

# PRINCIPLES FROM PATTERNS

## Geometry - Algebra II - Trigonometry

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*Geometry is all about measuring lines, angles, surfaces, solids, velocities and their interrelationships. In this study, you will act as a consultant, designer-planner, and builder. The projects will range from designing a tree fort in your back yard to planning the construction of a sidewalk and home on the hilly streets of San Francisco to charting the path of the earth around the sun. In the process you will learn the principles as well as the vast usage of geometry in everyday life. Geometry is used by graphic animators, artists, photographers, interior designers, engineers, architects, builders, construction teams, surveyors and doctors just to name a few.*





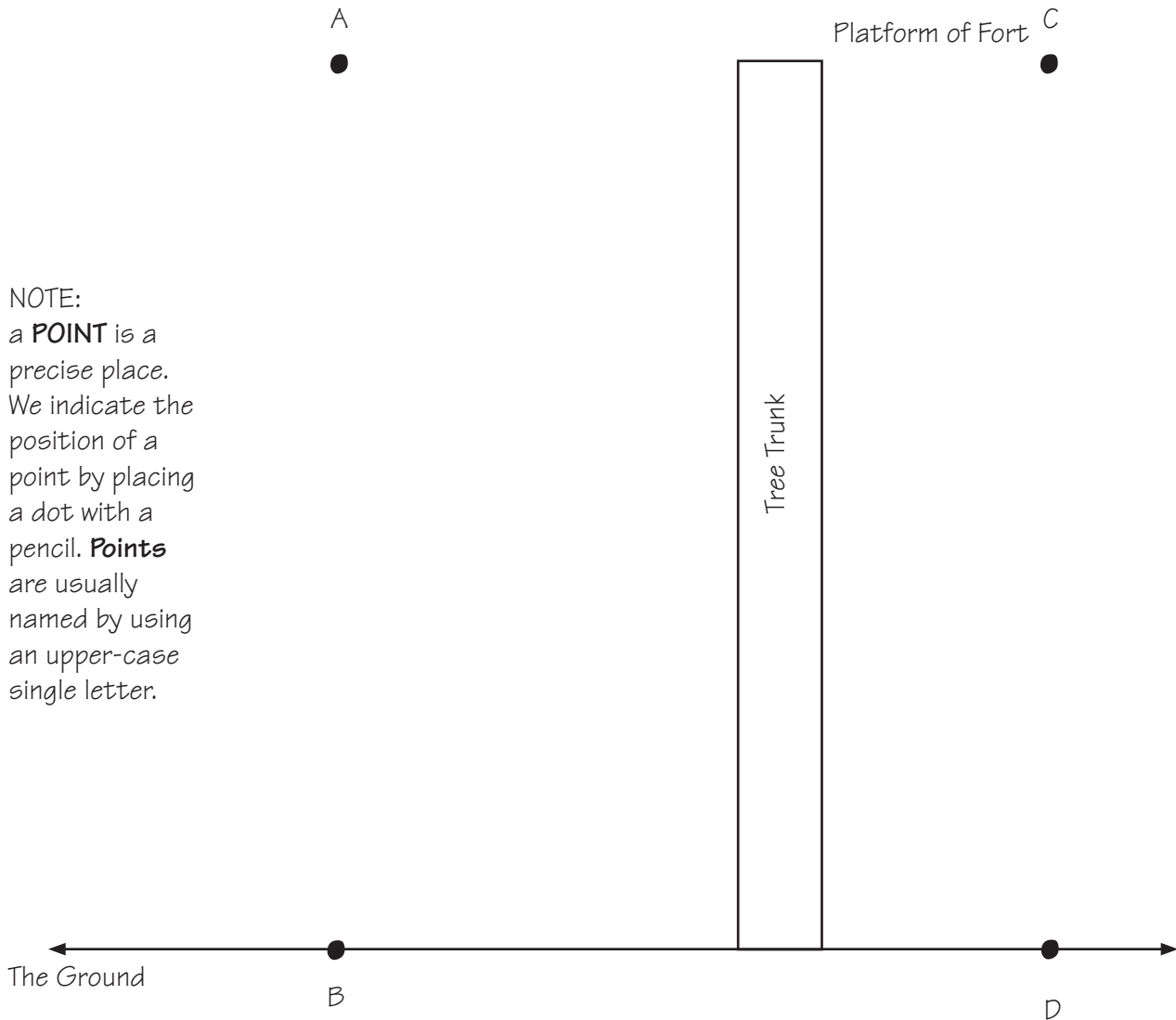
# Chapter 1: A Portion of a Line

## EXPLORING THE IDEA

Your little brother asks you to help him make a tree fort.  
You decide to make a rough sketch of what it would look like.

Draw a straight line going through Points B and D connecting the arrows.  
This will represent the ground.

Next, draw a straight line going from Point A to Point C. This will represent the platform for the fort.



NOTE:  
a **POINT** is a  
precise place.  
We indicate the  
position of a  
point by placing  
a dot with a  
pencil. **Points**  
are usually  
named by using  
an upper-case  
single letter.

In the figure above, the line passes through the points B and D, and goes perfectly straight in both directions forever as indicated by the arrows in both directions. A line has no ends.

GO TO <http://www.mathopenref.com/line.html> for more information about the **LINE**.

Using your ruler **darken the section** of the line between Point B and Point D.

## EXPLAINING THE IDEA

The darkened portion of the line connecting Point B to Point D is called a

### LINE SEGMENT

and is represented by drawing a line over the letters representing the endpoints.

$\overline{BD}$  is read **line segment BD**.

A line segment is simply a part of the line.

It has two **ENDPOINTS**. In this case Point B and Point D.

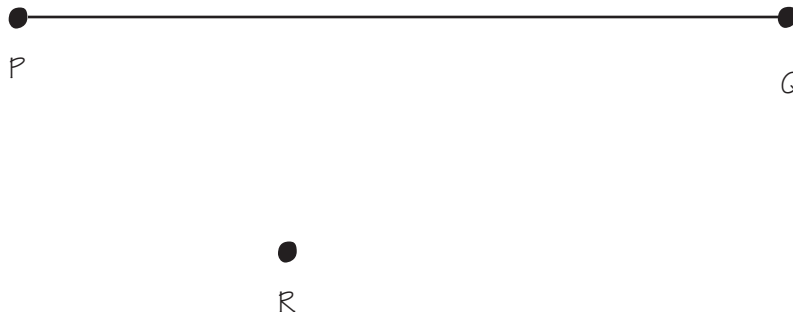
The **LINE SEGMENT** is named by its **ENDPOINTS**

How would you symbolize the line segment connecting Point A to Point C? \_\_\_\_\_ .

### COPYING A LINE SEGMENT

Using a compass and a straight edge make a copy of line segment PQ that has R as one of its endpoints.

1. Set the compass point on the point P of the line segment to be copied.
2. Set the pencil end of the compass on point Q.  
The compass width is now equal to the length of the line segment PQ.
3. Without changing the compass width, place the compass point on the the point R of the new line segment you will draw. Draw an arc roughly where the other endpoint will be.
4. Pick a point S on the arc. point S will be the other endpoint of the new line segment.
5. Draw a line from R to S.



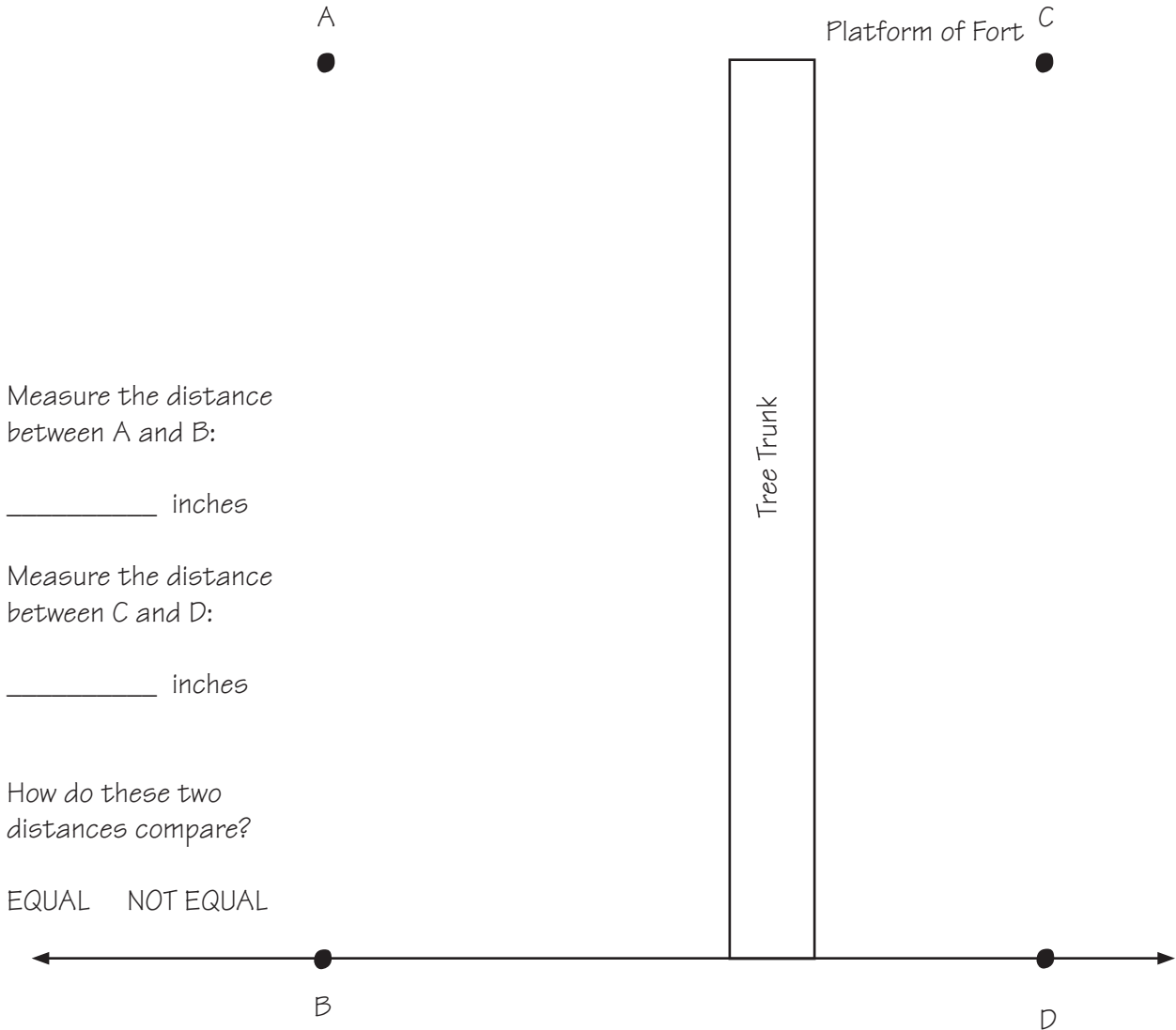
6. With a ruler measure line segment PQ and line segment RS.

Are they equal or not equal in length?    EQUAL        NOT EQUAL

GO TO <http://www.mathopenref.com/linesegment.html> for more information about **LINE SEGMENTS**.

## EXPANDING THE IDEA Part 1

Draw a line connecting Point A to Point B, and another line connecting Point C to Point D



Write an equation representing the relationship between  $\overline{AB}$  and  $\overline{CD}$ .

AB  CD

The distance between line segment AC and line segment BD is always equal. No matter how far you extend line AC and line BD the two lines will always be an equal distance apart. When two line segments are equally spaced they are said to be

### PARALLEL

Parallel lines are represented by this symbol: ||

In the above problem, line segment  $\overline{AC}$  is parallel to line segment  $\overline{BD}$ .

It is written  $\overline{AC} \parallel \overline{BD}$ .

## Drawing Parallel Lines

Using a ruler and pencil in the space below, draw two lines that are parallel.

How did you do? Where you able to draw two lines that are parallel?

1. Start with a line segment  $AB$  and a point  $Q$  off the line.
2. Draw any line through  $Q$  that crosses the line segment  $AB$  at any angle, forming the point  $R$  where it intersects the line  $AB$ . Now you have a new line segment  $QR$ .
3. With the compass width set to less than half the distance between  $R$  and  $Q$ , place the point on  $R$ , and draw an arc across both lines.
4. Without adjusting the compass width, move the compass to  $Q$  and draw a similar arc to the one in step 3.
5. Set compass width to the distance where the lower arc crosses line segment  $AB$  and line segment  $QR$ .
6. Move the compass to where the upper arc crosses line segment  $QR$  and draw an arc across the upper arc, forming point  $S$ .
7. Draw a straight line through points  $Q$  and  $S$ .
8. Line segment  $AB$  is parallel to line segment  $QS$ .

Q .



Using the information on the previous page,  
construct another set of two lines that are parallel.

P •



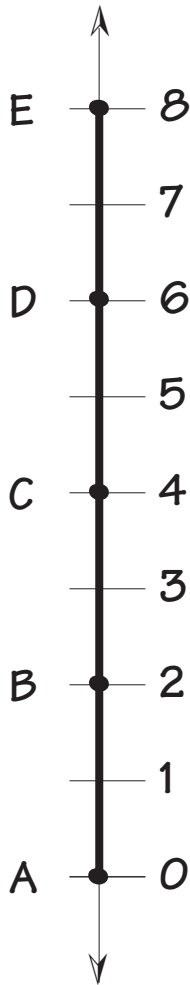
1. What are several characteristics of parallel lines?

For more information about constructing parallel lines through a point,

GO TO: <http://www.mathopenref.com/constparallel.html>

## EXPANDING THE IDEA PART 2

Find the number of spaces between each of the following line segments ...



The length of  $\overline{AB}$  is 2 spaces or units found by  $2 - 0 = 2$

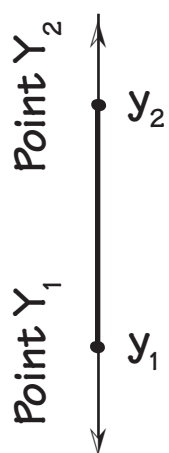
The length of  $\overline{AC}$  is \_\_\_\_\_ spaces or units found by  $4 - 0 =$

The length of  $\overline{BE}$  is \_\_\_\_\_ spaces or units found by  $8 - 2 =$

The length of  $\overline{CE}$  is \_\_\_\_\_ spaces or units found by  $8 - 4 =$

What mathematical operation is used to determine the length of a line segment?

Addition?      Subtraction?      Multiplication?      Division?



The diagram to the left shows a line segment  $\overline{Y_1Y_2}$ .  
 The numerical value of endpoint  $Y_1$  is represented by the lower case letter  $y_1$ .  
 The numerical value of endpoint  $Y_2$  is represented by the lower case letter  $y_2$ .  
 The equation used to determine the length of line segment  $\overline{Y_1Y_2}$  is ...

$$\overline{Y_1Y_2} = \underline{\hspace{2cm}}$$

Each point on a line has exactly one real number associated with it.  
 The distance between two points is determined by subtraction:

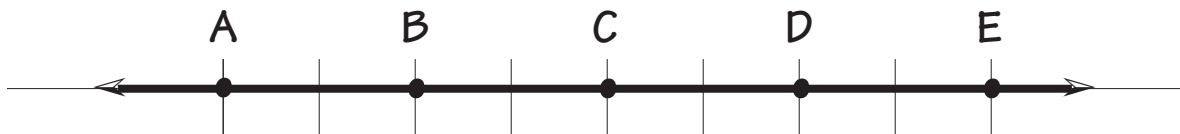
**The larger number minus the smaller number**

The distance between  $Y_1$  and  $Y_2$  is ...  $\overline{Y_1Y_2} = y_2 - y_1$

### EXPANDING THE IDEA Part 3

Instead of the line being vertical, the line is now horizontal.

Find the number of spaces between each of the following line segments ...



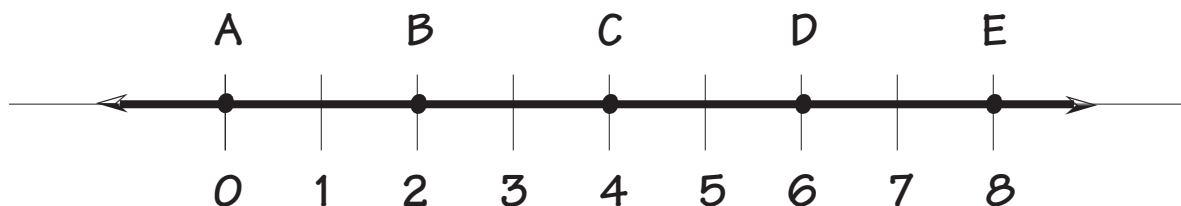
The length of  $\overline{AB}$  is 2 spaces.

The length of  $\overline{AC}$  is \_\_\_\_\_ spaces.

The length of  $\overline{BE}$  is \_\_\_\_\_ spaces.

The length of  $\overline{CE}$  is \_\_\_\_\_ spaces.

If the spaces are given numbers, then the line would look like this ...



The length of  $\overline{AB}$  is found by subtraction: Point B - Point A:  $2 - 0 = \square$

The length of  $\overline{BE}$  is found by subtraction: Point E - Point B:  $8 - 2 = \square$

The length of  $\overline{AC}$  is found by subtraction: Point C - Point A: \_\_\_\_\_

What mathematical operation is used to determine the distance of a line segment?

Addition?      Subtraction?      Multiplication?      Division?



The diagram above shows a line segment  $\overline{X_1X_2}$ . The numerical value of endpoint  $X_2$  is represented by the lower case letter  $x_2$ . The numerical value of the endpoint  $X_1$  is represented by the lower case letter  $x_1$ .

$$\overline{X_1X_2} = \underline{\hspace{2cm}}$$

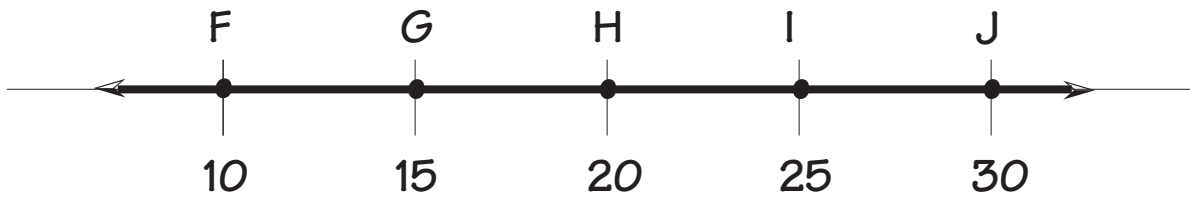
The distance between two points is determined by subtraction:

**The larger number minus the smaller number**

The distance between  $X_1$  and  $X_2$  is ...  $\overline{X_1X_2} = x_2 - x_1$

## EXPANDING THE IDEA Part 4

Finding the distance between two or more line segments ...



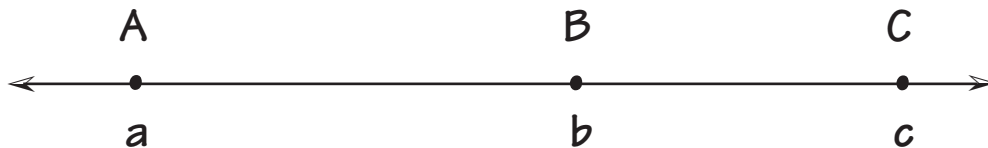
15. The length of  $\overline{FH}$  is equal to  $\overline{FG} + \overline{GH}$  which is equal to  $(5) + (5) = \square$
16. The length of  $\overline{GI}$  is equal to  $\overline{GH} + \overline{HI}$  which is equal to \_\_\_\_\_ .
17. The length of  $\overline{HJ}$  is equal to  $\overline{HI} + \underline{\hspace{1cm}}$  which is equal to \_\_\_\_\_ .
18. The length of  $\overline{FI}$  is equal to  $\overline{FG} + \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$  which is equal to \_\_\_\_\_ .
19. What mathematical operation is used to determine the distance between two or more line segment?

Addition?

Subtraction?

Multiplication?

Division?



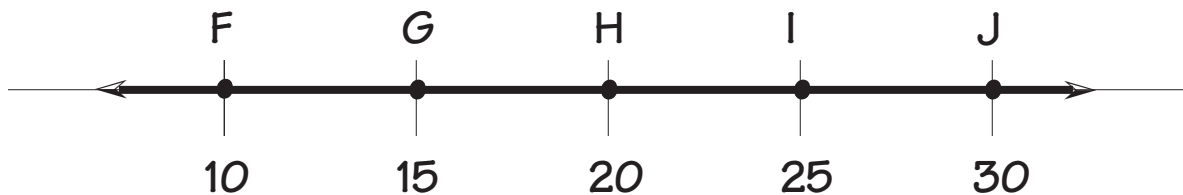
20. The diagram above shows three line segments:  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{AC}$ . Write the general equation expressing the length of line segment  $\overline{AC}$ :

$$\overline{AC} = \underline{\hspace{1cm}} + \underline{\hspace{1cm}}$$

The WHOLE Line Segment (AC) is made up of the two PARTS (AB + BC).  
(Refer to Making Math Meaningful Level 2)

## EXPANDING THE IDEA Part 5

Finding the Distance Half Way Between Endpoints...



21. What is the point halfway between the line segment FJ? \_\_\_\_\_ .

Therefore, the distance from F to H is equal to the distance from H to J.

The length of line segment FH is one-half the length of line segment FJ

22. What is the point halfway between the line segment GI \_\_\_\_\_ .

23. Explain how the point halfway between the endpoints can be found.



24. M is halfway between  $\overline{AC}$ . That means that  $\overline{AM}$  is \_\_\_\_\_ to  $\overline{MC}$ .

The point halfway between the two endpoints of a line segment is called the

**MIDPOINT.**

The midpoint always divides the line segment into two equal parts.

The midpoint is equal to one-half the distance of the line segment.

Line segment AM is what fractional part of line segment AC? \_\_\_\_\_

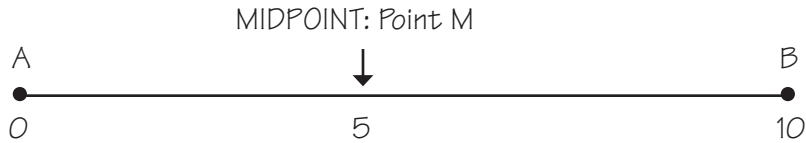
Write a math equation showing this relationship between  $\overline{AM}$  and  $\overline{MC}$  :

$$AM = \boxed{\phantom{00}} MC$$

For problems a - e the left endpoint is 0.

Find the MIDPOINT of each of the line segments given below:  
Make a sketch of each line segment showing the midpoint.

- a. If line segment AB is 10, then the midpoint is  $\frac{1}{2} AB = \frac{1}{2} (10) = 5$  .



How does the distance from point A to the midpoint, point M compare with the distance from the midpoint, point M, to point B?    EQUAL    NOT EQUAL

- b. If line segment BC is 4, then the midpoint is \_\_\_\_\_ .
- c. If line segment AG is 22, then the midpoint is \_\_\_\_\_ .
- d. If line segment DE is 9, then the midpoint is \_\_\_\_\_ .
- e. If line segment FG is 15, then the midpoint is \_\_\_\_\_ .

f. Explain how to find the midpoint of a line segment.

Did you add, subtract, multiply, divide, or some combination of these operations?

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**Try these two problems. Make a drawing to confirm your answers!**

g. If the left end point of line segment CF is 4, and the right end point is 10, then the midpoint is \_\_\_\_\_ .

h. If the right endpoint of line segment BE is 8, and the left end point is 2, then the midpoint is \_\_\_\_\_ .

Adjust your explanation of how to determine the MIDPOINT OF A LINE SEGMENT to include problems g and h.

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The midpoint can be determined by using either one of two methods:

METHOD 1:

Find one - half of the line segment and move that many spaces from either endpoint.

METHOD 2:

Find the average of the values for the two endpoints.

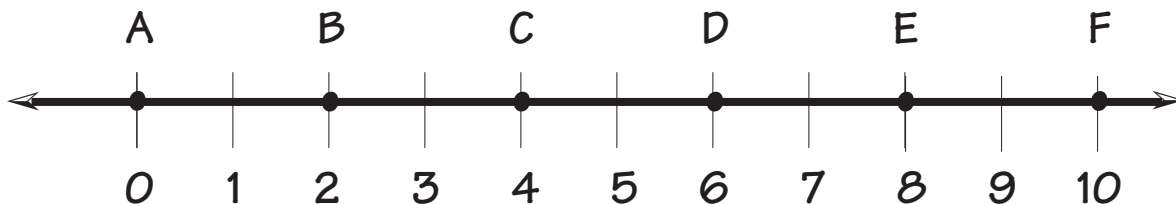
For problems i and j, the left endpoint is not at 0.

i. If the left end point of line segment CF is 4, and the right end point is 10, then the midpoint is \_\_\_\_\_ .

Did you think?      The length of the line segment is  $10 - 4 = 6$ ;  
                                   $1/2$  of 6 is 3;  
                                  The midpoint is 3 spaces to the right of 4 which is 7.

Or did you think?      The average of the two endpoints is  
                                  the sum of the two numbers  $4 + 10 = 14$   
                                  divided by 2       $14 \div 2 = 7$

Check your answer by using the number line below:



j. If the right endpoint of line segment BE is 8, and the left end point is 2, then the midpoint is \_\_\_\_\_ .

Did you think?      The length of the line segment is \_\_\_\_\_ ;  
                                   $1/2$  of \_\_\_\_\_ is \_\_\_\_\_ ;  
                                  The midpoint is \_\_\_\_\_ spaces to the right of \_\_\_\_\_ which is \_\_\_\_\_.

Or did you think?      The average of the two endpoints is  
                                  the sum of the two numbers \_\_\_\_\_ + \_\_\_\_\_ = \_\_\_\_\_  
                                  divided by 2      \_\_\_\_\_  $\div$  2 = \_\_\_\_\_

Check you answer by using the number line above.

The distance from one endpoint to the midpoint is EQUAL NOT EQUAL to the distance from the other endpoint to the midpoint.

For problems k - m, find the midpoint using either method. Confirm your answer with a sketch.

k. If the left end point of a line segment is 5, and the right end point is 11, then the midpoint is \_\_\_\_\_ .

l. If the left endpoint of a line segment is 12, and the right end point is 20, then the midpoint is \_\_\_\_\_ .

m. If the line segment is 10 inches and the left endpoint is at 5 inches, then the midpoint is \_\_\_\_\_ .

What method did you use to find the midpoint?

For problems n - q, you might need to draw the line segments in order to find the midpoint.

n. If the line segment is 12, and if the left endpoint is at 4, then the right endpoint is at \_\_\_\_\_ and the midpoint is at \_\_\_\_\_ .

o. If the line segment is 18, and if the left endpoint is at 1, then the right endpoint is at \_\_\_\_\_ and the midpoint is at \_\_\_\_\_ .

p. If the line segment is 4, and if the right endpoint is at 5, then the left endpoint is at \_\_\_\_\_ and the midpoint is at \_\_\_\_\_ .

A CHALLENGE!

q. If the line segment is 24, and if the right endpoint is at 20, then the left endpoint is at \_\_\_\_\_ and the midpoint is at \_\_\_\_\_ .

GO TO: <http://www.mathopenref.com/midpoint.html> for more information about MIDPOINT.

## EXPANDING THE IDEA PART 6

Look closely at the three diagrams below ...



25. Describe how the three diagrams are different ...

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26. How would you define a LINE.

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27. How would you define a LINE SEGMENT.

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28. How is the diagram on the far right different from a LINE SEGMENT and a LINE?

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A portion of a line that has only one endpoint and continues forever in one direction is called a

### RAY.

A ray is named by the letter of its endpoint and any other point along the ray.

The symbol  $\rightarrow$  written above the two letters is used to represent a ray.

29. Using the letters A and B, represent a LINE: \_\_\_\_\_

30. Using the letters A and B, represent a LINE SEGMENT: \_\_\_\_\_

31. Using the letters A' and B', represent a RAY: \_\_\_\_\_

32. Label the diagrams above as either a LINE, a LINE SEGMENT, or a RAY.

GO TO: <http://www.mathopenref.com/ray.html> for more information about RAYS.

## EXPANDING THE IDEA PART 7

COLLECTING YOUR THOUGHTS:

### A LINE:

Draw a line going through Point A and Point B and extending in each direction with no end.

Define a line.

Symbolically, represent the line going through point A and point B.

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### A LINE SEGMENT:

Draw a line segment going through point C and point D.

Define a line segment.

Symbolically, represent the line going through point C and point D.

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### A RAY:

Draw a ray starting at point E and going through point F.

Define a ray.

Symbolically, represent the ray starting at point E and going through point F.

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### THE LENGTH OF A LINE SEGMENT:

Explain how to find the distance between the endpoints of a line segment.

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### THE MIDPOINT OF A LINE SEGMENT:

What is meant by the midpoint of a line segment?

Explain how to find the midpoint of a line segment.

## EXPANDING THE IDEA PART 8

SUMMARY:

A LINE



a **line** is a quantity which extends in two directions without end, but is without width or thickness.

It is represented by ...  $\longleftrightarrow$   
 $\overleftrightarrow{AB}$

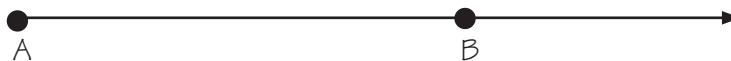
A LINE SEGMENT:



a **line segment** is a part of a line that is bounded by two end points, and contains every point on the line between its end points.

It is represented by ...  $\overline{AB}$

A RAY:



a portion of a line that has only one endpoint and continues forever in the other direction.

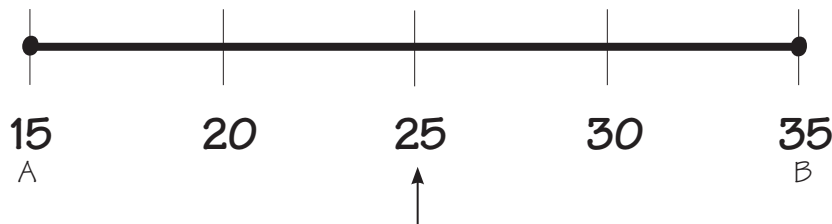
It is represented by ...  $\overrightarrow{AB}$

THE LENGTH OF A LINE SEGMENT



is the distance between the two endpoints and is found by subtraction:  $B - A$

THE MIDPOINT:



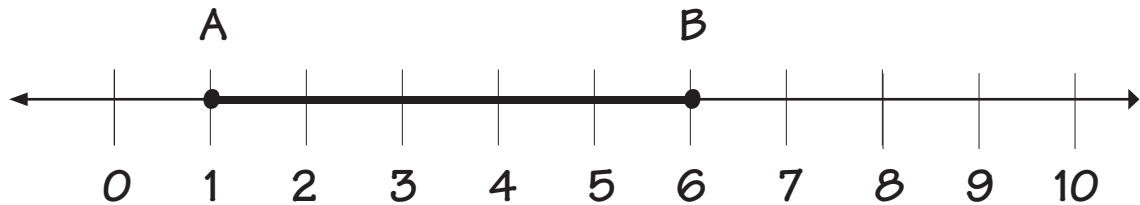
the point halfway between the two endpoints of a line segment:  $\frac{B + A}{2}$  or  $\frac{B - A}{2} + A$

## EXPANDING THE IDEA PART 9

Look at line segment AB below.

1. What value is the left endpoint? \_\_\_\_\_

2. What value is the right endpoint? \_\_\_\_\_



Line segment AB contains 1 and 6 and all the values between \_\_\_\_\_ and \_\_\_\_\_ .

3. Name two or three of these values. \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_ .

Line segment AB contains 1 and 6 and all the values in between.

The closed or solid endpoints means that the endpoints are included.

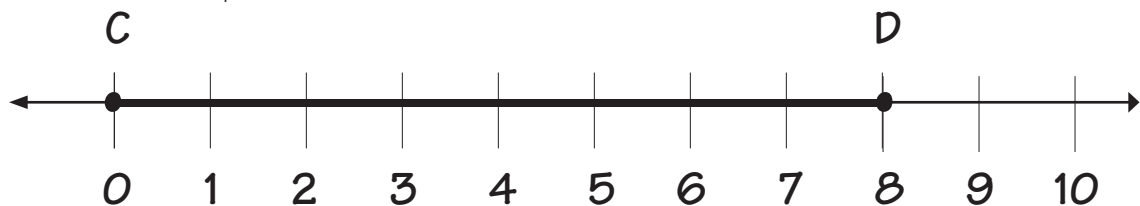
Line segment AB contains all the values greater than or equal to 1 and less than or equal to 6.

This is represented as

$1 \leq x \leq 6$  or  $[1, 6]$ , where the  $[ ]$  indicate the endpoints are included in the interval.

4. Look at line segment CD in the diagram below.

Give the values of the endpoints and several values included in the line segment.



5. Endpoint values: \_\_\_\_\_ , \_\_\_\_\_

6. Name several values included in the line segment. \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_

7. Line segment CD contains \_\_\_\_\_ and \_\_\_\_\_ and all the values between \_\_\_\_\_ , \_\_\_\_\_ .

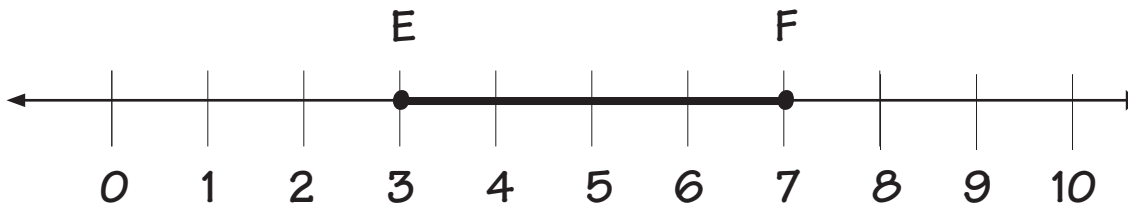
Line segment CD contains all the values greater than or equal to 0 and less than or equal to 8.

This is represented as

$0 \leq x \leq 8$  or  $[0, 8]$ , where the  $[ ]$  indicate the endpoints are included in the interval.

Look at line segment EF in the diagram below.

Give the values of the endpoints and several values included in the line segment.



8. Endpoint values: \_\_\_\_\_ , \_\_\_\_\_
9. Name several values included in the line segment. \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_
10. Line segment EF contains \_\_\_\_\_ and \_\_\_\_\_ and all the values between \_\_\_\_\_ , \_\_\_\_\_ .

Line segment EF contains all the values greater than or equal to 3 and less than or equal to 7.

11. How is this represented?

$$3 \leq x \leq 7 \text{ or } [3, 7]$$

Look at line segment GH below.



11. Notice the endpoints of each line segment.  
How is this line segment different from the previous line segments?

In line segment AB the endpoints are solid. In line segment GH the endpoints are open.

Why do you think the endpoints are represented in two different ways? \_\_\_\_\_

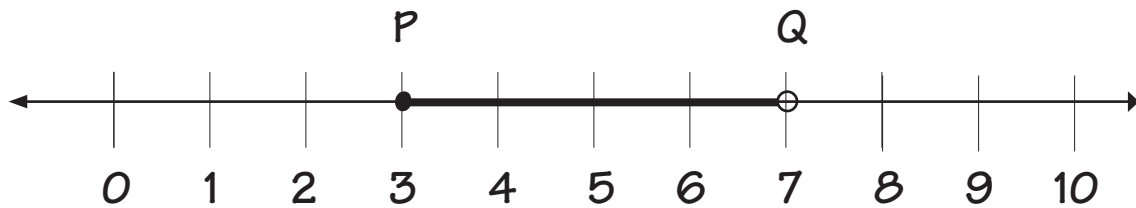
The open endpoints means that the endpoints are not included.

The interval of line segment GH is greater than one and less than 6.

This interval is represented as

$$1 < x < 6 \text{ or } (1, 6), \text{ where the } ( ) \text{ indicates that the endpoints are not included.}$$

Look at line segment PQ in the diagram below.



8. Endpoint values: \_\_\_\_\_ , \_\_\_\_\_

The left endpoint is included as indicated by the closed circle,  
but the right endpoint is not included as indicated by the open circle.

9. Name several values included in the line segment. \_\_\_\_\_ , \_\_\_\_\_ , \_\_\_\_\_

All values of  $x$  that are true for the relationship are called the DOMAIN.

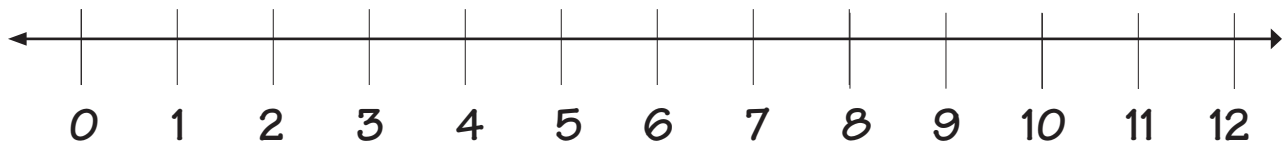
The domain of line segment PQ contains all the values greater than or equal to 3 and less than 7.

The domain of this line segment is represented by

$3 \leq x < 7$  or  $[3, 7)$  where the  $[$  indicates that the endpoint is included  
and the  $)$  indicates that the endpoint is not included.

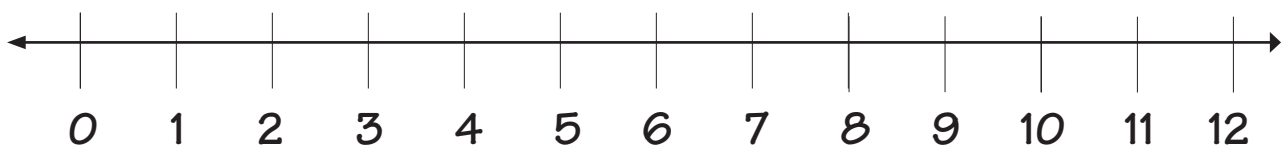
Given the domain, draw the line segment.

10. Domain:  $1 \leq x \leq 12$  or  $[1, 12]$



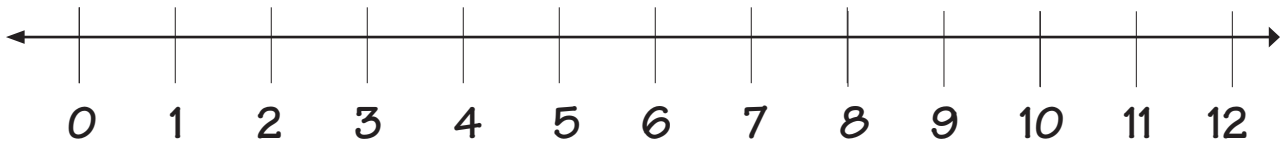
Given the domain, draw the line segment.

11. Domain:  $4 < x \leq 8$  or  $(4, 8]$



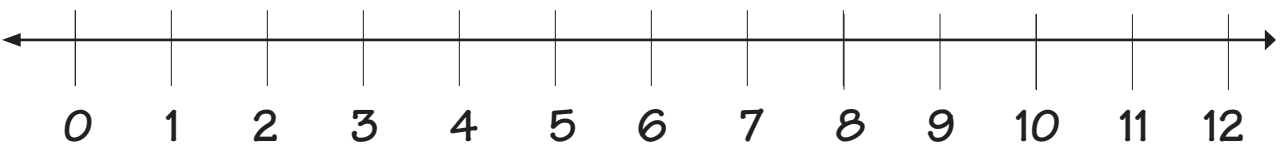
Given the domain, draw the line segment.

12. Domain:  $5 < x < 10$  or  $(5, 10)$



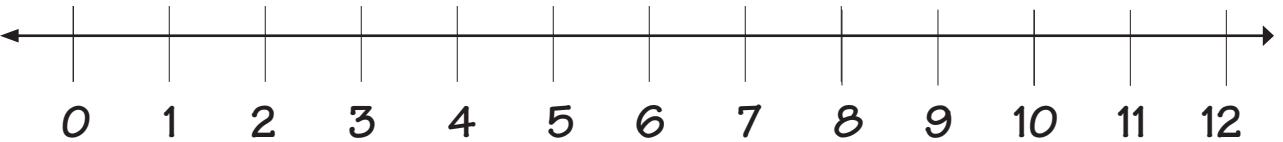
Given the domain, draw the line segment.

13. Domain:  $[2, 9]$



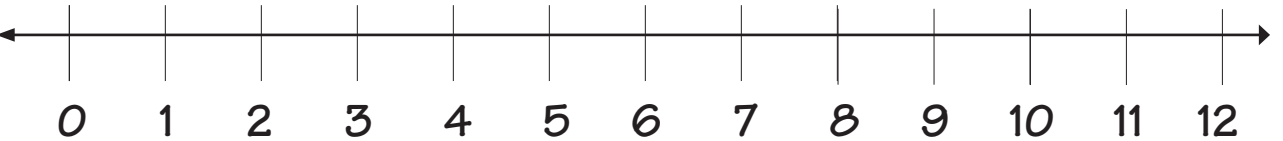
Given the domain, draw the line segment.

14. Domain:  $(7, 11)$



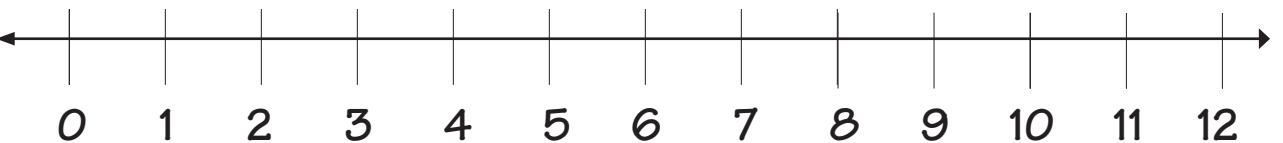
Given the domain, draw the line segment.

15. Domain:  $(0, 7]$



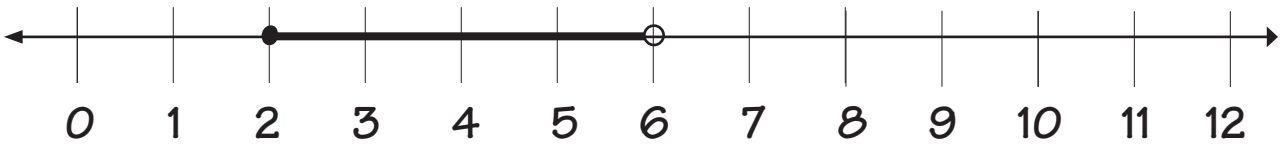
Given the domain, draw the line segment.

16. Domain:  $(0, 7]$



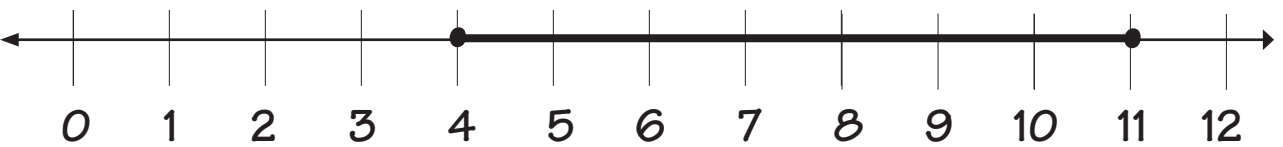
Given the line segment, state the domain.

12. Domain:  $\underline{\hspace{1cm}} \leq x < \underline{\hspace{1cm}}$  or  $[\underline{\hspace{1cm}}, \underline{\hspace{1cm}})$



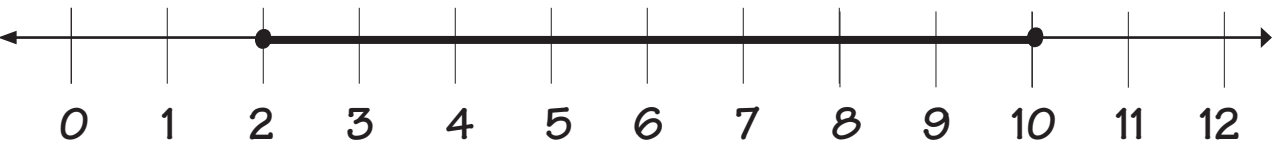
Given the line segment, state the domain and find the midpoint.

13. Domain:  $\underline{\hspace{1cm}}$  or  $\underline{\hspace{1cm}}$  Midpoint:  $\underline{\hspace{1cm}}$



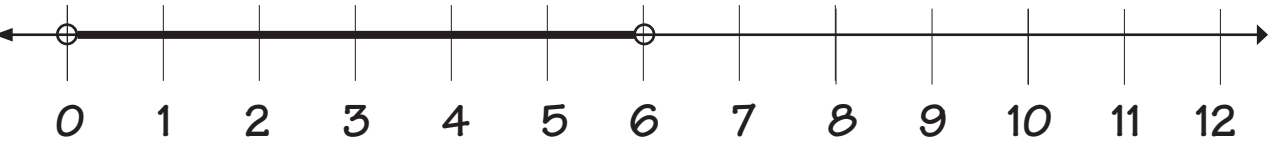
Given the line segment, state the domain and find the midpoint.

14. Domain:  $\underline{\hspace{1cm}}$  or  $\underline{\hspace{1cm}}$  Midpoint:  $\underline{\hspace{1cm}}$



Given the line segment, state the domain.

15. Domain:  $\underline{\hspace{1cm}}$  or  $\underline{\hspace{1cm}}$



Given the line segment, state the domain.

16. Domain:  $\underline{\hspace{1cm}}$  or  $\underline{\hspace{1cm}}$

