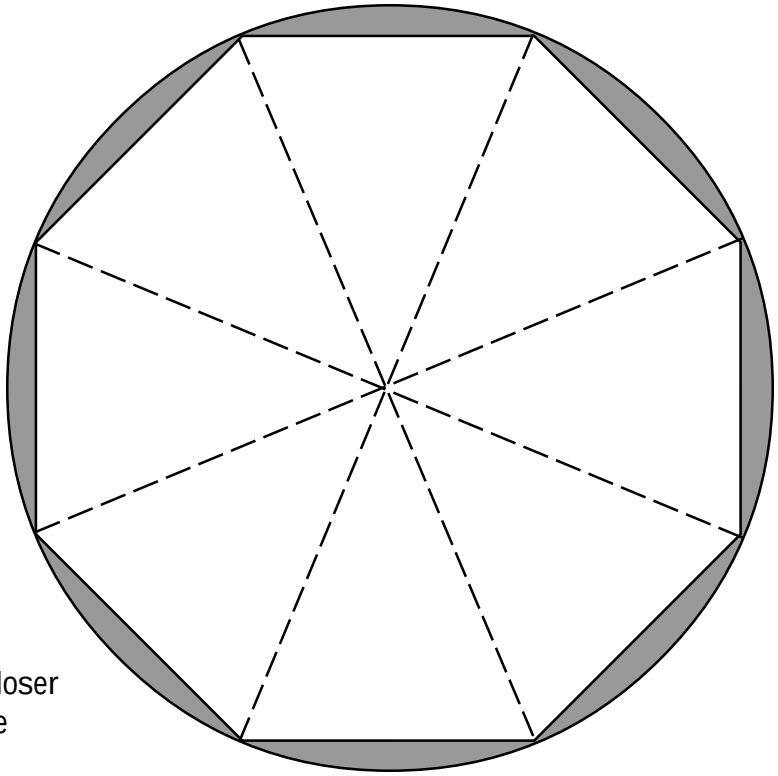
BASE: _____

HEIGHT: _____

Calculate the
area of the triangle: _____

Calculate the value of the eight
sided polygon: _____

Color the space between
the octagon and the
edge of the circle. The area of
the octagon plus the shaded area
is equal to the area of the circle.



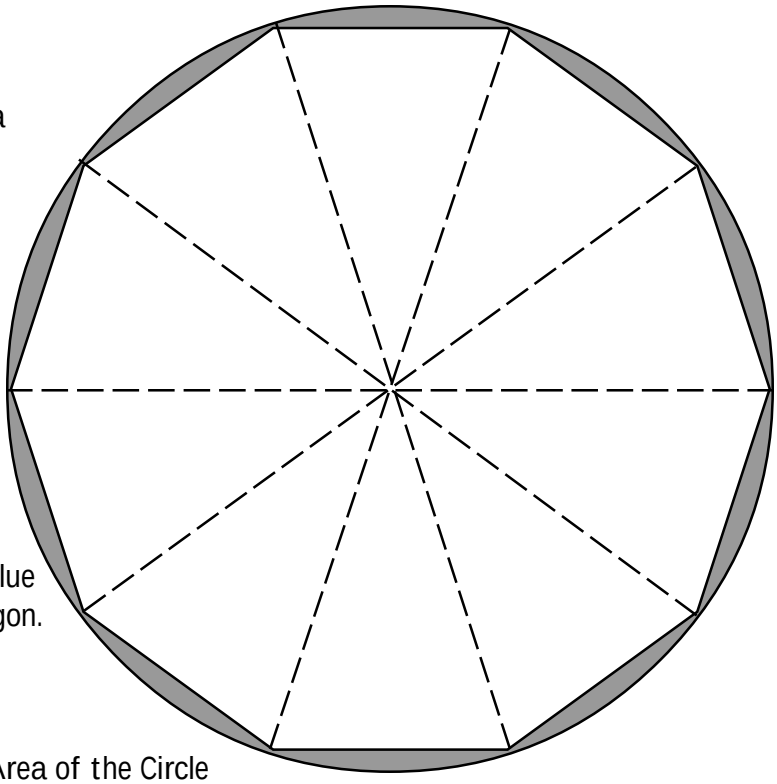
Does the area of the octagon give a closer
approximation of the area of the circle
than the previous calculations?

YES NO

If a ten sided polygon (called a decagon)
is placed within the circle, and if the area
of this polygon can be calculated, will
this value be even closer to the area
of the circle? YES NO

Shade the area between the decagon
and the circle. Is the shaded area
becoming less and less?
YES NO

The decagon is separated into 10
congruent triangles. Calculate the area
of one triangle and then multiply that value
by 10 to determine the area of the decagon.

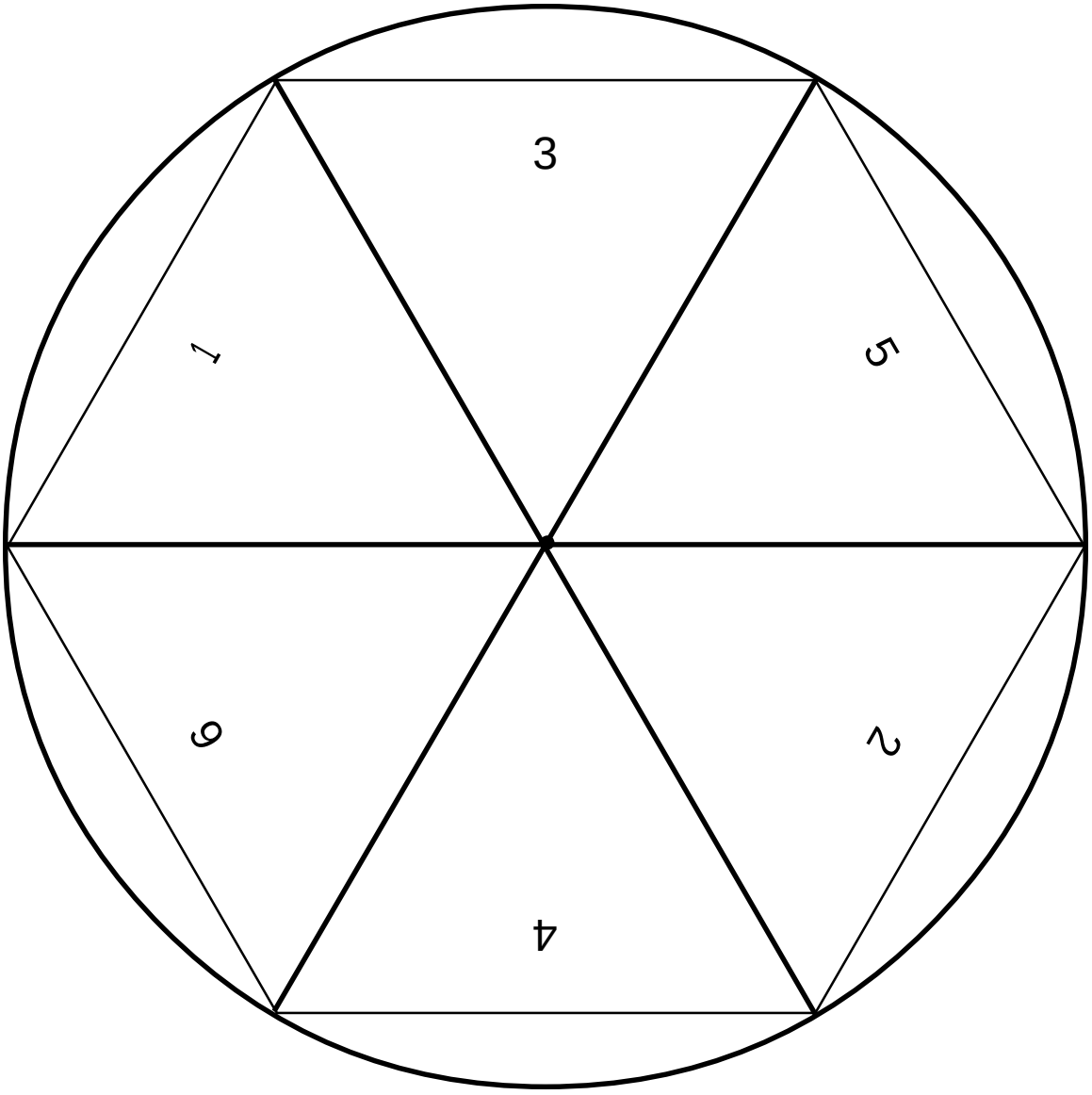


Area of Decagon + Shaded Area = the Area of the Circle

Is there a mathematical relationship involving the diameter or radius of the circle, the circumference, and pi that will allow us to determine the area of the circle?

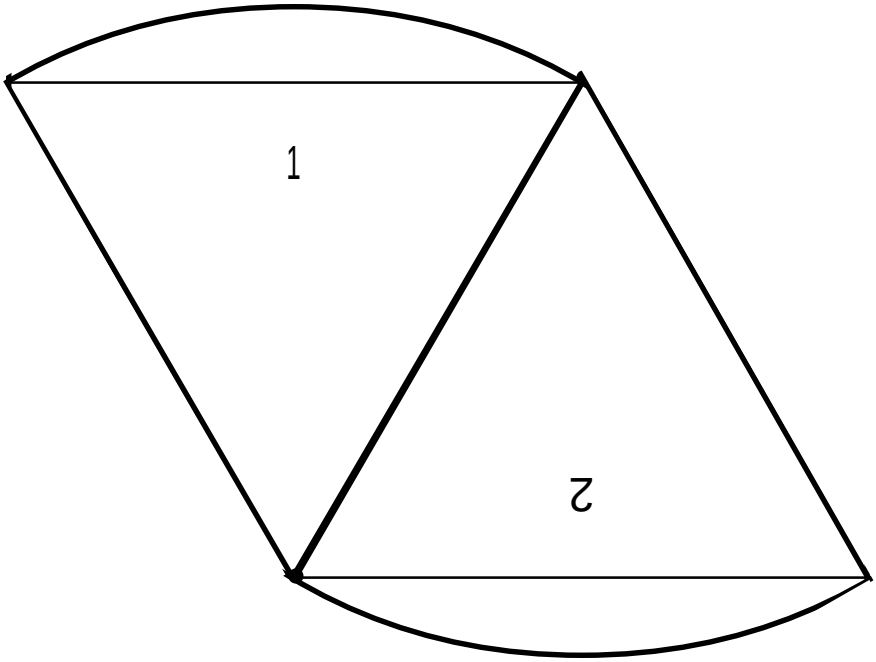
The circle below encircles a hexagon. The area of the hexagon plus the area between the hexagon and the circle equals the area of the circle.

Cut out each pie shape.
Arrange them according to the pattern on the following page.



Place Pie Shape Number 1 next to Pie Shape Number 2 as shown below.

Continue placing each pie shape in the numerical order in the same pattern.



Describe the shape.... _____

_____

_____ .

How is this shape different from the last one? _____

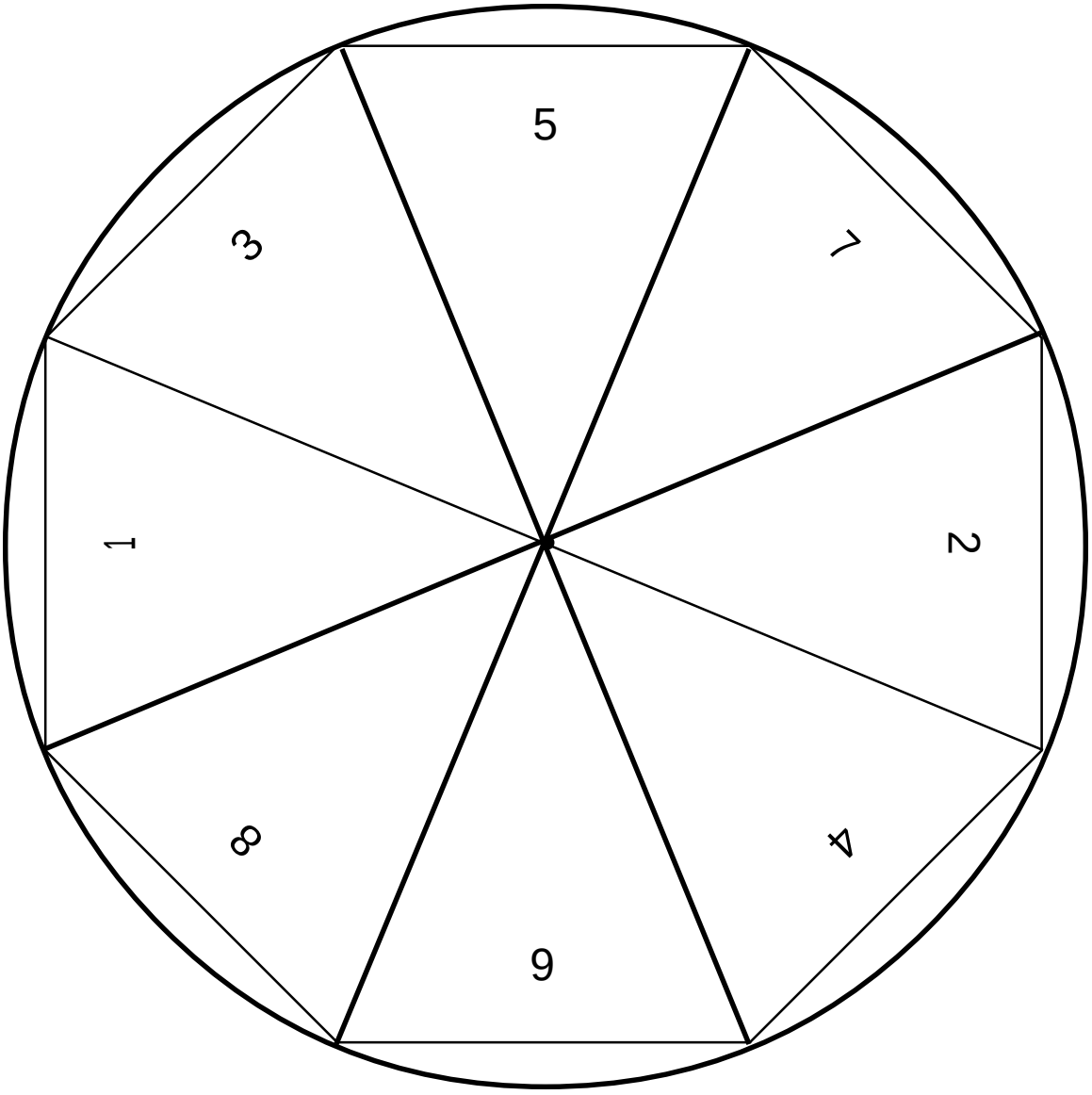
_____ .

The distance across the top is equal to what fractional part of the circle? _____

The circle below encircles an octagon.

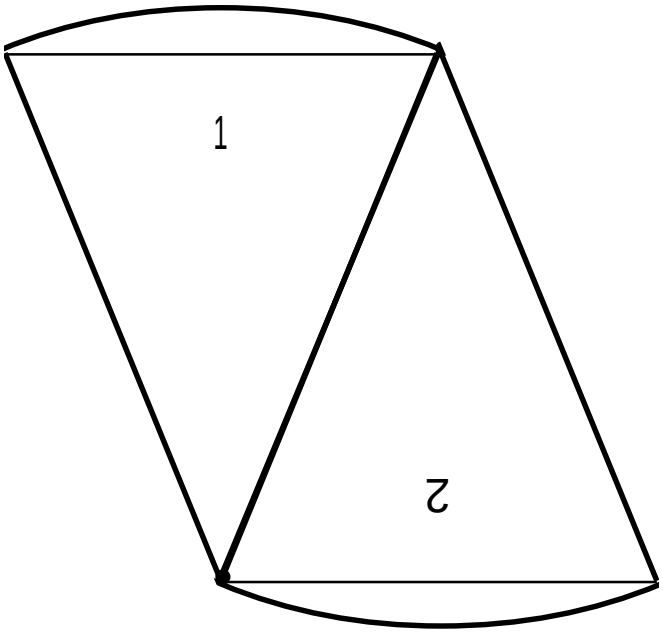
The area of the octagon plus the area between the octagon and the circle equals the area of the circle.

Cut out each pie shape.
Arrange them according to the pattern on the following page.



Place Pie Shape Number 1 next to Pie Shape Number 2 as shown below.

Continue placing each pie shape in the numerical order in the same pattern.



Describe the shape.... _____

_____

_____ .

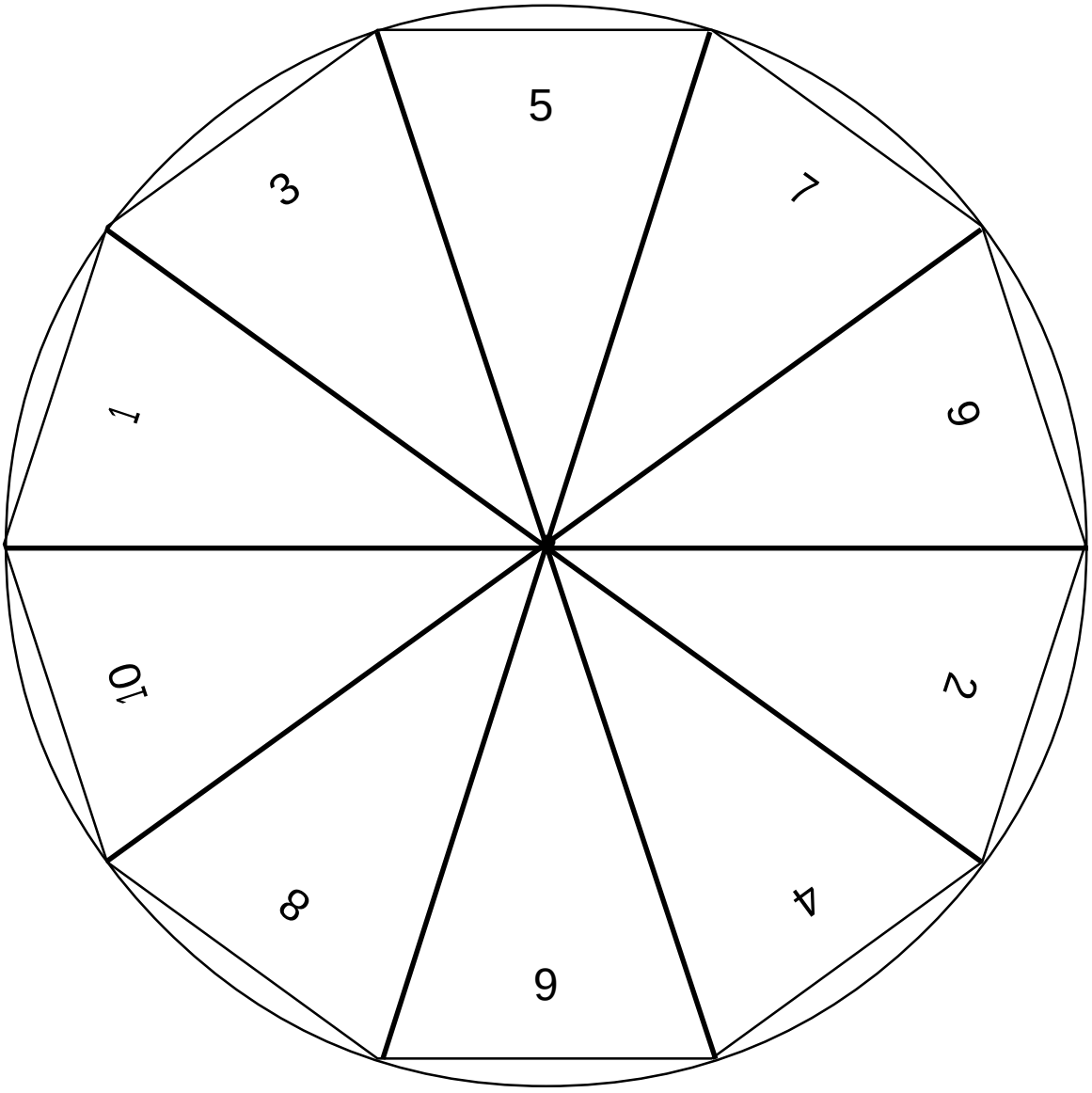
The distance across the top is equal to what fractional part of the circle? _____

The short dimension is equal to the _____ of the circle.

The circle below encircles an octagon.

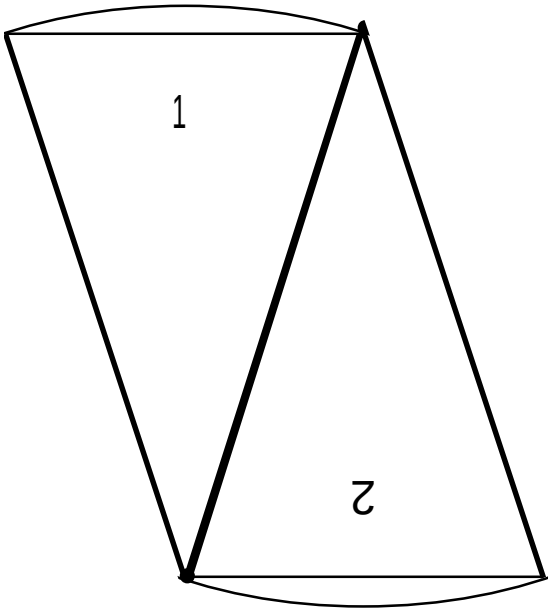
The area of the octagon plus the area between the octagon and the circle equals the area of the circle.

Cut out each pie shape.
Arrange them according to the pattern on the following page.



Place Pie Shape Number 1 next to Pie Shape Number 2 as shown below.

Continue placing each pie shape in the numerical order in the same pattern.



Describe the shape.... _____

_____

_____ .

The distance across the top is equal to what fractional part of the circle? _____ .

The short dimension is equal to the _____ of the circle.

The circle below encircles a polygon with sixteen sides.

The area of this polygon is getting closer and closer to the actual area of the circle le.

Cut out each pie shape and arrange them according to the previous pattern.

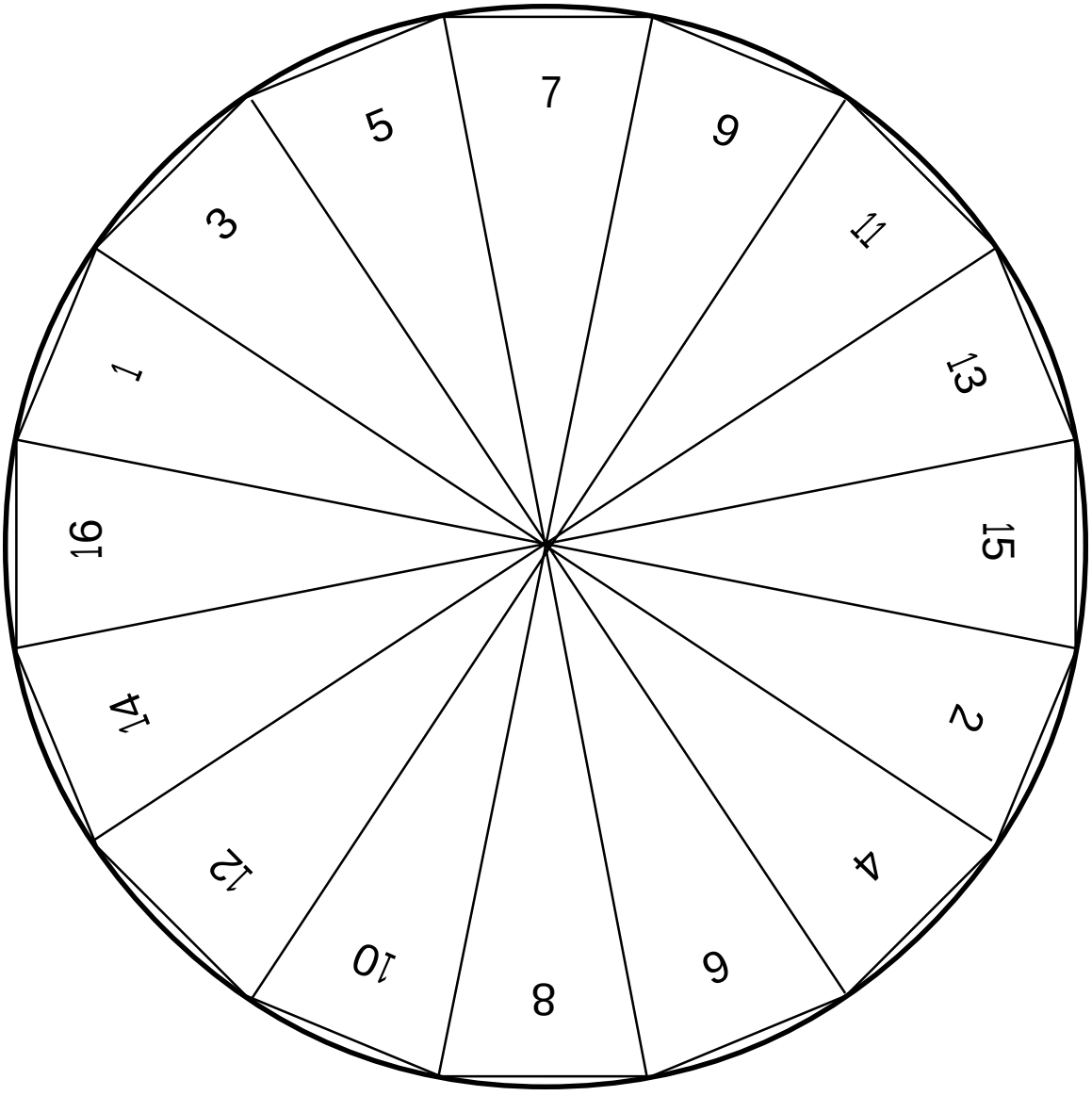
Compare this shape with the previous shapes. What changes do you notice? _____

_____

_____ .

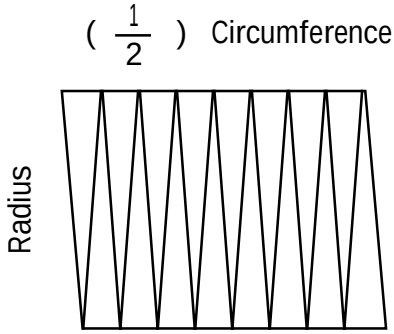
What is the long dimension equal to? _____ .

What is the short dimension equal to? _____ .



As the number of sides of the polygon increases, the area of the polygon gets closer and c loser to the area of the circle. You should also have recognzed that the shape that results from placing the individual pie pieces next to each other in the alternating pattern gets closer and closer to a rectangle.

The long dimension of this rectangle is equivalent to one half the circumference of the circle. And the short dimension of this rectangle is equivalent to the radius of the circle.



This diagram shows the circle cut into 16 pie pieces.

Therefore, the area of this rectangle formed from these individual pie pieces could be written …

$$\text{Area} = \left(\frac{1}{2}\right)(\text{Circumference})(\text{radius})$$

Since the circumference of a circle is equal to pi times the diameter, then the equation becomes…

$$\text{Area} = \left(\frac{1}{2}\right)(\pi)(d)(\text{radius})$$

And since the diameter is equivalent to two times the radius, then the equation becomes…

$$\text{Area} = \left(\frac{1}{2}\right)(\pi)(2)(\text{radius})(\text{radius})$$

If this equation is simplified … the area of any circle becomes …

$$\text{Area} = (\pi)(\text{radius})^2$$

THEREFORE, the area of the circle is equal to pi times the radius squared.

If the radius of the circle is known, then the area of the circle is found by multiplying pi times the square of the radius.

Regular Tetrahedron - 4 faces
Regular Icosahedron - 20 faces
Regular Octahedron - 8 faces
Regular Hexahedron - 6 faces
Regular Dodecahedron - 12 faces

POLYHEDRON	VERTICES (V)	EDGES (E)	FACES (F)
Octahedron	6	12	8
Tetrahedron	4	6	4
CUBE	8	12	6
PRISM	6	9	5
Pyramid	5	8	5

V - E + F = 2

CHAPTER 8: No Longer Flat

EXPLORING THE IDEA

To this point in your study of geometry the figures you have studied have all been flat. These figures have only two dimensions. Go to your kitchen and choose cereal box, a box of oatmeal or can, a square box of tissues, and a ball. Look closely at these items.

Describe each one. Write three things about each.

ITEM:	DESCRIPTION	DRAWING
Item Number 1 …	•	
	•	
	•	
	•	
	•	
Item Number 2 …	•	
	•	
	•	
	•	
	•	
Item Number 3 …	•	
	•	
	•	
	•	
	•	
Item Number 4 …	•	
	•	
	•	
	•	
	•	

Using a ruler and protractor, find two more ways to describe each item.

Did you include information about …

- the number of sides?
- the number of edges?
- the number of corners (where the edges come together)?
- the length of each side?
- the angles where the sides meet at the corners?

Did you include words like … circle … polygon … parallelogram … rectangle … square … etc.?

If you did not include this information, add it to your list.

After you put the items back away, ask a friend to read each description and then draw what they think the item is. Space for the drawing is provided next to the description.

How did they do? Was there drawing close to the real object?

Show them the objects after they have drawn them.

Talk about your description and their drawings. How could you have given a better description?

Several years ago I read the following description in a geometry book describing a geometric solid.…

> This object is a regular polyhedron. It is convex, has congruent regular polygons for faces and has the same number of polygons meeting at each vertex. This object is a regular hexahedron, made up of six congruent regular quadrilaterals.

In the space below try you best to draw the shape described above.…

Objects are not restricted to flat surfaces. In fact, most objects take up space. There are so many different solid shapes in the world. Some shapes, such as shoe boxes, have only flat sides. Others are round like balls, or partly round like baseball bats. This aspect of geometry is called Solid Geometry. The created world and the world created by man are filled with geometric solids.

If you have a magnifier, pour out some salt onto a dark paper or surface. Look carefully through the magnifier at the salt. What do you see? Though the salt crystals may vary in size, each salt crystal has a regular pattern or shape. Describe each crystal of salt. What shape do you see? How are the shapes alike? … different?

Next, look at a small amount of sugar with the magnifier.

How is the shape of each salt crystal different from that of the sugar?

Many different types of crystals form regular uniform shapes. This can be seen at the following web site: www.

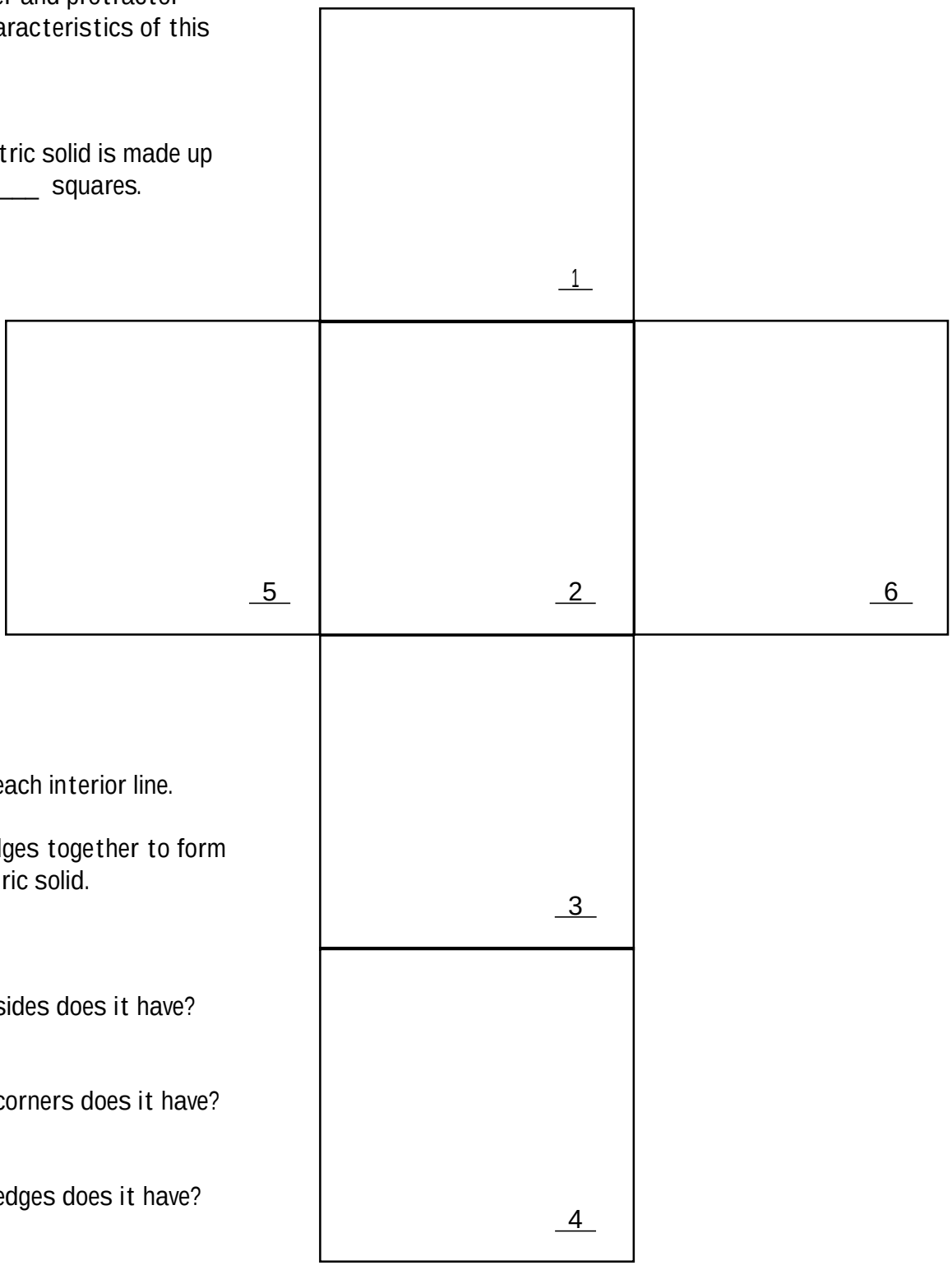
Construct each of the following geometric solids....

NUMBER 1:

Cut along the outside edge of this shape.

Using a ruler and protractor find the characteristics of this shape.

This geometric solid is made up of _____ squares.



Fold along each interior line.

Tape the edges together to form the geometric solid.

How many sides does it have?

How many corners does it have?

How many edges does it have?

NUMBER 2:

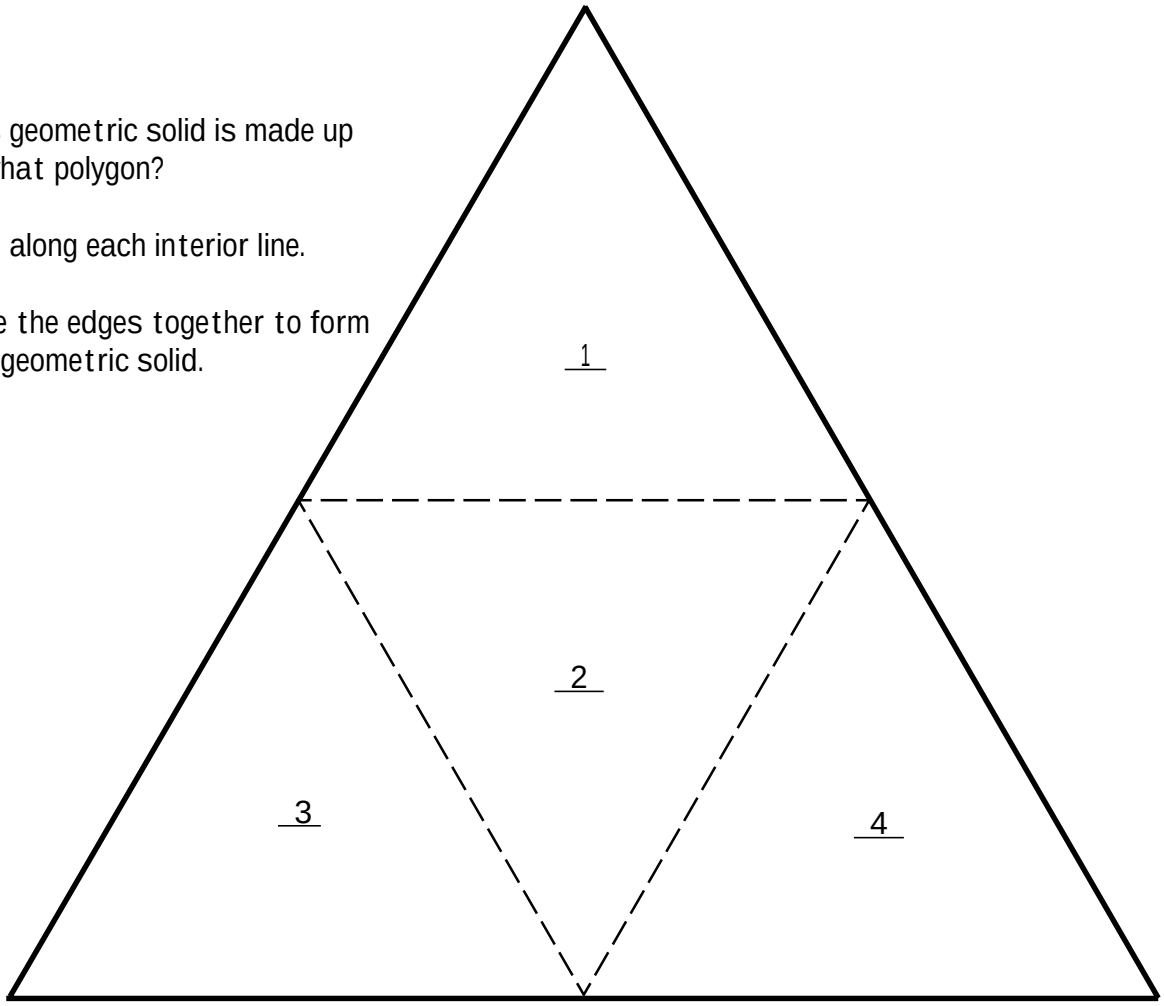
Cut along the outside edge of this shape.

Using a ruler and protractor find the characteristics of this shape.

This geometric solid is made up of what polygon?

Fold along each interior line.

Tape the edges together to form the geometric solid.



How is this geometric solid like the others you have constructed? How is it different?

How many sides does it have? _____

How many corners does it have? _____

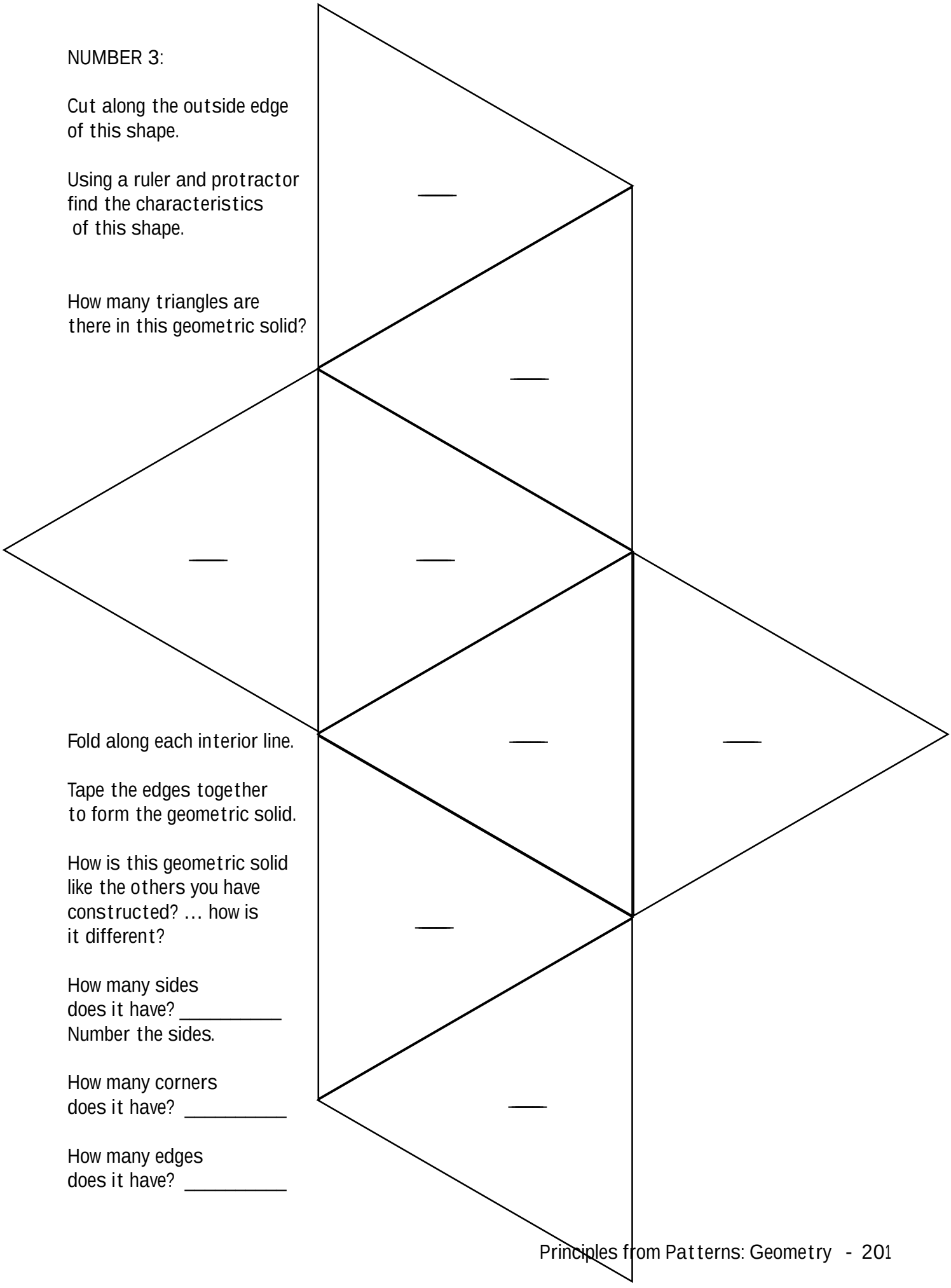
How many edges does it have? _____

NUMBER 3:

Cut along the outside edge of this shape.

Using a ruler and protractor find the characteristics of this shape.

How many triangles are there in this geometric solid?



Fold along each interior line.

Tape the edges together to form the geometric solid.

How is this geometric solid like the others you have constructed? … how is it different?

How many sides does it have? _____
Number the sides.

How many corners does it have? _____

How many edges does it have? _____

NUMBER 4:

Cut along the outside edge of this shape.

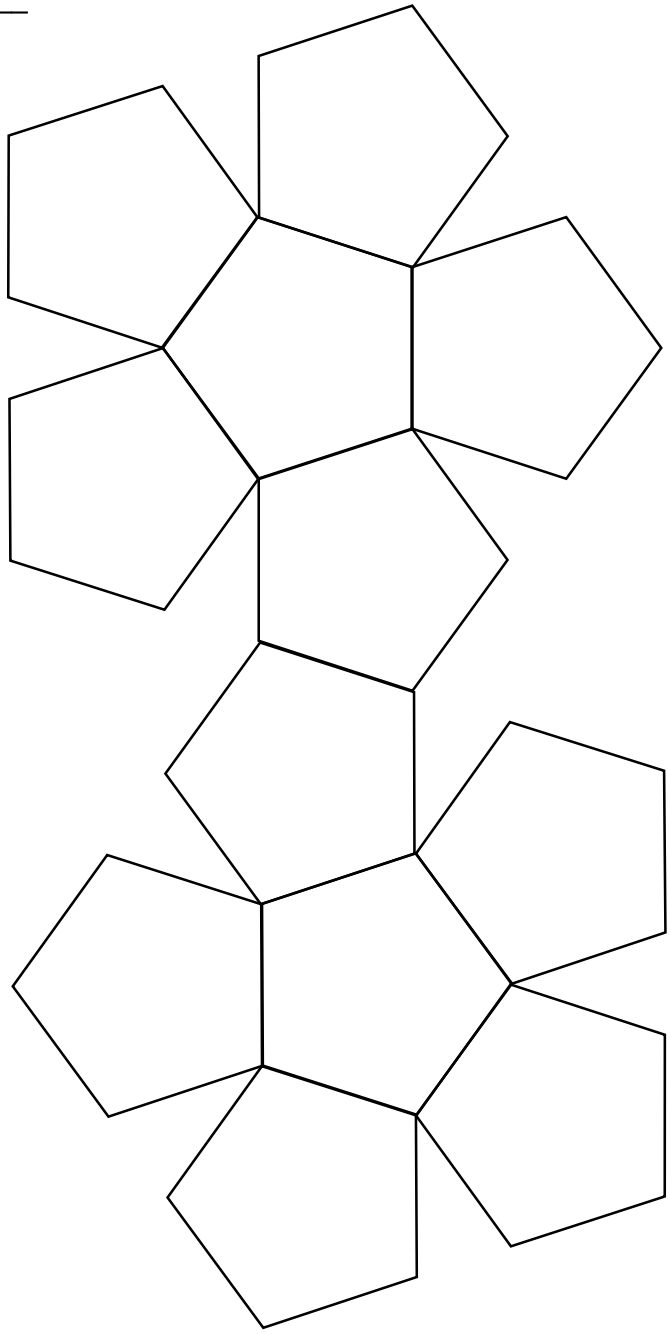
This geometric solid is made up of what polygon?

How many pentagons are there? _____

Using a ruler and protractor find the characteristics of this shape.

Fold along each interior line.

Tape the edges together to form the geometric solid.



How is this geometric solid like the others you have constructed? How is it different?

How many sides does it have? _____

Number the sides.

How many corners does it have? _____

How many edges does it have? _____

NUMBER 5:

Cut along the outside edge of this shape.

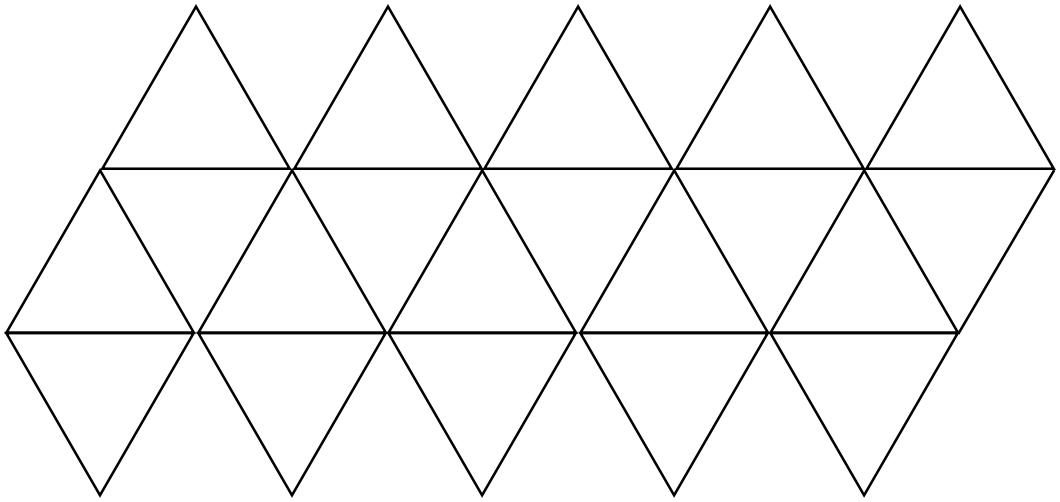
This geometric solid is made up of what polygon?

How many small triangles are there? _____

Using a ruler and protractor find the characteristics of this shape.

Fold along each interior line.

Tape the edges together to form the geometric solid.



How is this geometric solid like the others you have constructed? How is it different?

How many sides does it have? _____
Number the sides.

How many corners does it have? _____

How many edges does it have? _____

EXPLAINING THE IDEA

There are three major characteristics to notice when describing any geometric solid.

Look carefully at each of the solids you have constructed.

You will recognize each geometric solid by looking at the flat surfaces. Notice the specific number of flat surfaces for each geometric solid. Flat polygon surfaces are called **FACES**.

The segment where two faces meet is called the EDGE.

The corner of the geometric solid is the point of intersection of three or more edges..
This is called the VERTEX.

A geometric solid formed by polygons that enclose a specific space is called a

POLYHEDRON.

A REGULAR POLYHEDRON is a polyhedron that is uniform in structure. That means that all faces, all edges and all vertices are alike in every way. If any part of the polyhedron is not uniform in shape or structure, then it is considered an IRREGULAR POLYHEDRON.

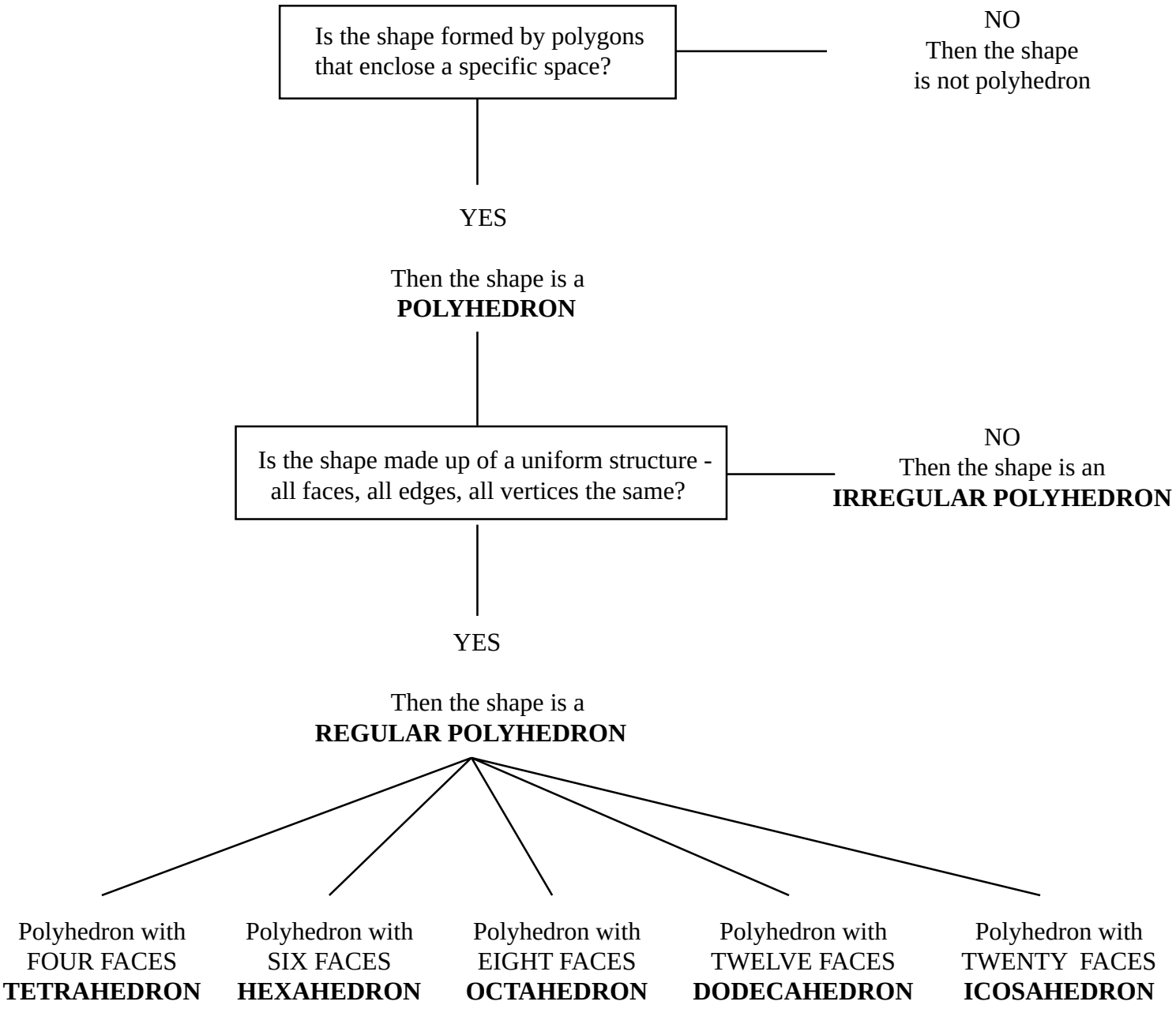
EXPANDING THE IDEA - PART 1

When describing a polyhedron it is very important to describe these three characteristics: Faces, Edges, and Vertices. Complete the chart below for the polyhedrons you have constructed....

POLYHEDRON	VERTICES (V)	EDGES (E)	FACES (F)	NAME
Number 1	_____	_____	_____	_____
Number 2	_____	_____	_____	_____
Number 3	_____	_____	_____	_____
Number 4	_____	_____	_____	_____
Number 5	_____	_____	_____	_____

Look for the mathematical relationship involving VERTICES, EDGES, AND FACES?
(HINT: the relationship involves both addition and subtraction whose answer is 2.)

You are ready to give a formal definition to each of the various shapes that you constructed. Each definition will contain the answer to each of the following questions.…



Look at the six geometric solids that you constructed.

One is an IRREGULAR POLYHEDRON. Which one? _____

Name each of the REGULAR POLYHEDRONS that you constructed.

Remember the definition of the geometric shape that was quoted from another textbook? Read the definition again.

> This object is a regular polyhedron. It is convex, has congruent regular polygons for faces and has the same number of polygons meeting at each vertex. This object is a regular hexahedron, made up of six congruent regular quadrilaterals.

This definition describes one of the six geometric solids you constructed? Which one?

To answer this question look carefully at the definition and at each geometric solid.

What is the first characteristic? A REGULAR POLYHEDRON.

Are any of the solids eliminated because they are not regular polyhedrons? YES NO

Which geometric solid fits all the characteristics described in the definition?
Why do you think so?

Look back at the chart describing the three major characteristics of the six geometric solids.

VERTICES (V) EDGES (E) FACES (F)

There are six different ways that VERTICES, EDGES, and FACES can be related mathematically. The only stipulation is that the relationship involves both addition and subtraction.

For example, Vertices + Edges + Faces = ☐ is not a possibility because subtraction was not used.

For example, the following is just one possibility of six….

	VERTICES	+	EDGES	-	FACES	=	☐
Tetrahedron	4	+	6	-	4	=	6
Hexahedron	8	+	12	-	6	=	10
Octahedron	6	+	12	-	8	=	12
Dodecahedron	20	+	30	-	12	=	38
Icosahedron	12	+	30	-	20	=	22

Is this the proper pattern? YES NO NOT SURE

Write all six possible equations. The first one is done for you.

1. Vertices - Edges + Faces = ☐

2.

3.

4.

5.

6.

Now that you have written allthe possible mathematical relationships, use the values from your study to determine if one gives a pattern. To determine which pattern is correct, place the numbers from your previous study into the equations

.

EXPANDING THE IDEA - PART 2

Construct the following geometric solids.
Keep a record of the three major characteristics:

VERTICES (V) EDGES (E) FACES (F)

Decide if the solid is a polyhedron or not.
If it is a polyhedron, determine if it is a regular polyhedron or an irregular polyhedron.

Finally, see if the mathematical relationship

VERTICES - EDGES + FACES = 2

is true for these geometric solids.

THE PYRAMID ...

Cut the shape below.... Fold along inside edges.... Tape the edges.

How many VERTICES, EDGES, AND FACES does this shape have? V = _____ E = _____ F = _____

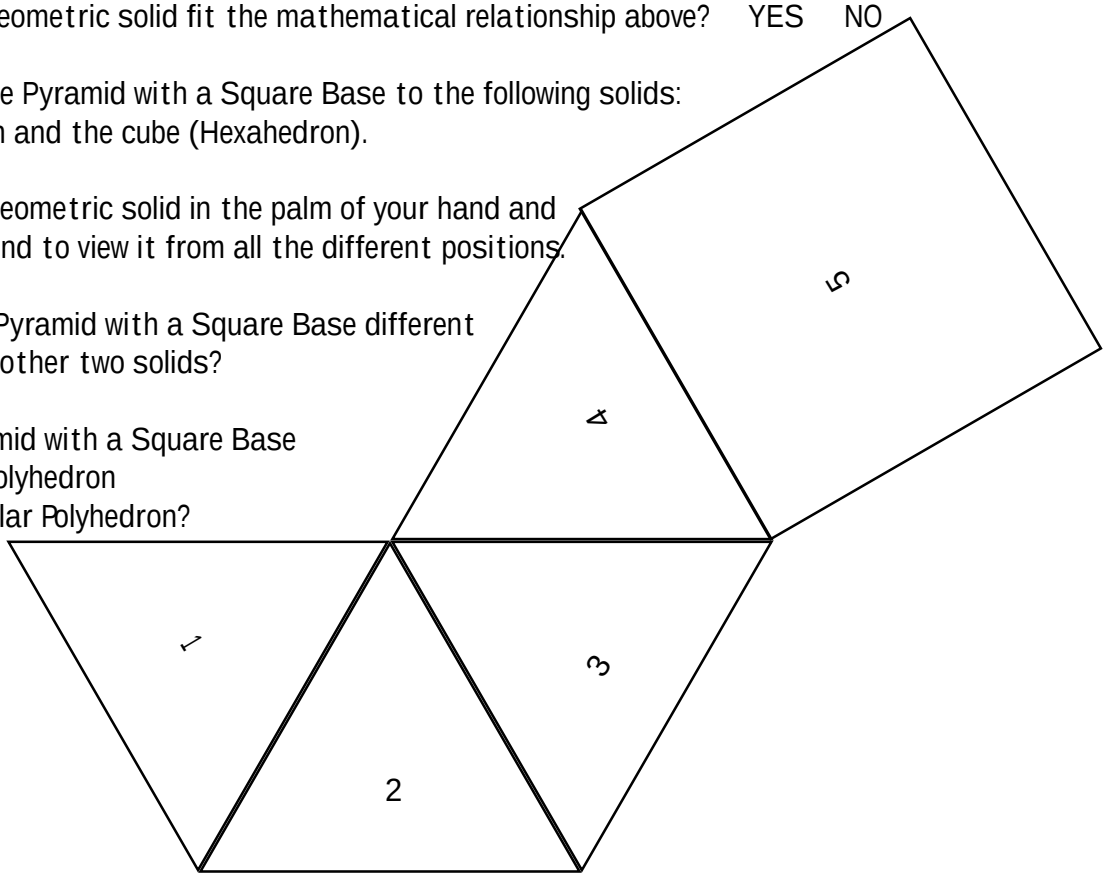
Does this geometric solid fit the mathematical relationship above? YES NO

Compare the Pyramid with a Square Base to the following solids:
Tetrahedron and the cube (Hexahedron).

Hold each geometric solid in the palm of your hand and turn it around to view it from all the different positions.

How is the Pyramid with a Square Base different from these other two solids?

Is the Pyramid with a Square Base
a Regular Polyhedron
or an Irregular Polyhedron?



THE PYRAMID …

Cut the shape below.… Fold along inside edges.… Tape the edges.

How many VERTICES, EDGES, AND FACES does this shape have? V = _____ E = _____ F = _____

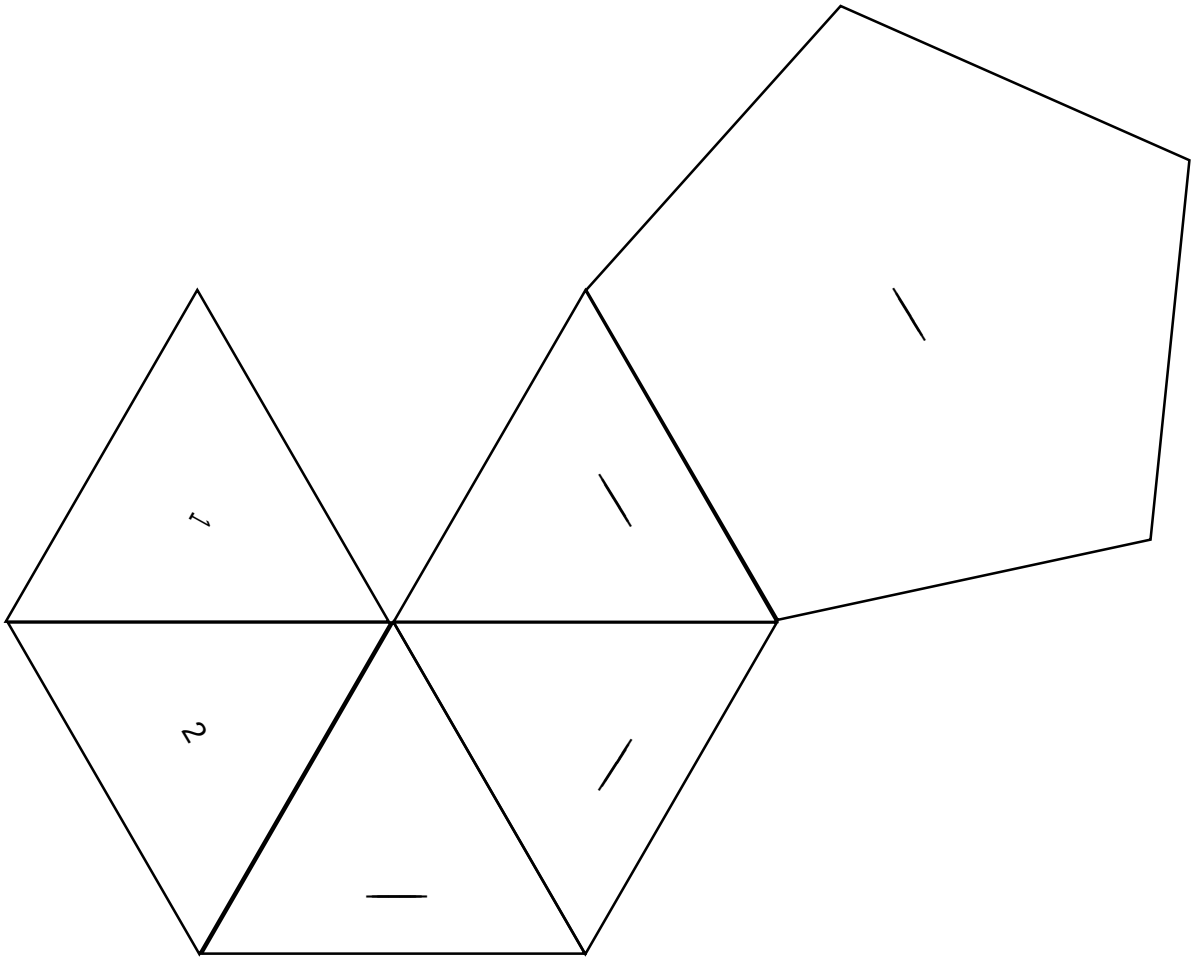
Number the sides.

Does this geometric solid fit the mathematical relationship? YES NO

Compare the is pyramid with the last pyramid. Hold each pyramid in the palm of your hand and turn it around to view it from all the different positions.

How are they alike? How are they different?

Is the Pentagonal Pyramid a Regular Polyhedron or an Irregular Polyhedron?

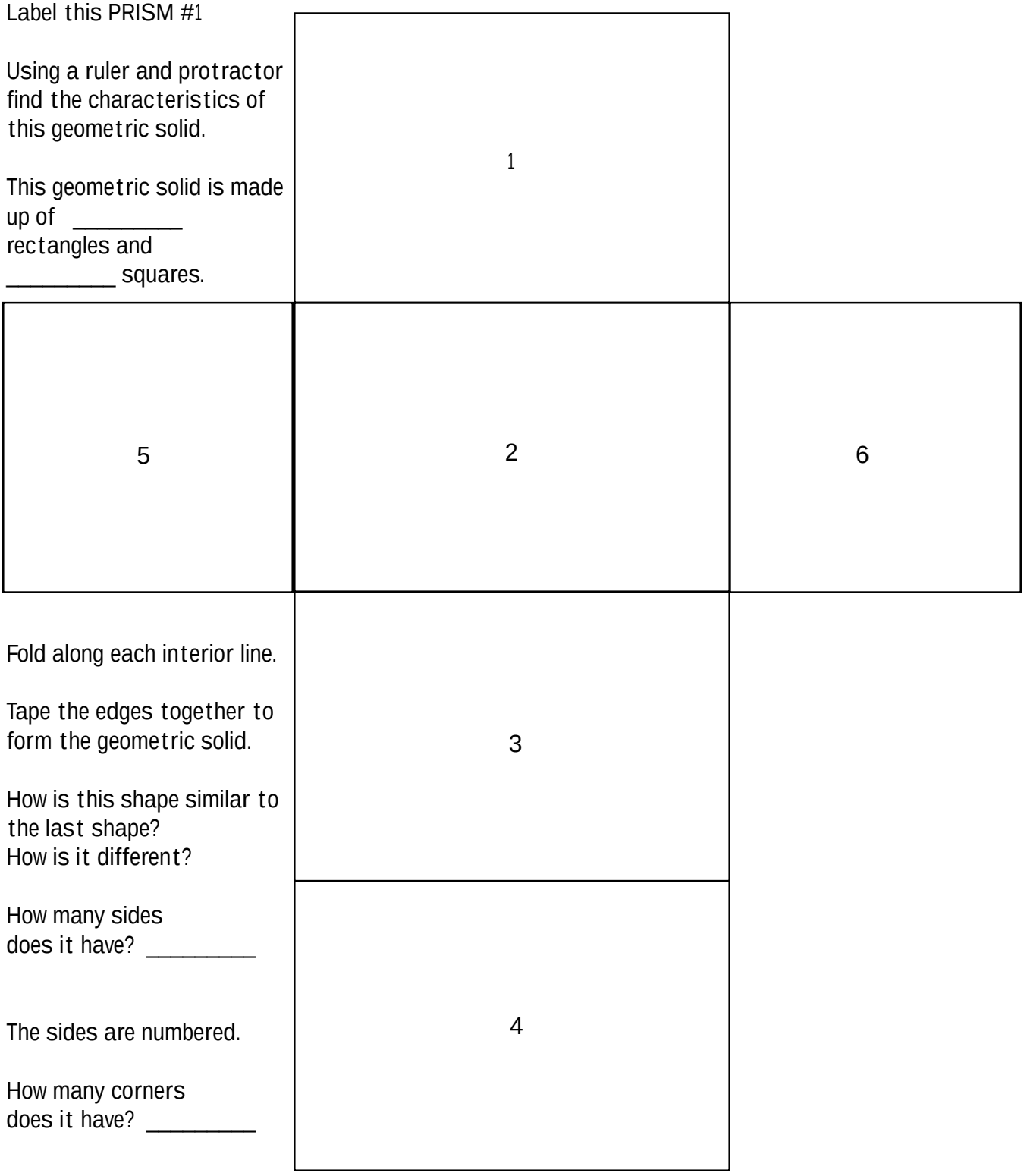


THE PRISM… NUMBER 1

Cut along the outside edge of this shape.
Label this PRISM #1

Using a ruler and protractor find the characteristics of this geometric solid.

This geometric solid is made up of _____ rectangles and _____ squares.



Fold along each interior line.

Tape the edges together to form the geometric solid.

How is this shape similar to the last shape?
How is it different?

How many sides does it have? _____

The sides are numbered.

How many corners does it have? _____

How many edges does it have? _____

THE PRISM… NUMBER 2

Cut the shape below.… Fold along inside edges.… Tape the edges.

Label this PRISM # 2

How many VERTICES, EDGES, AND FACES does this shape have? V = _____ E = _____ F = _____

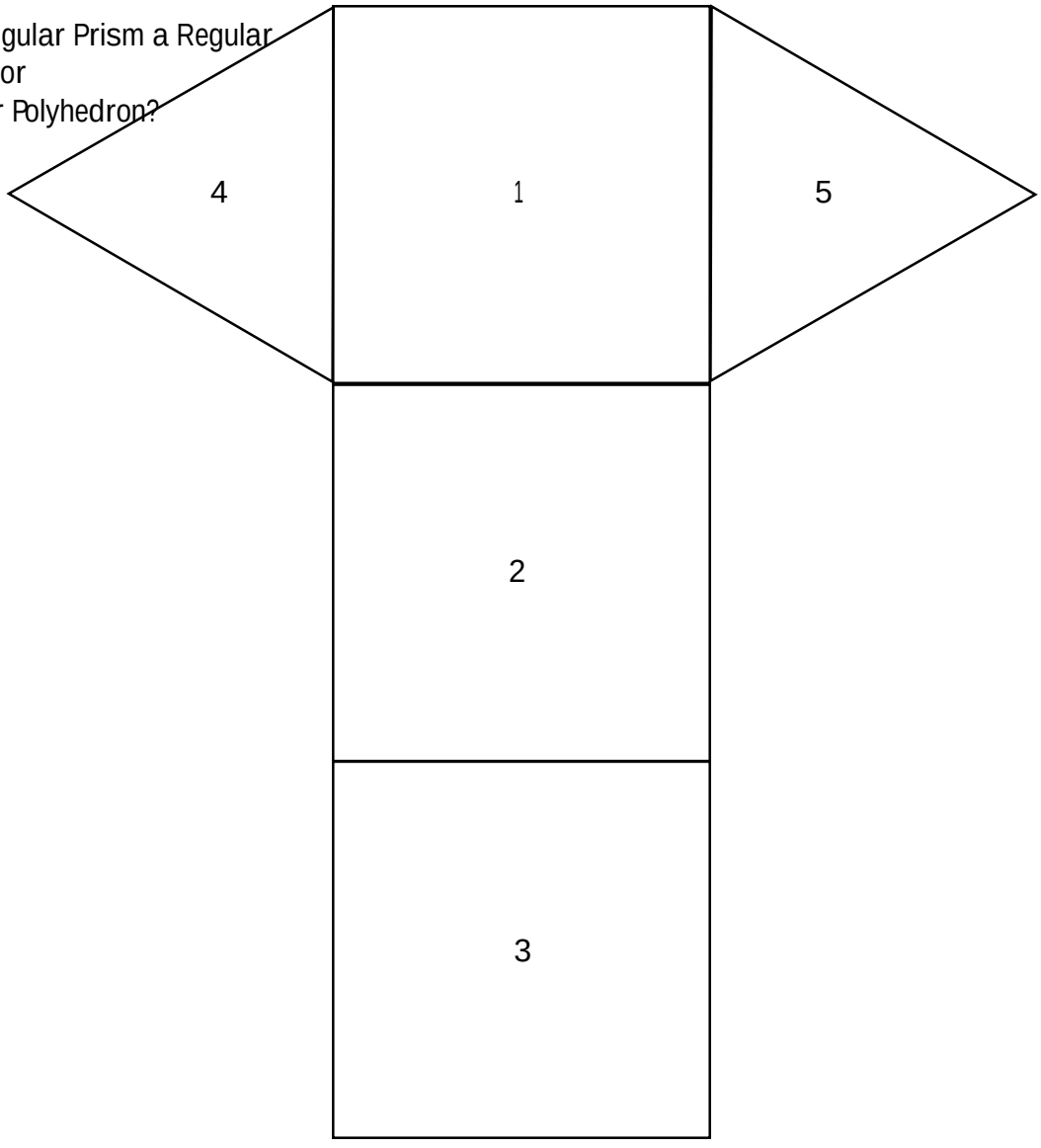
The sides are numbered.

Does this geometric solid fit the mathematical relationship? YES NO

Compare the prism with the two pyramids. Hold these geometric solids in the palm of your hand and turn them around to view from all the different positions.

How are they alike? How are they different?

Is the Triangular Prism a Regular Polyhedron or an Irregular Polyhedron?



THE PRISM… NUMBER 3

Cut the shape below.… Fold along inside edges.… Tape the edges.

Label this PRISM #3

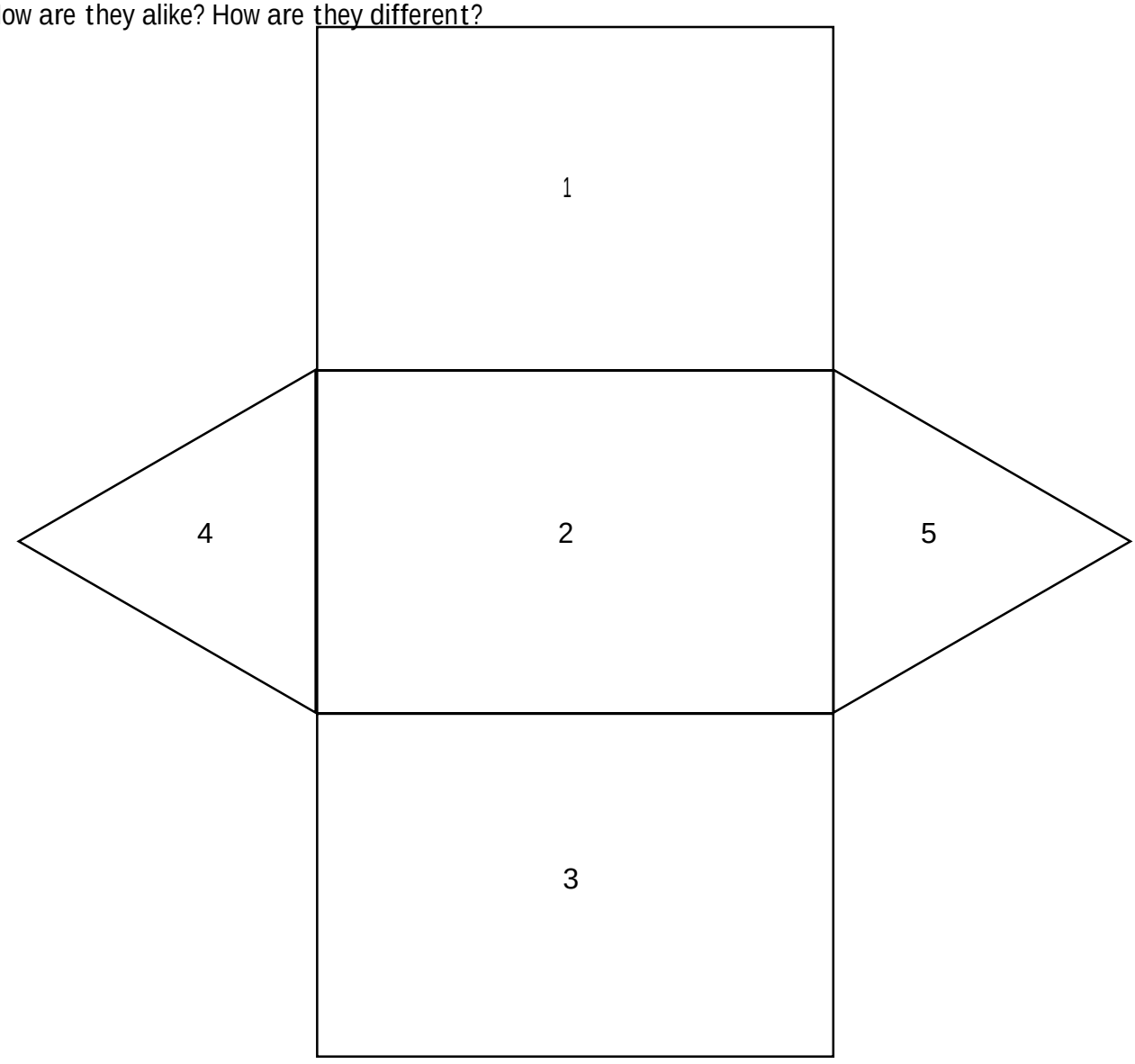
How many VERTICES, EDGES, AND FACES does this shape have? V = _____ E = _____ F = _____

The sides are numbered.

Does this geometric solid fit the mathematical relationship? YES NO

Compare the prism with the other prism. Hold these geometric solids in the palm of your hand and turn them around to view from all the different positions.

How are they alike? How are they different?



THE PRISM…NUMBER 4

Cut the shape below.… Fold along inside edges.… Tape the edges.

Label this PRISM #4

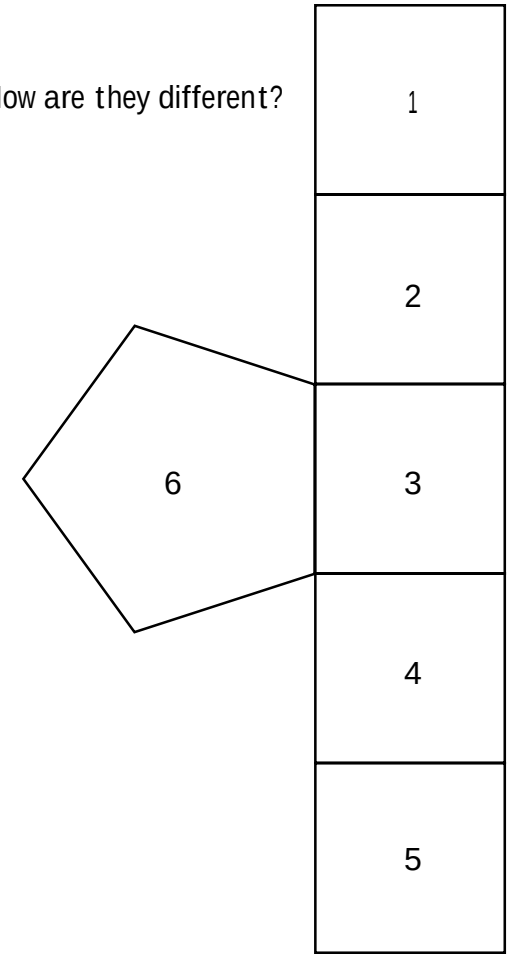
How many VERTICES, EDGES, AND FACES does this shape have? V = _____ E = _____ F = _____

The sides are numbered.

Does this geometric solid fit the mathematical relationship? YES NO

Compare the prism with the other prisms.
Hold these geometric solids in the
palm of your hand and
turn them around to view
from all the different positions.

How are they alike? How are they different?



THE PRISM… NUMBER 5

Cut the shape below.… Fold along inside edges.… Tape the edges.

Label this PRISM #5

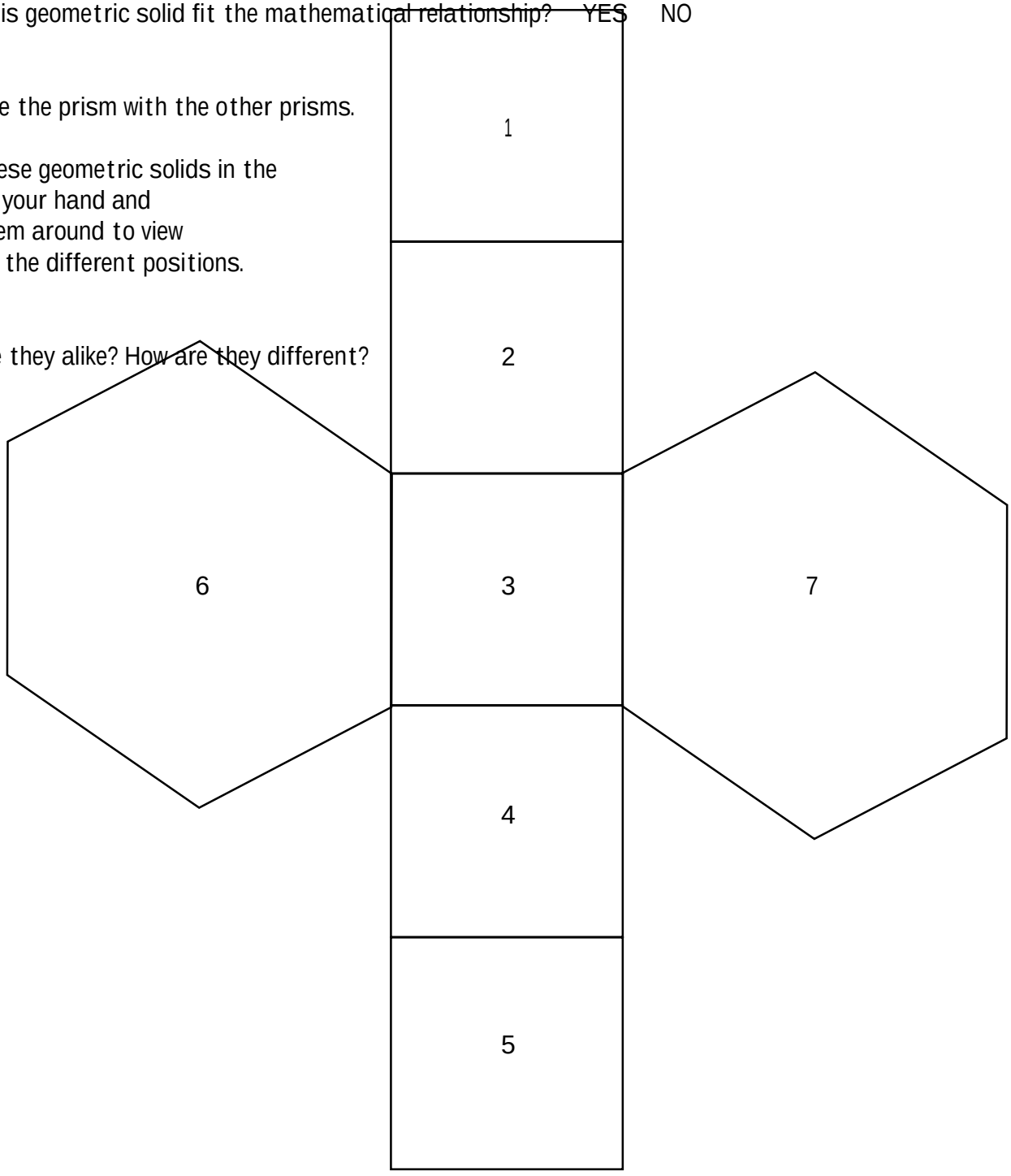
How many VERTICES, EDGES, AND FACES does this shape have? V = _____ E = _____ F = _____

Does this geometric solid fit the mathematical relationship? YES NO

Compare the prism with the other prisms.

Hold these geometric solids in the palm of your hand and turn them around to view from all the different positions.

How are they alike? How are they different?



THE CONE…

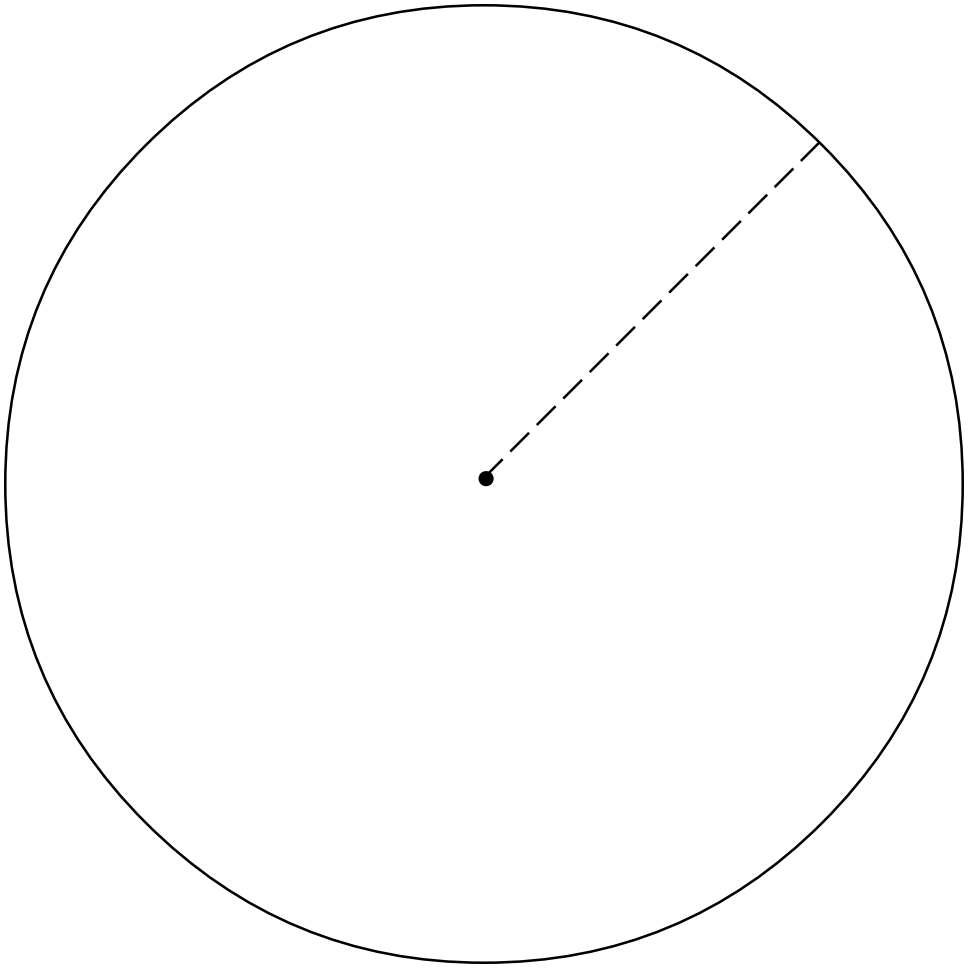
Cut the shape below.… Also cut along the dashed line.… Rotate the paper making a cone shape. Tape the edge of the paper. Set the cone onto another sheet of paper with the point up. Draw around the base of the cone. Cut out the resulting circle. Tape the circle to the base of the cone.

Compare this geometric solid to the others you have constructed.
How is like the others? … How is it different?

Is this shape a polyhedron? YES NO Explain your answer.…

Does this geometric solid have VERTICES, EDGES, AND FACES?

Is this geometric solid a regular polyhedron? YES NO Not Sure



THE SPHERE …

Find several round objects to add to the collection of geometric solids you have constructed. The objects could include ping-pong balls, a globe, some oranges would be round, a baseball or golf ball to name a few.

Compare these geometric solids to those you have constructed.
How are they like the others? … How are they different?

A round geometric solid is called a SPHERE.

Does the sphere have VERTICES, EDGES, AND FACES? YES NO

Would you consider a sphere to be a polyhedron? YES NO Explain your answer.….

EXPANDING THE IDEA - PART 3

Set all the geometric solids from the previous activity on the table.

Separate these solids into two groups.

On what characteristic or what basis did you make the first separation?

Did you separate them on the basis of whether or not they were polyhedrons or not polyhedrons?

If not, please do so now.

Which geometric solids are in the not-polyhedron grouping?
Which solids are in the polyhedron grouping?

Write the names in the chart below under the appropriate groupings....

ALL OBJECTS in Your Collection	
POLYHEDRONS	NOT POLYHEDRONS

Next, separate the polyhedrons into two groups.

Which geometric solids are in each group?

Did you group them on the basis of whether the solid is regular or irregular.

Write the names in the chart below under the appropriate groupings....

ALL OBJECTS in Your Collection		
POLYHEDRONS		NOT POLYHEDRONS
REGULAR	NOT REGULAR (IRREGULAR)	Cones Spheres

Find five geometric solids in your home. Place them into your groupings. Write the name of each object in the chart above.

EXPANDING THE IDEA - PART 4

Have you ever looked into a cereal box and wondered just how big it is? Or perhaps you have needed to paint your bedroom and wondered just how much paint you need to buy? Or maybe you have wondered just how much space is in your bedroom?

Is there some way to determine the size of a geometric solid? How much space does it contain and how much area is on the surface? The two areas of study are

the SURFACE AREA of a geometric solid, and

the VOLUME of a geometric solid.

THE SURFACE AREA
Of these two areas the first is much easier to determine.

Look at the first geometric solid you constructed. It is called a cube. All faces of a cube are equal.

How many faces does a cube have? _____

What is the area of each face?
(HINT: What is the mathematical relationship involving length, width, and area?)

_____

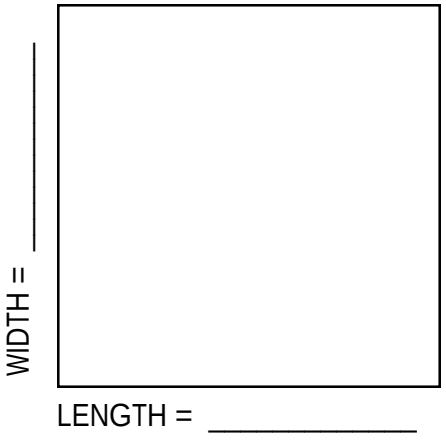
If you could determine the surface area of one face, then how could you determine the surface area of the complete cube? Write the mathematical relationship to determine the surface area of the cube?

_____

Measure the sides of one face of the cube
Record the measurements on the diagram.

The area of one face is found by multiplying the length by the width. Calculate the area of one face.

_____



THE SURFACE AREA

Since there are six faces on a cube,
multiply the surface area of one face by 6
to find the surface area of the complete cube.
What is the surface area of the cube you constructed?

_____

Find the geometric solid you constructed in the shape of a cereal box. This geometric solid is a rectangular prism.

How many faces does it have? _____

Are all the faces the same dimensions? YES NO

How could you determine the area of each face?
(HINT: What is the mathematical relationship involving length, width, and area?)

_____

If you numbered the faces from 1 - 6 (Face 1, Face 2, Face 3, Face 4, Face 5, and Face 6), write a mathematical equation involving only addition to determine the surface area of a rectangular box.

_____

If you add the area of each face, the sum would be the surface area of the rectangular box.

Use the measurements for each face to determine the surface area of each face....

FACE 1: Length: _____ Width: _____ AREA: _____

FACE 2: Length: _____ Width: _____ AREA: _____

FACE 3: Length: _____ Width: _____ AREA: _____

FACE 4: Length: _____ Width: _____ AREA: _____

FACE 5: Length: _____ Width: _____ AREA: _____

FACE 6: Length: _____ Width: _____ AREA: _____

THE SURFACE AREA

Since there are six faces on a rectangular prism, then the surface area of the complete prism is found by adding the surface areas of each face. Find the surface area of this prism.

_____

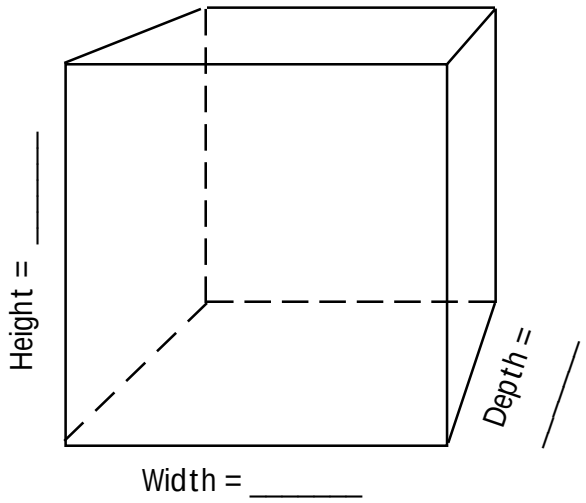
What about the space within these two geometric solids? How much space is there? In other words, what is the volume of each solid? Is it possible to determine the volume of a geometric solid? For example, have you ever wondered just how much water is in your bathtub when it is full? Since most bathtubs have the shape of a rectangular prism, if we can find a relationship involving the three dimensions - height, depth, and width - and the volume of a prism, then the volume of water within a bathtub could also be determined.

The VOLUME of the CUBE....
Look closely at the cube you constructed. There is a drawing representing the cube you constructed below. If you could open it up and pour sand or some liquid into it, how much would it hold?

Write the dimensions of the cube on the drawings....

What is the mathematical relationship involving height, width, depth, and volume of a cube?

_____



Another way to think about volume ...

How many 1 x 1 x 1 inch cubes could you place with this cube?

_____

Construct 12 - 1 x 1 x 1 inch cubes using the patterns on the following pages.

After they are constructed, carefully open one face of the 2 x 2 x 2 inch cube.

How many 1 x 1 x 1 cubes will fit into the larger cube? _____

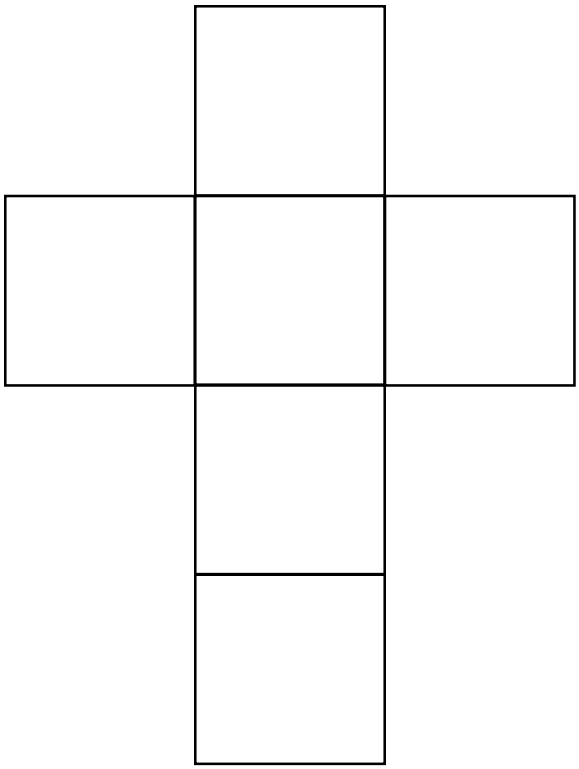
Is that what you had expected? YES NO

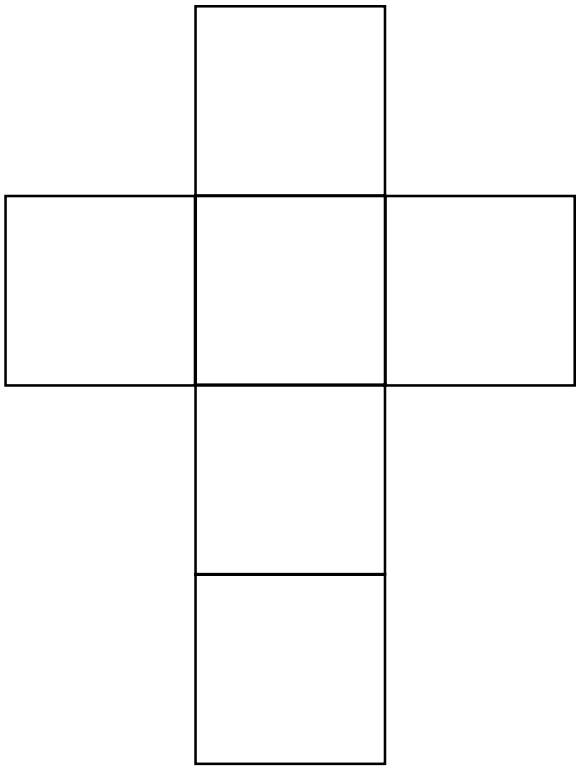
Height	Width	Depth	=	VOLUME
2	2	2	=	8

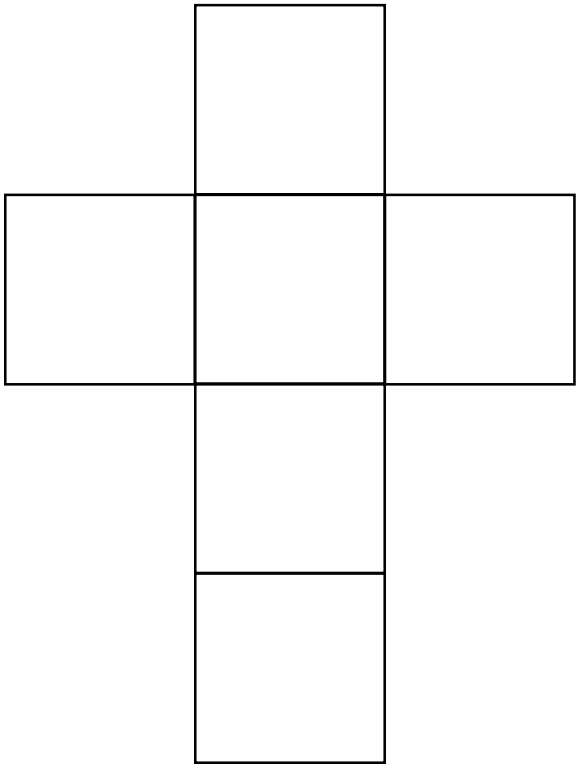
What mathematical relationship involving height, width, and depth will equal the volume of a cube?

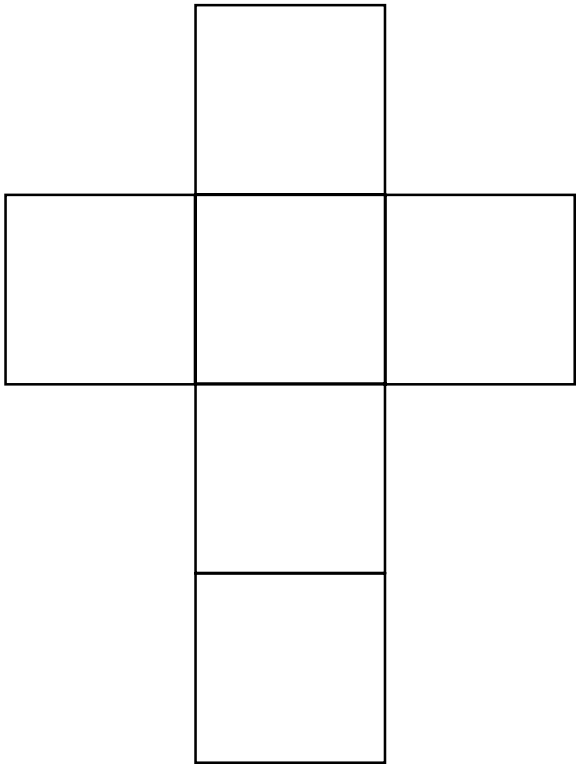
addition subtraction multiplication division

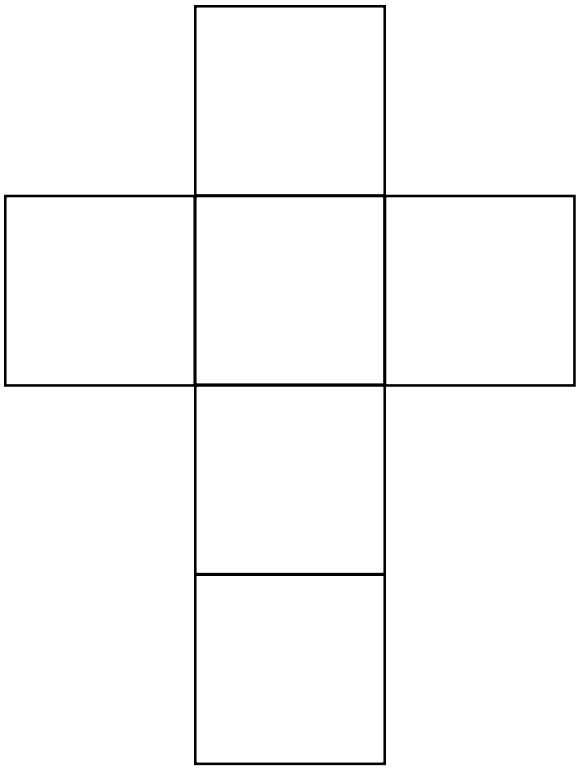
Place the proper mathematical symbol between height, width, and depth in the equation above.

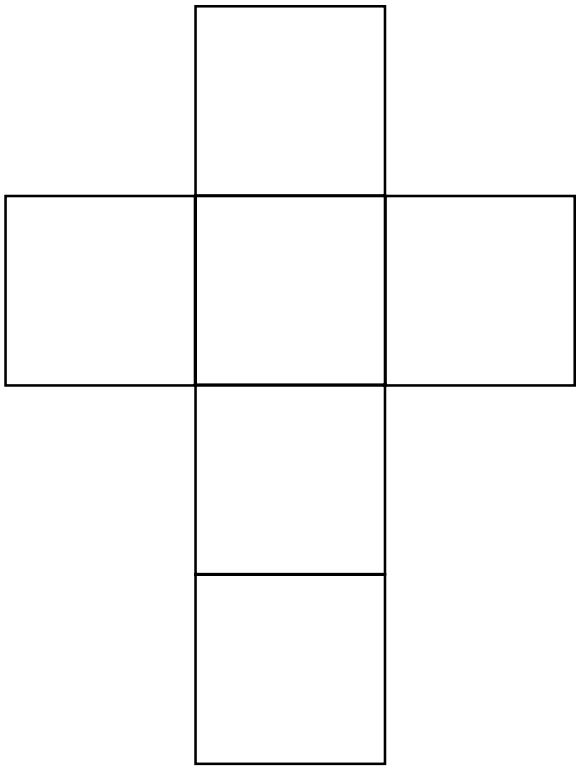


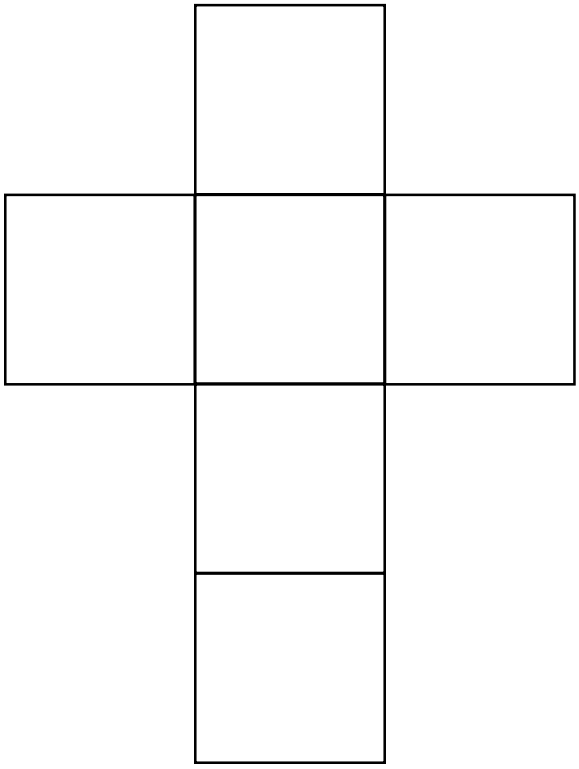


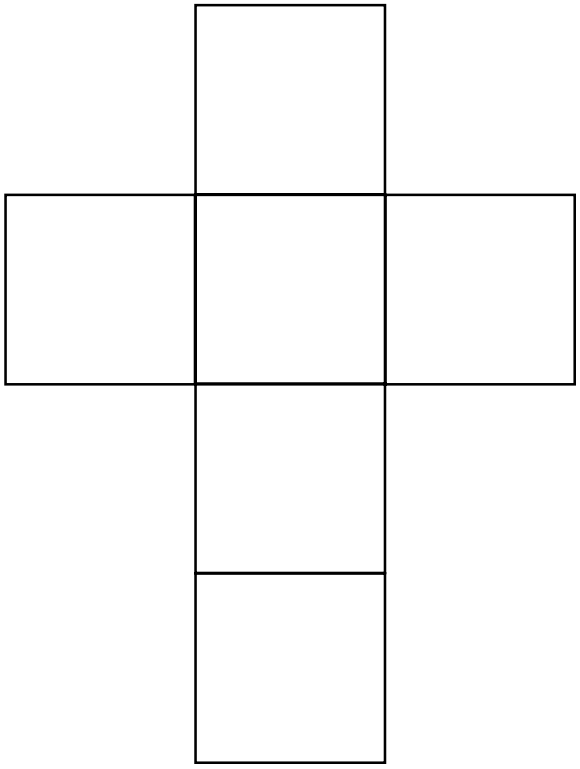


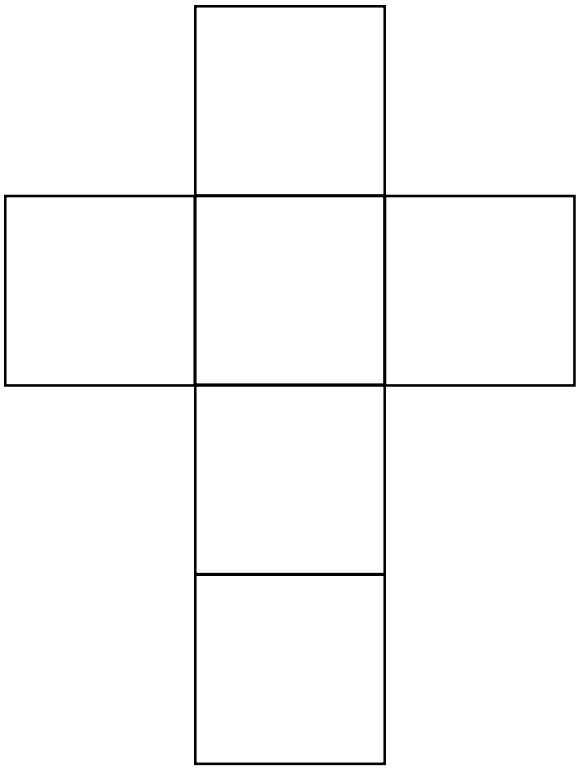


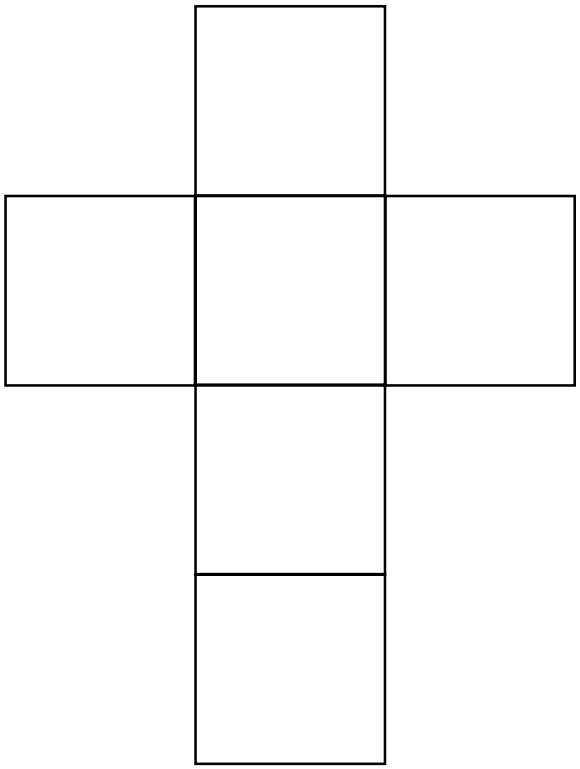


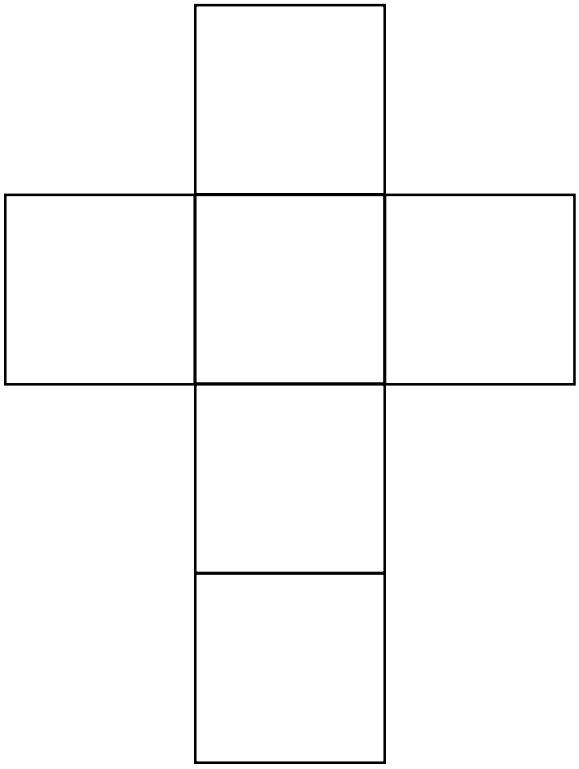


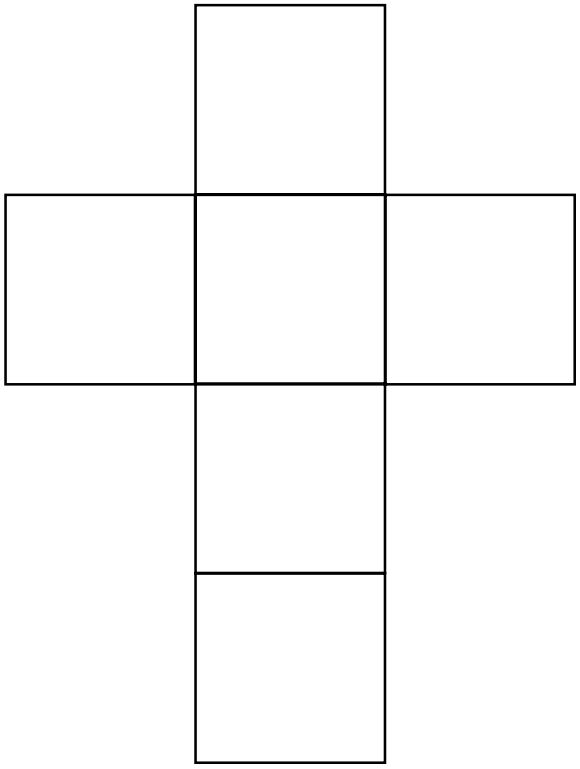




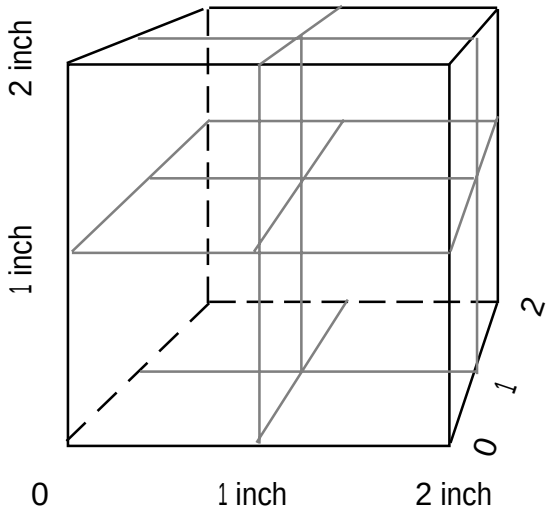








This drawing represents the 2 by 2 by 2 inch cube you constructed. The gray lines represent the 1 by 1 by 1 cubes. Using a ruler draw these same lines on your large cube.



Think of the large cube as having two layers. How many small cubes are on the bottom layer?

_____

How many small cubes are on the top layer?

_____

How many 1 by 1 by 1 cubes does it take to fill up the space of the 2 by 2 by 2 cube?

_____

Can you see a mathematical relationship involving the height, width, and depth of the cube and its volume? If so, write the equation below....

_____

The VOLUME of the RECTANGULAR PRISM
How can you use this relationship to determine the volume of a rectangular prism?

Look at the rectangular prism you constructed.
What are the dimensions of the rectangular prism you constructed?
Use a ruler to measure the height, width, and depth.

Height = _____ Width = _____ Depth = _____

How many little 1 by 1 by 1 cubes could you place within this rectangular prism? _____ Try placing the little 1 by 1 by 1 cubes into this geometric solid. How many fit? _____
Is this what you had expected? YES NO

Can you see a mathematical relationship involving the height, width, and depth of the rectangular prism and its volume? If so, write the equation below....

SUMMARY At this point what do you know?

THE CUBE: THE PRISM

2 something 2 something 2 something EQUALS 8 2 something 2 something 3 something EQUALS 12

What mathematical operation can you substitute for the SOMETHING to make the expression of the left side of the equal symbol equal to the value on the right side of the equal symbol?

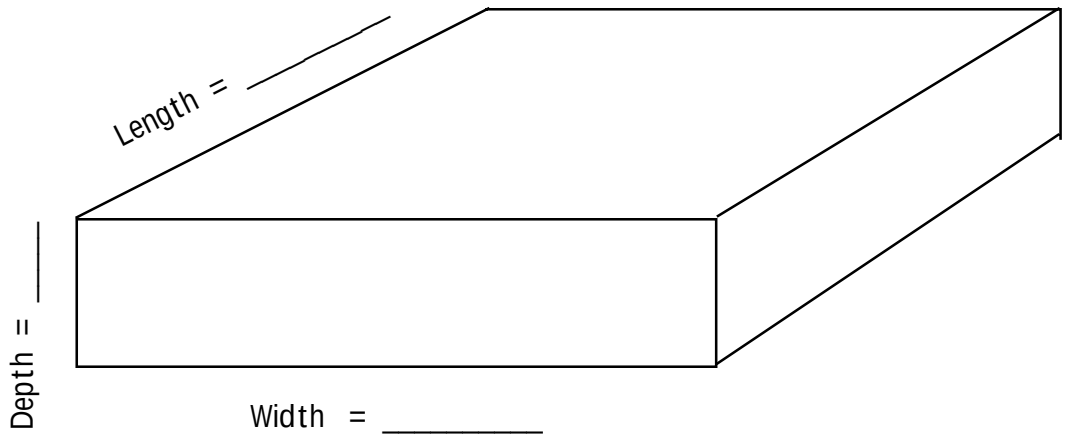
Addition Subtraction Multiplication Division

What is the mathematical relationship involving length, width, depth, and volume of a cube or rectangular prism? To better understand this relationship close your geometry book and describe the type of geometric solid it is. After making your observation, record your decision in the space provided…

My GEOMETRY BOOK is a … CUBE or RECTANGULAR PRISM

Measure the length, width, and depth of your book to the nearest inch. Record your measurements in the space provided and on the sketch of the book below.

Length = _____ Width = _____ Depth= _____



What is the volume of this book? In this case, is it possible to place small cubes in the space as was done in the previous two examples? YES NO

Is there a mathematical relationship involving length, width, depth, and volume?
And if so, can that relationship be determined?

Of course, the reason it is impossible to place little 1 by 1 by 1 cubes in the space is because pages fill up the space. It is as if the volume within this rectangular prism has been cut into many many thin pieces. It is this fact, that actually helps us to determine the volume.

If you knew the surface area of each page and the thickness of each page, then you could determine the volume of this rectangular prism.

Determine the surface area of the front cover….

Surface Area of Front Cover = Length X Width = _____ X _____ = _____

But try to measure the thickness of the front cover.

Most likely, you found it too thin to measure accurately.

Open the book to the first page. Measure the length and width of the first page to the nearest inch and record your measurements….

Length = _____ Width = _____

The surface area of the first page is …

Surface Area of the first page = Length X Width = _____ X _____ = _____

Again, try to measure the thickness of this page.

It too is to small to determine.

How does the surface area of the front cover compare with the surface area of the first page?

Equal or Not Equal

Find the surface area of the next several pages.

Page 2 Surface Area = _____ Page 3 Surface Area = _____

How does the surface area of each page of this book compare? EQUAL or NOT EQUAL

How does the fact that the surface area of each page is equal, help to find the volume of the book?

_____

_____ .

If you continued to measure the surface area of each page and if you could measure the thickness of each page, how would that help you determine the volume of the prism?

_____ .

What measurement is the depth of the book actually making? _____

_____ .

The sum of all he thicknesses of the slices (the pages), is the depth of the book. By cutting the volume of this rectangular prism into many very thin slices (the pages), the volume has not changed. Each page has a very small volume. The sum of all the slices of the book (that is, all the pages), makes up the whole volume. From this knowledge, state the mathematical relationship involving the length, width, and depth to the volume.

Length Width Depth = VOLUME

What mathematical symbol would you place between Length and Width, and between Width and Depth so that the Volume could be determined?

+ - x ÷

There is a mathematical relationship involving length, width, and depth and the volume of a cube or rectangular solid.

LENGTH X WIDTH X DEPTH = VOLUME

Go to your pantry and select several different boxes. Some of these items should represent the geometric solids of a cube while others the rectangular prism.

Take the measurements of each container and calculate both the SURFACE AREA and the VOLUME. Take measurements to the nearest inch.

CONTAINER 1:

Height = _____ Width = _____ Depth = _____

SURFACE AREA = _____

VOLUME = _____

CONTAINER 2:

Height = _____ Width = _____ Depth = _____

SURFACE AREA = _____

VOLUME = _____

CONTAINER 3:

Height = _____ Width = _____ Depth = _____

SURFACE AREA = _____

VOLUME = _____

CONTAINER 4:

Height = _____ Width = _____ Depth = _____

SURFACE AREA = _____

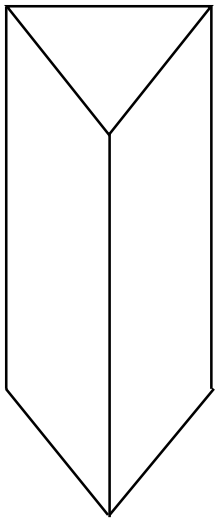
VOLUME = _____

Look at the other prisms you have constructed....

SURFACE AREA of Prism #2
Set the two triangular prisms on the table. Look closely at this geometric solid.
Below is a sketch of a triangular prism.

Triangular prisms have five polygon faces. These have ...

- three identical rectangular faces,
- a triangular base,
- and an identical triangular top.



How would you find the surface area of a triangular prism?

_____

_____ .

To calculate the surface area of the triangular prism it is
necessary to find the sum of the surface areas of each polygon face.

Measure the length and width of each polygon face of each triangular prism....

PRISM NUMBER 2

Rectangular Face 1: Length _____ Width _____ Surface Area _____

Rectangular Face 2: Length _____ Width _____ Surface Area _____

Rectangular Face 3: Length _____ Width _____ Surface Area _____

Triangular Face 4: Base _____ Height _____ Surface Area _____

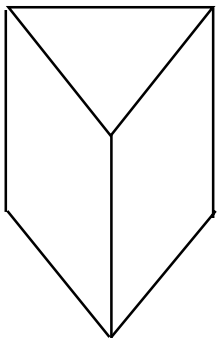
Triangular Face 5: Base _____ Height _____ Surface Area _____

Surface Area of Rectangular Face = Length X Width

Surface Area of Triangular Face = $\frac{1}{2}$ (base X height)

SURFACE AREA of Prism #3

This prism is also a triangular prism.



This prism also has …

- three identical rectangular faces,
- a triangular base,
- and an identical triangular top.

How would you find the surface area of a triangular prism?

_____

_____ .

To calculate the surface area of the triangular prism it is necessary to find the sum of the surface areas of each polygon face.

Measure the length and width of each polygon face of each triangular prism.…

PRISM NUMBER 3

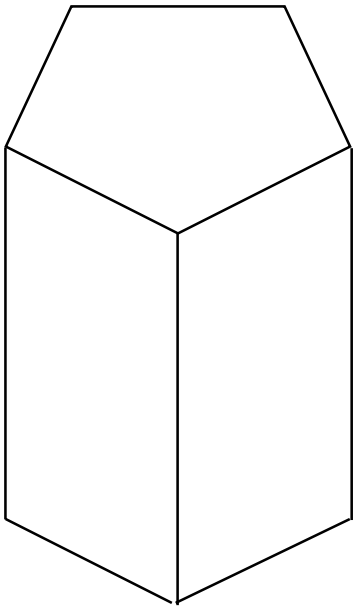
Rectangular Face 1:	Length	_____	Width	_____	Surface Area	_____
Rectangular Face 2:	Length	_____	Width	_____	Surface Area	_____
Rectangular Face 3:	Length	_____	Width	_____	Surface Area	_____
Triangular Face 4:	Base	_____	Height	_____	Surface Area	_____
Triangular Face 5:	Base	_____	Height	_____	Surface Area	_____

Surface Area of Rectangular Face = Length X Width

Surface Area of Triangular Face = $\frac{1}{2}$ (base X height)

SURFACE AREA of Prism #4

Below is a sketch of a Pentagonal Prism.



Pentagonal prisms have seven polygon faces. These have …

- five identical rectangular faces,
- a triangular base,
- and an identical triangular top.

How would you find the surface area of a Pentagonal Prism?

_____

_____ .

To calculate the surface area of the Pentagonal Prism it is necessary to find the sum of the surface areas of each polygon face.

Measure the length and width of each polygon face of each Pentagonal Prism….

PRISM NUMBER 4

Rectangular Face 1: Length _____ Width _____ Surface Area _____

Rectangular Face 2: Length _____ Width _____ Surface Area _____

Rectangular Face 3: Length _____ Width _____ Surface Area _____

Rectangular Face 4: Length _____ Width _____ Surface Area _____

Rectangular Face 5: Length _____ Width _____ Surface Area _____

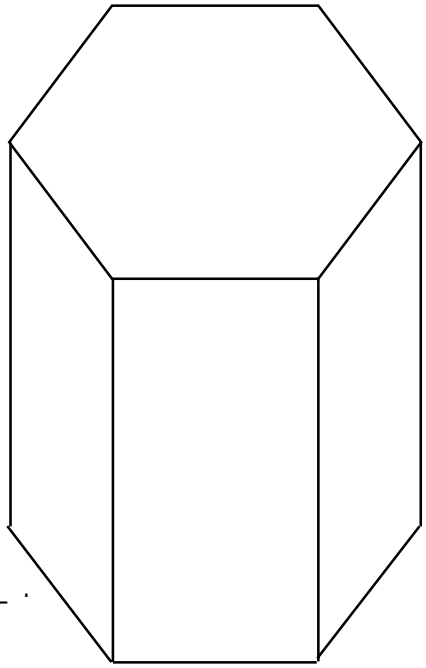
Pentagonal Face 6: Base _____ Height _____ Surface Area _____

Pentagonal Face 7: Base _____ Height _____ Surface Area _____

If you need help in determining the Surface Area of a Pentagon Face…. GO TO Page XXX

SURFACE AREA of Prism #5

Below is a sketch of a Hexagonal Prism.



Hexagonal Prisms have eight polygon faces. These have …

- six identical rectangular faces,
- a hexagonal base,
- and an identical hexagonal top.

How would you find the surface area of a Hexagonal Prism?

_____

_____ .

To calculate the surface area of the Hexagonal Prism it is necessary to find the sum of the surface areas of each polygon face.

Measure the length and width of each polygon face of each Hexagonal Prism….

PRISM NUMBER 5

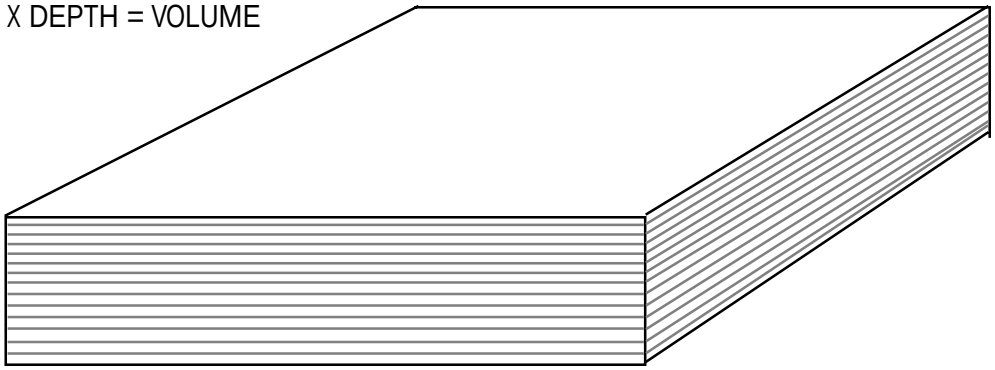
Rectangular Face 1:	Length	_____	Width	_____	Surface Area	_____
Rectangular Face 2:	Length	_____	Width	_____	Surface Area	_____
Rectangular Face 3:	Length	_____	Width	_____	Surface Area	_____
Rectangular Face 4:	Length	_____	Width	_____	Surface Area	_____
Rectangular Face 5:	Length	_____	Width	_____	Surface Area	_____
Pentagonal Face 6:	Base	_____	Height	_____	Surface Area	_____
Pentagonal Face 7:	Base	_____	Height	_____	Surface Area	_____
Pentagonal Face 8:	Base	_____	Height	_____	Surface Area	_____

If you need help in determining the Surface Area of a Hexagonal Face…. GO TO Page XXX

RETURNING TO THE VOLUME OF A PRISM

You will remember that the volume of your geometry book (a rectangular prism) was based upon the idea of finding the volume of each of the many slices (the pages of the book). Therefore, the volume of the rectangular prism is found by multiplying the surface area of each slice (Length X Width) times the combined thickness of the solid (the Depth).
The mathematical relationship is then,

LENGTH X WIDTH X DEPTH = VOLUME

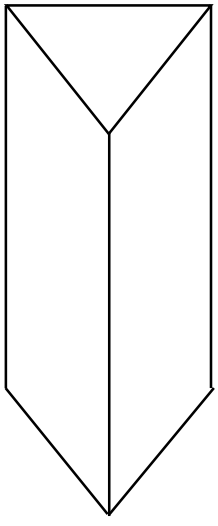


Explain how this relationship can be used to determine the volume of any and every prism.

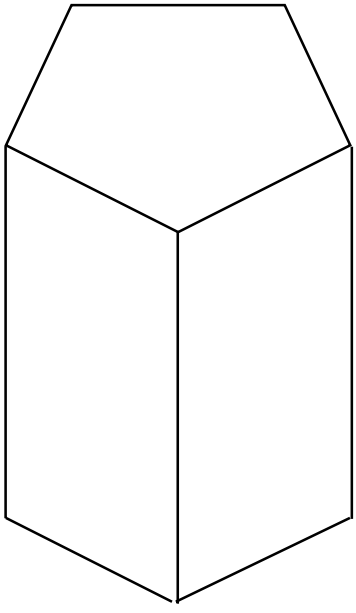
_____

_____

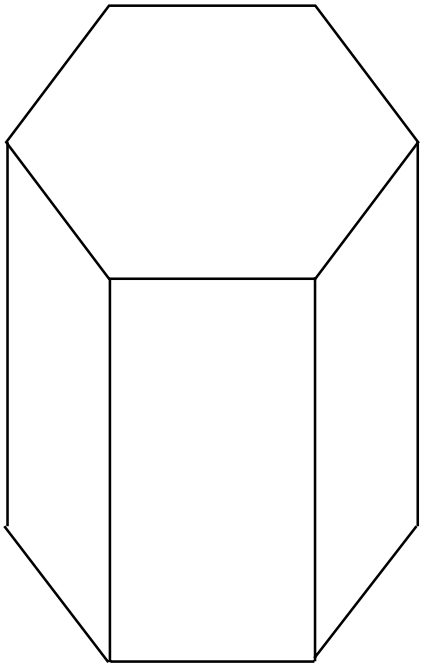
_____ .



Triangular Prism



Pentagonal Prism

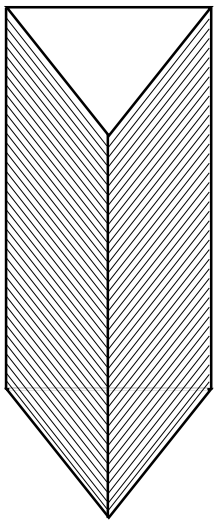


Hexagonal Prism

The volume of the three other prisms you constructed — the Triangular Prism, the Pentagonal Prism, and the Hexagonal Prism — can also be thought of as many very tiny slices. The VOLUME of all prisms is the relationship of the SURFACE AREA of each slice multiplied by the total height of the prism.

VOLUME = SURFACE AREA of the base of the prism X HEIGHT

If you think of the Triangular Prism as being made up of many very thin triangles, then the volume of this prism is simply the surface area of the triangle times the height of the prism.



VOLUME = SURFACE AREA of TRIANGLE X HEIGHT of PRISM

And since the surface area of the triangle is one-half the base times height,

then the mathematical relationship determining the volume of a triangular prism is …

$$\text{VOLUME} = \frac{1}{2} \text{ BASE } X \text{ HEIGHT}$$

Calculate the volume of the two triangular prisms you constructed earlier....

PRISM #2

PRISM # 3

Using this concept of volume from the previous prisms, explain how to determine the volume of both the Pentagonal Prism and the Hexagonal Prism. Illustrate your answer with the drawings below as well. Finally, determine the volume of each prism.

Pentagonal Prism

_____

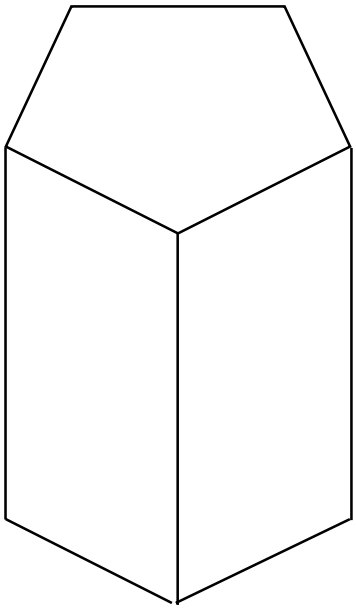
_____

_____

_____

_____

_____



Hexagonal Prism

_____

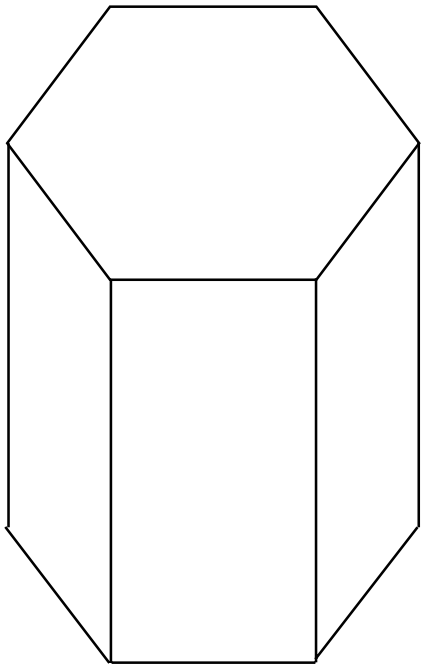
_____

_____

_____

_____

_____



Take out a round 'Quaker Oats' container and two cans of vegetables or fruits from your pantry....

This geometric solid is not a prism, but rather a cylinder.

Is it possible to find the volume of such a geometric shape?
Give your thoughts on how you could find the volume of the cylinder....

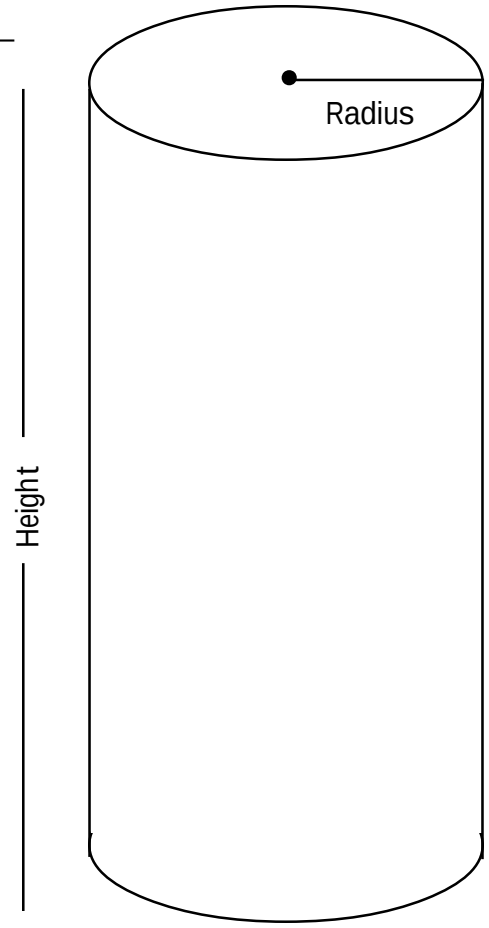
_____

_____

_____

_____

_____ .



Did you explain the volume of the cylinder as the surface area of many small slices of the circular face? If not, do so now.

_____

_____

_____ .

Using the several cylinders from your pantry, measure the height and radius of each and record this information on the following page.

Next, calculate the surface area of a circular face....

$$\text{Surface Area} = \pi r^2$$

Finally, calculate the volume of the cylinder....

$$\text{Volume} = \text{Height} \times \text{Surface Area of the Circular Face}$$

$$\text{Volume} = \text{Height} \times \pi \, \text{Radius}^2$$

Container 1

Height: _____

Radius: _____

Surface Area of Circular Face: _____

Volume = Height x π Radius^2 = _____

Container 2

Height: _____

Radius: _____

Surface Area of Circular Face: _____

Volume = Height x π Radius^2 = _____

Container 3

Height: _____

Radius: _____

Surface Area of Circular Face: _____

Volume = Height x π Radius^2 = _____

SURFACE AREA of other Geometric Solids

Place the following geometric solids on the table....

Tetrahedron • Octahedron • Dodecahedron • Icosahedron

What basic shape makes up the face of each solid?

Tetrahedron _____

Octahedron _____

Dodecahedron _____

Icosahedron _____

The figure below was the pattern used to construct the tetrahedron.
It is made up of 4 triangles.
If you know the surface area of one triangle,
then how would you find the surface area of the tetrahedron? _____

How do you calculate the area of a triangle?

_____

BASE = _____

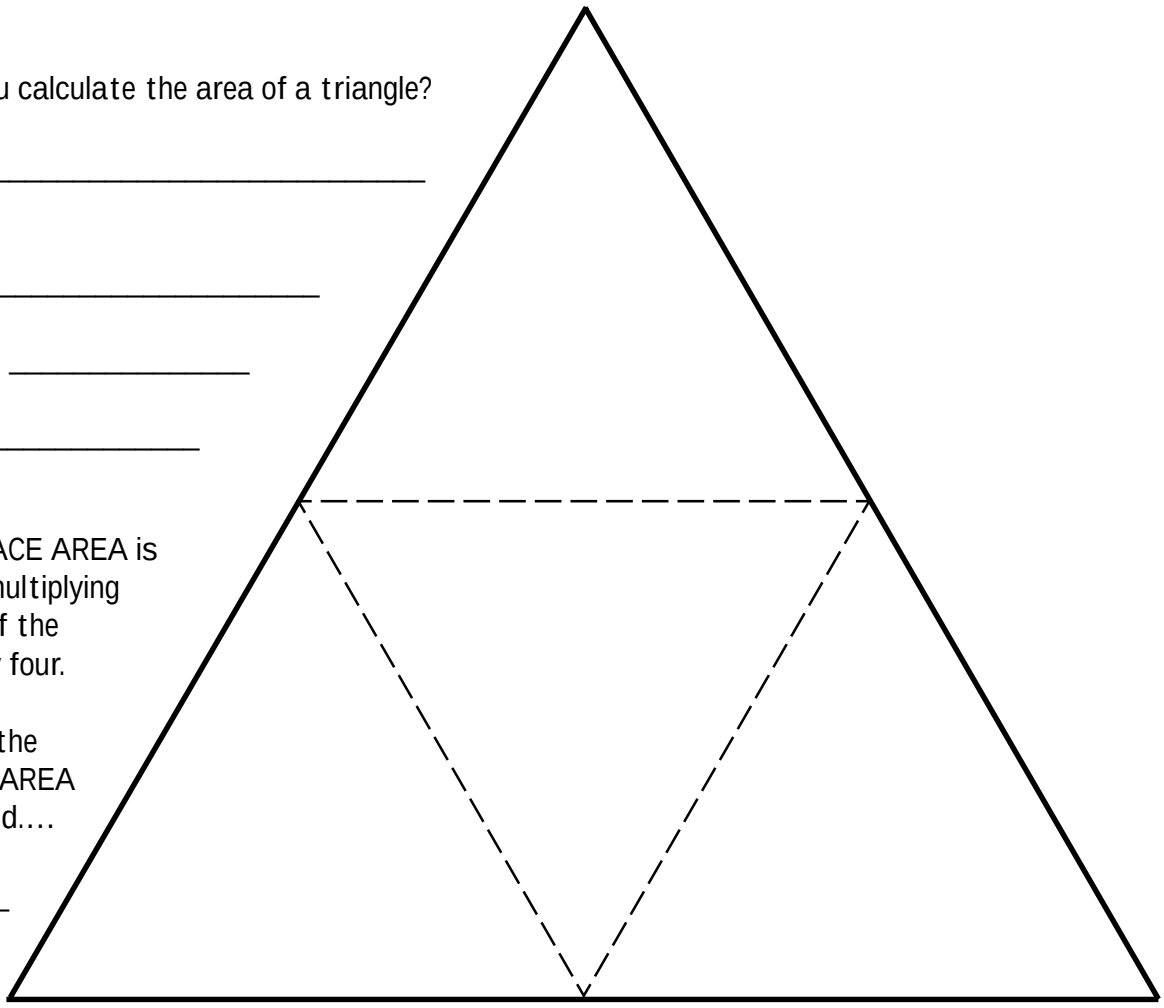
HEIGHT = _____

AREA = _____

The SURFACE AREA is found by multiplying the area of the triangle by four.

Calculate the SURFACE AREA of this solid....

_____



Next, look at the octahedron.

What polygon faces make up this geometric solid? _____

How many triangular faces make up the octahedron? _____

Explain how to calculate the surface area of the octahedron.
What mathematical relationship would you use to determine the surface area?

SURFACE AREA of OCTAHEDRON = _____

The octahedron is made up of 8 triangular polygons.
Take the appropriate measurements to determine the area of the triangular face....

BASE = _____

HEIGHT = _____

AREA of the Triangular Face = _____

The SURFACE AREA of the octahedron is found by multiplying the area of the triangle by eight.

Calculate the SURFACE AREA of the geometric solid you constructed....

_____

Calculate the surface area of the Icosahedron....

To calculate the surface area of the dodecahedron, what must be known? _____

_____ .

The dodecahedron is made up of how many polygon faces? _____

What kind of polygon faces make up the dodecahedron? _____

Can you calculate the surface area of one pentagon? YES NO

Explain the mathematical relationship to determine the surface area of pentagon? _____

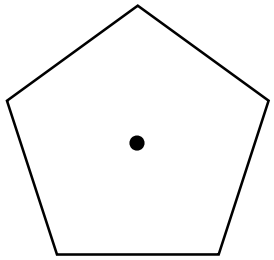
_____

_____ .

The pentagon on the right is taken from the pattern used to construct the dodecahedron.

BASE = .844 inches

HEIGHT = .594 inches



Calculate its surface area of one pentagon:

_____

The SURFACE AREA of the dodecahedron is 12 times the area of one polygon face.

SURFACE AREA = 12 X _____

Volume of Other Geometric Solids

Place the tetrahedron, prism number 1, and the two pyramids you constructed on the table....

What basic shape makes up the face of each solid?

Tetrahedron _____

Octahedron _____

Dodecahedron _____

Icosahedron _____

CHAPTER 9: The Cone

EXPLORING THE IDEA

Obtain the following items:

- a kitchen funnel, plastic
- a sugar ice cream cone
- a cone shape paper cup
- 'Play-Doh'

Describe the shape of the funnel, sugar ice cream cone. and a cone shape paper cup.... ____

_____

_____ .

Make a sketch of the general shape in the space below....

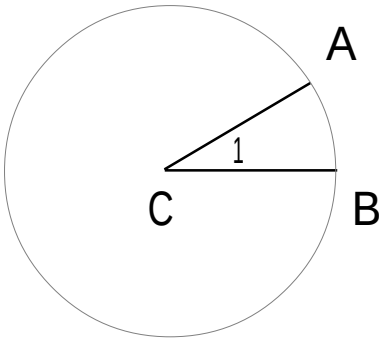
Use the 'Play-Do' and the kitchen funnel.

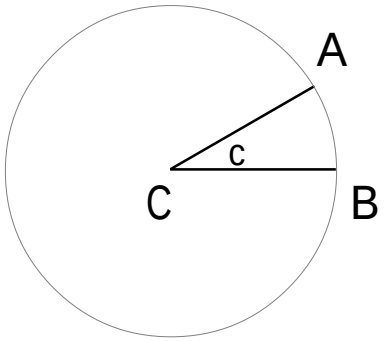
APPENDIX 1

Measuring an Angle

An angle is represented in one of three ways:

- By the letter of the vertex (the common end point of two line segments),
- By the number or letter located in the interior of the angle,
- By the three points that make the triangle.





The angle in the diagrams above could be represented in any of the following ways:

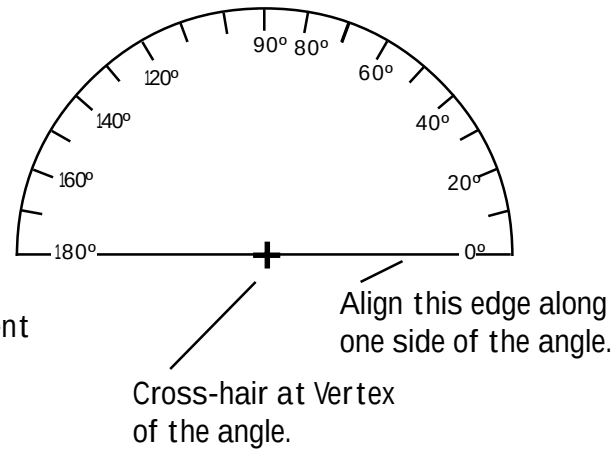
SYMBOL	BY The …
• ∠C	Letter of the vertex. read “angle C”.
• ∠1 or ∠c	Number or letter located in the interior of the angle read “angle 1” or “angle c”.
• ∠ACB or ∠BCA	Three points that make the angle, read “angle ACB” or “angle BCA”.

An angle is measured in degrees. A PROTRACTOR is used to measure the size of the angle.
Reading a Protractor:
The numbers along the each of the protractor are the degrees.

To measure an angle place the cross-hair at the vertex .

Align the straight edge of the protractor to one of the rays.

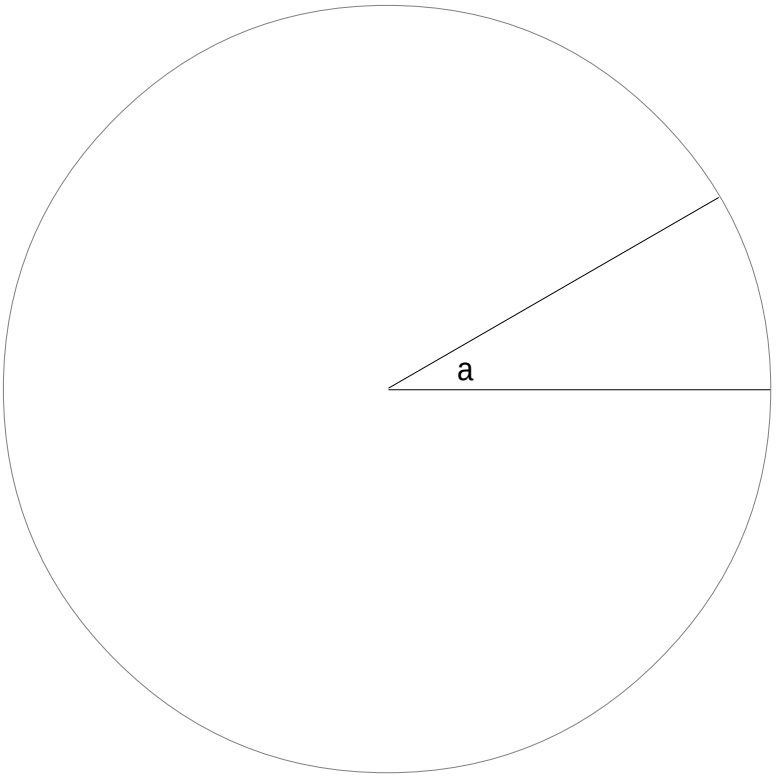
Notice where the other ray crosses the protractor. The number that aligns with this ray is the measurement of the angle in degrees.



Measuring an Angle

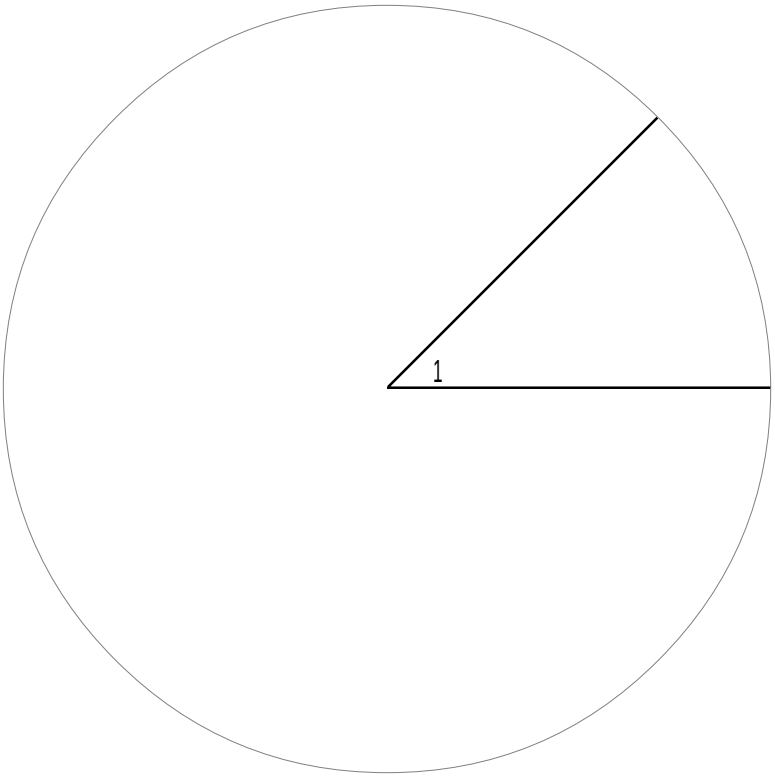
6. ∠A = _____

Using your pencil shade Angle A.
Next, using your protractor,
measure Angle a.



7. ∠1 = _____

Using your pencil shade Angle 1.
Next, using your protractor,
measure Angle 1.

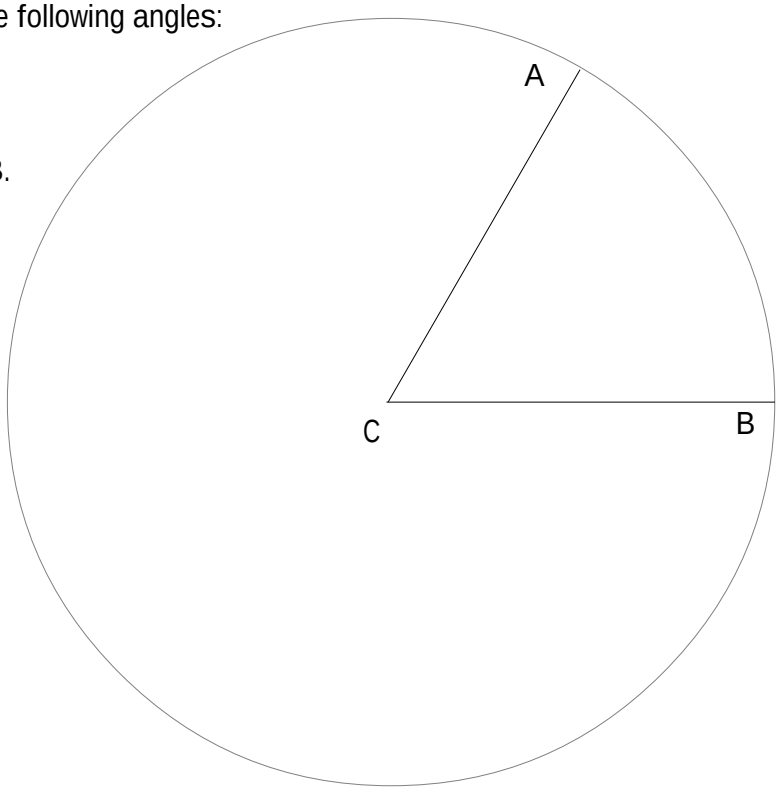


Measuring an Angle

Using your protractor measure the following angles:

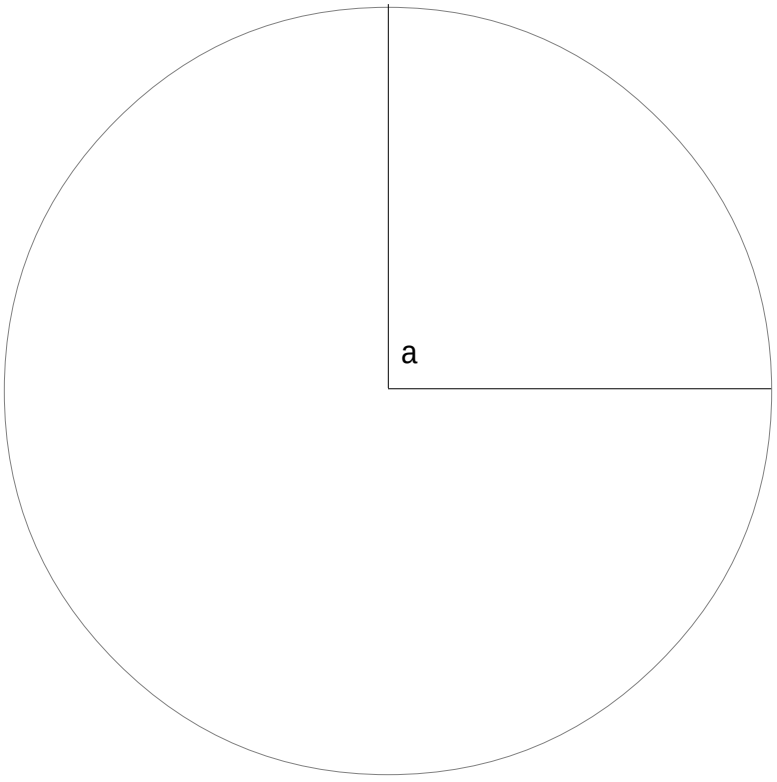
8. ∠ ACB = _____

Using your pencil shade Angle ACB.
Next, using your protractor,
measure Angle ACB.



9. ∠ a = _____

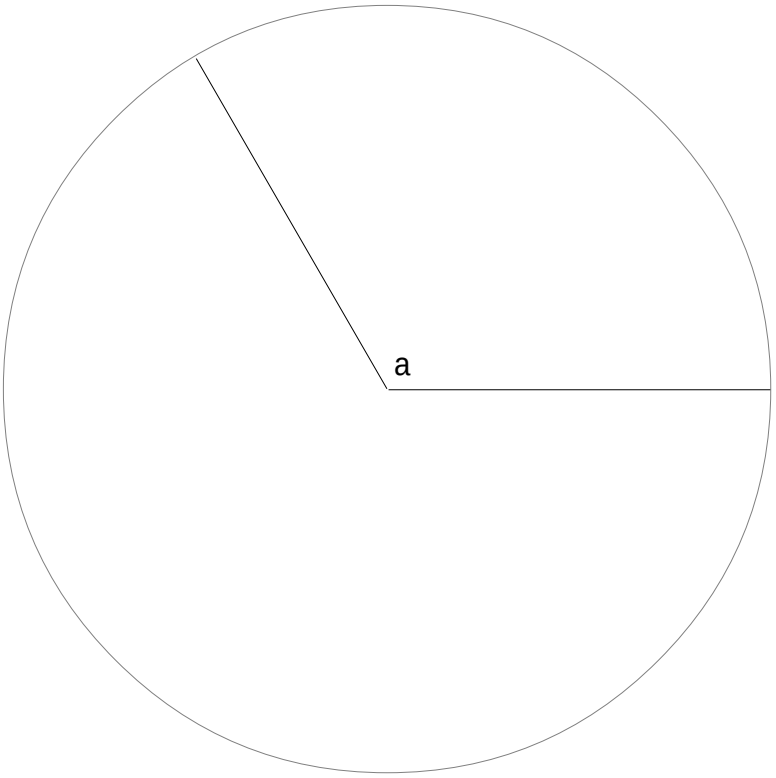
Using your pencil shade Angle a.
Next, using your protractor,
measure Angle a.



Measuring an Angle

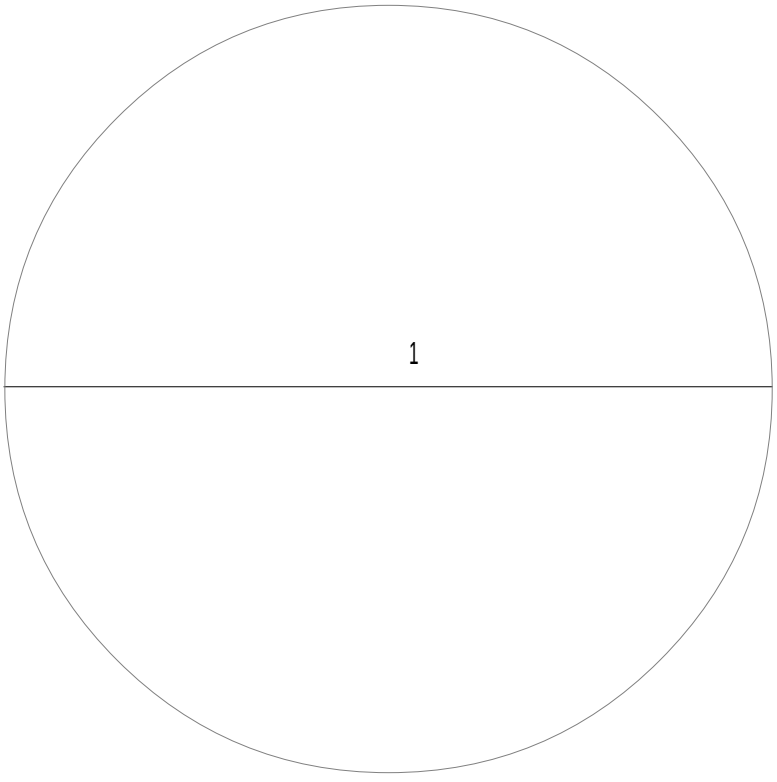
8. ∠A = _____

Using your pencil shade Angle A. Next, using your protractor, measure Angle a.



9. ∠1 = _____

Using your pencil shade Angle 1. Next, using your protractor, measure Angle 1.



It is important that you become very familiar with the size of certain angles. You must be able to recognize the following angles....

30º angle 45º angle 60º angle 90º angle 120º angle 180º angle

10. Without using your protractor, make a rough sketch of each of the following angles:

A. 30º angle

B. 45º angle

C. 60º angle

D. 90º angle

E. 120º angle

F. 180º angle

11. Using your protractor check how close your drawings are to the correct angle. If you are more than 5 degrees off, redraw the angle.